Online Learning in Educational Research
Volume 6, Issue 1, 227-243.
e_ISSN: 2798-2351
https://www.journal.foundae.com/index.php/oler/index
DOI: 10.58524/oler.v6i1.1225

Bridging Verbal Geometry Problems and Formal Proofs: A GeoGebra-Supported Element–Relation Detection Framework

I Putu Wisna Ariawan*
Universitas Pendidikan Ganesha, INDONESIA

I Made Ardana
Universitas Pendidikan Ganesha, INDONESIA

I Made Sugiarta
Universitas Pendidikan Ganesha, INDONESIA

Sariyasa
Universitas Pendidikan Ganesha, INDONESIA

Raphita Yanisari
Universitas Pendidikan Ganesha, INDONESIA

Agus Adiarta
Universitas Pendidikan Ganesha, INDONESIA

Dewa Gede Hendra Divayana
Universitas Pendidikan Ganesha, INDONESIA

Article Info

Article history:

Received: December 24, 2025
Revised: February 16, 2026
Accepted: March 19, 2026

Keywords:

Dynamic geometry software;
Element detection;
GeoGebra;
Image visualization;
Relation detection.

Abstract

Geometry proof problems presented without diagrams require students to transform verbal information into accurate visual representations before constructing formal deductive arguments. However, instructional approaches that explicitly support this transition remain limited. This study investigated the effectiveness of a GeoGebra-supported Element–Relation Detection (ERD) framework in strengthening undergraduate students' plane geometry proof performance. A quasi-experimental design was conducted with first-year mathematics education students assigned to four instructional conditions combining the ERD framework, conventional Polya-based instruction, and the presence or absence of GeoGebra support. Students' proof performance was evaluated through validated tasks measuring diagram construction, conjecture generation, proof organization, and argument validity. The findings indicate that learning environments supported by GeoGebra consistently produced stronger proof performance than conventional instruction without dynamic visualization. The ERD framework further assisted students in identifying essential geometric elements and relationships before constructing formal proofs, resulting in more coherent reasoning and fewer errors during the proof process. These findings suggest that the primary contribution of GeoGebra lies in facilitating accurate mathematical representations, while the ERD framework provides a structured pedagogical approach for bridging verbal geometric information and formal deductive reasoning. This study offers a practical instructional framework for improving geometry proof learning in technology-enhanced mathematics education.

To cite this article: Ariawan, I. P. W., Ardana, I. M., Sugiarta, I. M., Sariyasa, S., Silalahi, R. Y., Adiarta, A., & Divayana, D. G. H. (2026). Bridging verbal geometry problems and formal proofs: A GeoGebra-supported element–relation detection framework. *Online Learning in Educational Research, 6*(1), 227-243. https://doi.org/10.58524/oler.v6i1.1225

INTRODUCTION

Geometry remains a core domain of mathematics education because it connects spatial intuition, representation, deduction, and proof (Rizal et al., 2023). Recent reviews also show that contemporary geometry research is increasingly organized around five interrelated themes: geometric thinking and practice, geometric content, teacher education, argumentation and proof, and the use of digital tools (Jablonski & Ludwig, 2023). In other words, geometry teaching is no longer seen simply as learning about shapes and their properties; it is also seen as a platform for developing formal reasoning and mathematically justified explanations. In the realm of higher education,

* **Corresponding Author**
I Putu Wisna Ariawan, Universitas Pendidikan Ganesha, INDONESIA ✉ wisna.ariawan@undiksha.ac.id

particularly in mathematics education programs, the ability to prove serves a dual function: as an academic competency for students in understanding mathematical structures, and as a professional provision for prospective teachers to teach mathematics meaningfully, not merely procedurally.

However, students often struggle with proofs, particularly in constructing rigorous arguments, understanding proof structures, validating logical implications, and moving from informal reasoning to formal mathematical justification (Hoerudin, 2022; Laugwitz et al., 2025; Lessing & Ogbonnaya, 2026; Lestyanto et al., 2022; Shimizu, 2022; Yoon et al., 2024). More specifically, both students and prospective mathematics teachers experience difficulties in understanding relevant concepts, using definitions and notation, selecting appropriate facts or theorems, determining proof structures, initiating proofs, and constructing coherent deductive arguments (Hartono et al., 2025; Hein & Prediger, 2024; Pradana et al., 2025; Ridwan & Haeruddin, 2025). For example, a study of proof evaluation among prospective teachers showed that they frequently make erroneous decisions regarding theorems and proofs, use proof terms inappropriately, and tend to memorize proofs rather than understand how proofs work (Moralı & Filiz, 2023). These difficulties may also be associated with cognitive constraints such as working memory, inhibitory control, and cognitive flexibility, which can affect students' ability to organize proof steps and connect relevant concepts into coherent deductive arguments (Tapo & Rudhito, 2025).

Visualization is particularly central to geometric proof because students must transform verbal geometric conditions into meaningful representations before identifying relevant elements, inferring relationships, and organizing valid deductive arguments (Alfianti & Kuswanto, 2024; Sari et al., 2022). As argued by Schoenherr and Schukajlow (2024), visualization is not merely a decorative aid but an essential process through which students construct, interpret, and communicate mathematical meaning. While proof problems accompanied by visual representations provide explicit structural information, problems presented without visualizations require students to independently identify and represent the underlying geometric structure before deductive reasoning can begin, making them more difficult to solve (Anwar et al., 2023; Davor et al., 2026; Hlongwana et al., 2025; Menouer et al., 2025; Pratiwi et al., 2025; Ramírez-Uclés & Ruiz-Hidalgo, 2022). This challenge is further reflected in students' difficulties in constructing representations that support geometric reasoning (Agusfianuddin et al., 2024; Juman et al., 2022; Malik & Mursalin, 2018; Rahmawati et al., 2020), which may hinder the development of coherent proof arguments. In the Indonesian context, Anwar et al. (2021) demonstrated that prospective teachers' understanding of geometric proofs improved when instruction used multiple proof formats rather than a single, conventional format. Furthermore, Anwar et al., (2023) demonstrated that carefully structured learning trajectories can support emerging understanding of Euclid's proof structure.

The use of GeoGebra in geometry teaching has been reported to improve college students' logical reasoning and geometric drawing skills, as well as to support the connection between visual information and reasoning processes (Hamidah et al., 2025; Ikhsan et al., 2024; Rohmah et al., 2023). In proof-related contexts, GeoGebra has also been shown to support informal proofs, conjecture making, and opportunities for proof-oriented exploration (Martínez-zarzuelo et al., 2025; Putra et al., 2023 Rizos & Gkrekas, 2023). In teacher education, recent research on dynamic geometry software further demonstrates that these tools can improve specific content knowledge and support the design of exploratory tasks (Hovik & Nolan, 2024; Segal et al., 2021).

However, despite these promising findings, an important instructional gap remains. Existing interventions have not sufficiently targeted the one important difficulty in proof construction, namely, students' ability to detect and encode geometric elements and relations before developing a proof framework. In particular, there are still limited learning strategies that explicitly require students to identify elements such as points, lines, angles, and given or required information, as well as relations such as parallelism, perpendicularity, congruence, and conditional "if–then" structures. This gap becomes more critical in plane geometry problems that are presented only in narrative form without accompanying image visualizations. A second gap concerns the role of GeoGebra technology in computational validation, as Kovács et al. (2024) indicate that GeoGebra's automated reasoning features can be used to examine the validity of geometric statements by making the underlying verification steps explicit. This condition creates an opportunity to position GeoGebra not merely as a visualization tool, but as a medium for detecting, testing, tracing, and validating relations. In this approach, GeoGebra's output and features can be used as feedback for relation validation, while the structure of the argument remains constructed by students.

Therefore, a learning strategy is needed that explicitly trains students to extract elements and relations from narrative plane geometry problems without image visualizations, and then uses GeoGebra to construct dynamic representations that can be tested or traced. Through this strategy, students are guided to build more structured proof reasoning, verify geometric relations, minimize wild assumptions, and make use of GeoGebra's potential to connect visual representations with logical reasoning (Awi et al., 2025; Hamidah et al., 2025).

The novelty of this research lies in its conceptual-instructional contribution: it proposes a GeoGebra-assisted element–relation detection strategy as an explicit and sequential proof scaffolding. This scaffolding begins with element detection, including geometric objects and known or required information, followed by relation detection, including relationships between elements through construction, measurement, or transformation. The strategy places GeoGebra within a sequential proof-scaffolding process consisting of element detection, relation detection, dynamic construction, and relation validation. In this study, GeoGebra is therefore not used merely as a visualization tool, but as a relation-detection and validation environment that bridges visual and logical representations.

Accordingly, this study investigates the effect of a GeoGebra-supported element and relation detection (ERD) strategy on undergraduate students' ability to prove plane geometry problems presented without image visualizations. Specifically, the study compares proof performance across four instructional conditions: ERD with GeoGebra, ERD without GeoGebra, conventional instruction with GeoGebra, and conventional instruction without GeoGebra.

METHOD

Research Design

This study was quasi-experimental in design. The researchers had to work with intact classes and could not randomly assign individual students to treatment conditions. The study was conducted in the first semester of the 2024/2025 academic year in a plane geometry course for mathematics education students at Ganesha University of Education, a state university in Indonesia. The analysis was limited to post-intervention comparisons among instructional conditions; therefore, gain analysis was not conducted. This decision means that the findings should be interpreted as differences in final proof performance rather than as evidence of individual learning growth. Consequently, the absence of gain analysis is acknowledged as a limitation, and future research should employ a pre-test–post-test design to examine students' improvement more directly.

The treatment was implemented over 9 sessions, each lasting 100 minutes. The geometry topics covered included: fundamental theorems of plane geometry, congruence and similarity of triangles, and quadrilaterals. In the group given the element and relation detection strategy, students were explicitly guided to identify geometric elements and relations in textual problems before creating diagrams and planning proofs. In the GeoGebra-supported group, the diagramming and exploration phases were conducted using dynamic geometry software. In this activity, the lecturer acts as a guide and facilitator if students encounter difficulties.

To strengthen the consistency of research implementation and ensure that each group received relatively equal learning opportunities, all research groups were given two sets of practice questions through the Learning Management System (LMS). Each set consisted of four questions arranged with varying levels of complexity, from relatively easy to more difficult items, in order to encourage the gradual development of students' thinking skills. The same amount of practice was therefore provided to all groups, although students applied the learning strategy assigned to their respective treatment condition. For each set of questions, all groups were given the same three-day period to complete the tasks, followed by a three-day feedback period. Feedback was provided by the same instructor for all groups and focused on the quality of students' reasoning, the identification of relevant elements and relationships, and the appropriateness of the solution steps, without supplying direct answers. Instructor support was also kept comparable across groups and was limited to clarifying task instructions and procedural issues. Thus, the LMS-based assignments functioned not only as structured independent practice but also as a means of maintaining equivalence in practice intensity, time-on-task, feedback procedures, and instructor support across the four research groups.

Groups receiving treatment with GeoGebra assistance, namely Group 1 (ERD+GeoGebra) and Group 3 (Conv+GeoGebra), were required to create problem-solving visualizations using GeoGebra while working on the practice questions available on the LMS. These visualizations were designed according to the characteristics of the problem and the solution steps used. Specifically for Group 1 (ERD+GeoGebra), before creating their visualizations, they had to first determine the elements and relationships in the problem. This is intended to represent mathematical objects, relationships between elements, and emerging changes or patterns more clearly and dynamically. Meanwhile, for Group 3 (Conv+GeoGebra), they were asked to identify the known information in the problem and identify what is known about the problem. The obligation to create GeoGebra visualizations is intended to ensure that GeoGebra's use is truly part of the learning process, not simply an additional facility. Through this activity, students are guided to construct visual representations of mathematical problems, examine the relationships between objects, and support the reasoning process in finding solution strategies. Thus, GeoGebra serves as a tool for exploration, verification, and strengthening conceptual understanding.

Meanwhile, for the groups that did not use GeoGebra, namely Group 2 (ERD–GeoGebra) and Group 4 (Conv–GeoGebra), they were still required to create their problem visualizations manually without using GeoGebra software. For Group 2 (ERD–GeoGebra), before creating their visualizations, they first had to identify the elements and relationships in the problem. For Group 4 (Conv–GeoGebra), they simply identified what was known about the problem.

Although the study was quasi-experimental in nature, several steps were taken to maintain treatment fidelity and reduce potential bias. Treatment fidelity was monitored through structured lesson plans, observation sheets, and implementation checklists. These instruments were used to verify that each group adhered to the instructional procedure assigned to it. The learning duration, mathematical topics, problem tasks, and assessment instruments were held constant across groups, with variation limited to the intended treatment condition. These fidelity procedures were used to reduce potential threats arising from variations in implementation and to support a more cautious interpretation of group differences as being plausibly associated with the instructional treatments, while recognizing the limits of causal inference in a quasi-experimental design.

Students' initial geometry proof ability was assessed through a pre-test administered before the intervention. The pre-test data were not used as part of the main hypothesis testing, but were analyzed using one-way ANOVA solely to examine the baseline equivalence of the four classes before the treatments were implemented. This preliminary analysis was intended to determine whether the groups differed significantly in their initial ability, so that the subsequent comparison of learning outcomes could be interpreted with greater caution. After the equivalence of the classes had been examined, each group received the instructional treatment assigned to its respective condition: ERD assisted by GeoGebra, ERD without GeoGebra, conventional Polya-based instruction assisted by GeoGebra, or conventional Polya-based instruction without GeoGebra. Following the intervention, all groups completed the same post-test instrument, which was assessed using the same scoring rubric to maintain consistency and fairness in measurement. The hypothesis test was then conducted on the post-test results to examine whether students' geometry proof performance differed across the four instructional conditions. In addition, the learning process was monitored to reduce possible contamination among treatment conditions, particularly in relation to the use of GeoGebra and the manual construction of visual representations.

.

Participants

The population of this study consisted of 104 first-year undergraduate students distributed across five intact classes. Of these students, 82 were female, and 22 were male, with ages ranging from 18 to 20 years. All participants were newly enrolled university students and had not previously received specific instructional treatment related to geometric proof or the use of GeoGebra in higher education. This population characteristic was important because it helped ensure that students entered the study with relatively comparable formal learning experiences in proof construction and technology-assisted geometry learning at the university level. The use of intact classes also reflected the natural classroom setting of the study, which was consistent with the quasi-experimental design adopted in the research.

To ensure equivalence, the five classes were first given a test related to basic geometry concepts learned in high school. Assumption testing showed that the five classes were normally distributed and homogeneous in variance, and an initial one-way ANOVA revealed no significant differences among them ($p = 0.968$). At the preliminary stage, one-way ANOVA was used to examine whether the five available classes had equivalent initial competence in basic geometry before the experimental groups were selected. This procedure was more appropriate than multiple pairwise t-tests because it provided a single omnibus test and reduced the risk of inflated Type I error. From the intact population of five first-year classes, four classes were first randomly selected to participate in this study. Next, the four selected classes were randomly assigned to one of four learning conditions: ERD assisted by GeoGebra, ERD without GeoGebra, conventional Polya-based learning assisted by GeoGebra, and conventional Polya-based learning without GeoGebra. Thus, this study involved random class selection and class-level random assignment. Each class was given a different treatment as follows. Group 1: Geogebra-assisted Element and Relation Detection Strategy (ERD+GeoGebra) consisted of 21 students. Group 2: Element and Relation Detection Strategy without using GeoGebra (ERD–GeoGebra) consisted of 19 students. Group 3: Conventional (Polya) assisted by GeoGebra (Conv+GeoGebra) consisted of 21 students, and Group 4: Conventional (Polya) without using GeoGebra (Conv–GeoGebra) consisted of 18 students.

Data Collection Instruments

The outcome variable was students' ability to prove plane geometry problems presented without accompanying visualizations. The assessment used four proof tasks selected from ten previously developed items. The selection of the four questions used was based on the consideration that the questions were already distributed and represented topics previously discussed. The ten items were previously validated by two experts and tested on 50 students in the previous year who had completed plane geometry courses. The results of expert validation using the Gregory formula showed very good content validity (1.00). From the trial results, the ten items were valid, with strong internal consistency (Cronbach's alpha = 0.842). Students' answers were assessed using a previously developed rubric consisting of five aspects: (a) accuracy and completeness of the diagrams created, (b) initial proof steps, (c) formulation of conjectures, (d) coherence of the proof flow, and (e) validity of supporting arguments. Specifically, the accuracy and completeness of the diagrams are divided into two sub-aspects, namely: (a.1) image accuracy and (a.2) completeness of labels (Ariawan et al., 2024). This rubric-based assessment is consistent with the use of performance assessment in plane geometry, where students' reasoning process and justification are as important as the final answer (Ariawan, 2022).

Each aspect has a score range of 0 to 3. For each question, the assessment is conducted on all six aspects. Each aspect has a maximum score of 3. Since there are four questions to be completed, the maximum score that students can achieve in completing the test is 72. After nine instructional sessions had been completed in each group, the post-test was administered simultaneously to the four participating classes during the tenth meeting. This procedure was intended to ensure that all groups were assessed under comparable conditions and at the same stage after receiving their respective instructional treatments.

Data Analysis

Data were analyzed using IBM SPSS Statistics version 31.0.0.0 (117) with a statistical significance level set at $\alpha = 0.05$. Before the main analysis, pre-test scores were examined to determine baseline equivalence across the four intact classes before the instructional treatment was implemented. A one-way ANOVA was conducted on the pre-test scores, and the results indicated no statistically significant differences between the groups. Therefore, the pre-test analysis was used only to assess initial group comparisons and was not included as part of the main hypothesis test. The main hypothesis was tested using post-test scores using a one-way ANOVA after the completion of the instructional treatment. Before conducting the one-way ANOVA, assumptions of normality and homogeneity of variance were evaluated. Further testing was conducted using the Scheffé Post Hoc Method.

RESULTS AND DISCUSSION

Results

All four treatment groups met the assumptions for parametric analysis. Normality was tested using the Shapiro-Wilk test because each group had fewer than 50 participants (Mishra et al., 2019; Razali & Wah, 2011). The Shapiro–Wilk test results show a p-value above 0.05 for each group, with p = 0.349 for the ERD+GeoGebra group, p = 0.182 for the ERD–GeoGebra group, p = 0.502 for the Conv+GeoGebra group, and p = 0.904 for the Conv–GeoGebra group. Levene's test showed homogeneous variance across groups (p = 0.944). Table 1 presents a summary of descriptive statistics for the proof performance.

Table 1. A Summary of Descriptive Statistics for the Ability to Prove in the Four Treatment Groups

Group	N	Minimum	Maximum	Mean	Std. Deviation
ERD+GeoGebra	21	30.00	52.00	39.0952	5.79573
ERD–GeoGebra	19	25.00	46.00	32.4211	5.60075
Conv+GeoGebra	21	27.00	50.00	38.0476	6.01229
Conv–GeoGebra	18	20.00	41.00	30.5000	5.87367

The group treated with the GeoGebra-assisted element and relation detection strategy obtained the highest average score (M = 39.0952), followed by the group treated with the GeoGebra-assisted conventional strategy (M = 38.0476). The two non-GeoGebra groups showed significantly lower averages, with the group treated with the Conv–GeoGebra strategy obtaining the lowest average (M = 30.5000). The results of the one-way ANOVA test using IBM SPSS Statistics version 31.0.0.0 (117) are presented in Table 2.

Table 2. ANOVA Test Results

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	1033.043[a]	3	344.348	10.144	<.001	.289
Intercept	96437.885	1	96437.885	2840.983	<.001	.974
Groups	1033.043	3	344.348	10.144	<.001	.289
Error	2545.893	75	33.945			
Total	101759.000	79				
Corrected Total	3578.937	78				

a. R Squared = .289 (Adjusted R Squared = .260)

A one-way analysis of variance (ANOVA) revealed a statistically significant group effect on proof performance, $F(3,75) = 10.144$, $p < .001$. Based on the sum of squares reported in this study, the effect size was $\eta^2 = 0.289$, which shows that approximately 28.86% of the variance in students' proof scores is related to learning conditions. The results of the post hoc ANOVA test using the Scheffé post hoc method are presented in Table 3.

Table 3. Results of the Post Hoc ANOVA Test using the Scheffé Post Hoc Method

(I) Group	(J) Group	Mean Difference (I-J)	Std. Error	Sig.
ERD+GeoGebra	ERD–GeoGebra	6.67419*	1.84473	.007
	Conv+GeoGebra	1.04762	1.79802	.952
	Conv–GeoGebra	8.59524*	1.87144	< .001
ERD–GeoGebra	ERD+GeoGebra	-6.67419*	1.84473	.007

(I) Group	(J) Group	Mean Difference (I-J)	Std. Error	Sig.
	Conv+GeoGebra	-5.62657*	1.84473	.032
	Conv–GeoGebra	1.92105	1.91636	.800
	ERD+GeoGebra	-1.04762	1.79802	.952
Conv+GeoGebra	ERD–GeoGebra	5.62657*	1.84473	.032
	Conv–GeoGebra	7.54762*	1.87144	.002
	ERD+GeoGebra	-8.59524*	1.87144	< .001
Conv–GeoGebra	ERD–GeoGebra	-1.92105	1.91636	.800
	Conv+GeoGebra	-7.54762*	1.87144	.002

Based on observed means.
The error term is Mean Square(Error) = 33.945.
*. The mean difference is significant at the .05 level.

A post hoc Scheffé test confirmed these results. The ERD+GeoGebra group significantly outperformed the ERD–GeoGebra group (mean difference = 6.67, $p = 0.007$) and the Conv–GeoGebra (mean difference = 8.60, $p < 0.001$), but did not significantly differ from the Conv+GeoGebra group (mean difference = 1.05, $p = 0.952$). The Conv+GeoGebra group also outperformed the ERD–GeoGebra group (mean difference = 5.63, $p = 0.032$) and the Conv–GeoGebra (mean difference = 7.55, $p = 0.002$). No significant difference was found between the ERD–GeoGebra group and the Conv–GeoGebra group (mean difference = 1.92, $p = 0.800$).

Overall, the results showed a clear separation between GeoGebra-supported and non-GeoGebra-supported conditions. The element and relation detection routines yielded the highest mean scores only when combined with GeoGebra; without GeoGebra, they did not provide a statistically significant advantage compared to conventional instruction.

Discussion

The results indicated that the GeoGebra-assisted element detection and relation detection strategy produced the highest descriptive performance among the four treatments in students' ability to prove geometry problems without visualization. However, this finding requires a cautious interpretation. The treatment was statistically more effective than the two conditions without GeoGebra, indicating the important role of technology-mediated support in helping students construct and test geometric representations. Nevertheless, its difference from the conventional GeoGebra-supported treatment was not statistically significant. Therefore, this study does not support the claim that the routine application of element detection and relation detection strategies is superior in all circumstances. Rather, the findings suggest that students' ability to prove plane geometry problems without visualization improved substantially when they received GeoGebra-based support, whether through the structured element detection and relation detection strategy or through conventional GeoGebra-assisted instruction.

One important contribution of the detection routine is that it makes the hidden structure of verbal geometry problems explicit. In tasks without diagrams, students must first determine which mathematical objects are involved and how these objects are related before they can produce a meaningful representation. This process is directly reflected in the rubric dimension of diagram construction (Ariawan et al., 2024), because an accurate diagram depends on the student's ability to translate verbal conditions into appropriate geometric elements, positions, and relationships. It is also closely connected to proof validity, since a proof can only be logically sound when the claims, inferences, and theorem applications are grounded in correctly identified elements and relations. This is consistent with recent research emphasizing that visualization is a meaning-making process, not simply a decorative addition (Schoenherr & Schukajlow, 2024). By requiring students to separate "what is" from "how it relates," this routine can reduce the ambiguity of the initial reading phase and support the transition from problem statement to proof plan.

For example, when students solve the problem: In $\triangle ABC$, if the bisectors of the exterior angles of $\angle B$ and $\angle C$ intersect at D and DE is perpendicular to the extension of AB (E is on the extension of

AB), prove that AE = ½ the perimeter of △ABC (Problem 1). In this problem, the element detection and relation detection strategy functions as a cognitive scaffold that helps students transform a verbally stated geometric situation into a structured visual and relational representation. This is important because visualization-based instruction has been shown to improve mathematical problem-solving performance and reduce students' cognitive load by presenting abstract mathematical ideas in more accessible and organized forms (Ben-motreb & Al-makhalid, 2026).

First, students identify the relevant elements, namely points A, B, C, D, and E; triangle △ABC; line segments AB, BC, and CA; the extension of AB; and the exterior angles at B and C. Second, they identify the relations among these elements, namely that E lies on the extension of AB, g is the bisector of the exterior angle at B, h is the bisector of the exterior angle at C, g and h intersect at D, and DE is perpendicular to the extension of AB. This systematic identification supports visualization by directing students' attention to the relevant geometric objects and relations rather than allowing them to inspect the diagram randomly. This systematic identification supports visualization by directing students' attention to the relevant geometric objects and relations before they construct the diagram. From the perspective of cognitive load theory, this procedure can be understood as an instructional support that reduces unnecessary search by cueing essential information and helping students organize multiple verbal conditions into meaningful geometric chunks (van Nooijen et al., 2024). However, because cognitive load was not directly measured in this study, this interpretation should be understood as a design-based explanation rather than as an empirical claim about students' cognitive load levels.

The strategy may also reduce unnecessary cognitive processing. Without explicit element and relation detection, students may need to search simultaneously for points, lines, angle bisectors, perpendicularity, and congruent triangles, which can overload working memory. By making these elements and relations explicit, the strategy reduces extraneous load and allows students to allocate more cognitive resources to reasoning and proof construction. This interpretation is consistent with recent findings showing that visual scaffolding and self-generated sketches can reduce unnecessary processing and support reasoning in visually complex mathematical tasks (Muzaini et al., 2026).

GeoGebra may have supported this instructional process by providing visual and manipulable representations of geometric objects. Through GeoGebra-assisted exploration, students were able to construct, inspect, and test relationships among angle bisectors, perpendicular lines, and right triangles in a more concrete form. Although the present study did not directly measure underlying cognitive mechanisms, the observed role of GeoGebra can be interpreted as an instructional support for organizing visual information and relating it to deductive reasoning. This interpretation is consistent with recent studies showing that GeoGebra-based geometry learning can support students' visualization, geometric drawing, logical reasoning, pattern identification, deduction, and the connection between visual information and logical reasoning processes (Darmanova et al., 2025; Hamidah et al., 2025).

With a proof strategy that uses the perpendicular segment DE as a height drawn to the extension of AB, allowing the configuration to be analyzed through the right triangles formed around this perpendicular construction. Identifying the relevant elements and relations helps students recognize that the exterior angle bisectors at B and C, together with the perpendicular segment DE, generate right triangles whose relationships can be compared. This provides reasoning support because students can use the properties of angle bisectors and congruent right triangles to establish equalities among relevant segments. These equalities then function as intermediate links connecting AE with AB, BC, and CA, leading students toward the conclusion that AE = ½ (AB + BC + CA). In this sense, the strategy supports proof construction not merely by helping students "see" the figure, but by helping them organize visual information, manage cognitive load, formulate relevant conjectures, and construct a coherent chain of geometric inferences. This is consistent with evidence that GeoGebra can help students concretize mathematical arguments by dynamically examining triangle structures and can contribute to the development of proof and reasoning processes (Zengin & Broutin, 2025).

However, this study also shows that explicit detection alone is not enough. The group that only used element and relation detection did not perform significantly better than the conventional group. This finding is theoretically important because it suggests that identifying elements and relations at the verbal level does not automatically lead to stronger proof performance unless students can test, refine, and stabilize those identifications through representational work. In other words, without accurate and manipulable diagrams, the routine risks remaining declarative rather than productive.

The absence of significant differences between the element detection and relation detection strategy groups without GeoGebra and the Polya strategy group without GeoGebra should be interpreted with caution. One plausible explanation is that both strategies operated primarily as manual proof heuristics: students in both groups had to construct their own visual representations, identify relevant relationships, and organize the proof without dynamic technological support. This interpretation is consistent with Ariawan et al. (2024)those who emphasized that visualization supports analysis, intuition, and the preparation of evidence. The present finding may therefore suggest that, in the absence of GeoGebra, a more geometry-specific strategy, such as element and relation detection, does not necessarily yield an immediate statistical advantage over a general problem-solving strategy, such as Polya's. This result also complements Foster's (2023) the argument that Polya's general strategy is widely applicable but may become more effective when translated into domain-specific tactics. In this study, element and relation detection represent such a domain-specific tactic in geometry.

Nevertheless, alternative explanations should also be considered. The non-significant result may have been influenced by the duration of the treatment, the consistency and depth of strategy implementation, the statistical power available to detect small differences, or the sensitivity of the assessment rubric in capturing subtle distinctions in students' diagram construction, relational identification, and proof organization. Thus, the finding should not be read as evidence that the two manual strategies are pedagogically equivalent, but rather as indicating that, under the conditions of the present study, the ERD strategy without GeoGebra did not show a statistically distinguishable advantage over the Polya strategy without GeoGebra.

Furthermore, the findings support and refine the conclusions of Belay et al. (2024), who stated that successful proof construction depends not only on recognizing elements and relations but also on conceptual knowledge, systematic proof methods, understanding statements, selecting appropriate techniques, distinguishing premises from conclusions, providing valid reasons, and using definitions, properties, examples, and axioms. Similarly, in line with Winer & Battista (2022) this study indicates that students' intuitive or verbal reasoning may not be fully translated into formal written proof. Therefore, compared with previous studies, this research contributes by demonstrating that element and relation detection alone are insufficient to improve proof performance in the absence of GeoGebra. The strategy becomes more meaningful when supported by dynamic visualization, because such support helps bridge the gap between students' geometric intuition and the formal logical structure required in proof writing.

Thus, both strategies have adjacent functions in understanding problems and designing proofs, but without GeoGebra, students still have to visualize objects, find relations, choose theorems, and construct deductive arguments manually, so the element and relation detection strategy has not shown significant advantages over the Polya strategy. GeoGebra can improve the element-relation detection approach in visual representation, measurement, exploration of geometric properties, mathematical experiments, and interactive presentation of mathematical ideas (Iwani et al., 2023). But without this visual-dynamic support, this approach still relies on manual representational skills. This shows that GeoGebra seems to work as a principal cognitive central representational support.

The role of GeoGebra in supporting conceptual understanding, visualization, and exploratory learning in geometry has been reported repeatedly in recent literature (Gurmu et al., 2024; Liburd & Jen, 2021). The results of a meta-analysis conducted Suparman et al. (2024) further strengthen this view that GeoGebra has a strong positive influence on spatial visualization in geometry learning. In this study, both GeoGebra-supported groups substantially outperformed the non-GeoGebra-supported groups, indicating that dynamic visualization is not peripheral but central to proof success. These results align with evidence-oriented research on dynamic geometry environments. Putra et al. (2023) showed that GeoGebra can strengthen informal proofs because it allows students to establish relationships between constructs and mathematical arguments. Meanwhile, Rizos & Gkrekas (2023) demonstrated that dynamic geometry environments support the synergy between conjecture formulation, exploration, mathematical reasoning, and proof. For example, the visualization image for Question 1 can be made like Figure 1.

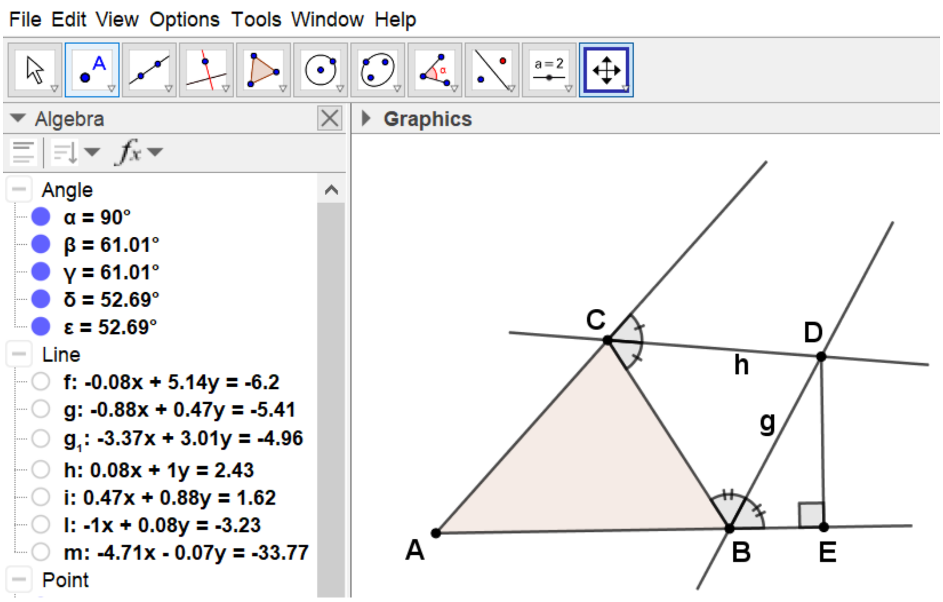

Figure 1. Visualization of Problem 1 using the GeoGebra Application

Figure 1 illustrates how GeoGebra was used to transform the verbally stated geometry problem into a visible and manipulable geometric configuration. The figure 1, makes the relevant points, segments, lines, auxiliary constructions, angle measures, and perpendicular relations explicit, allowing the structure of the problem to be inspected more carefully before a formal proof is developed. In this sense, the visualization does not function merely as a decorative addition, but as a representational bridge between the problem statement and the reasoning required to solve it.

The steps for creating a Visualization of Problem 1 using the GeoGebra Application can be described in a flowchart as in Figure 2.

```
graph TD
   A([Start]) --> B[Create Points A, B, and C]
   B --> C[Create Polygon ABC]

   subgraph "Extensions"
   C --> D[Draw Line through A and B]
   D --> E[Draw Line through B and C]
   end

   subgraph "Parallel & Perpendicular Construction"
   E --> F[Create Parallel Line to AB through C]
   F --> G[Place Point D on the Parallel Line]
   G --> H[Connect Segment BD]
   H --> I[Draw Perpendicular Line from D to AB]
   I --> J[Define Intersection Point E]
   end

   subgraph "Measurements"
   J --> K[Measure Angles:]
   K --> L["1. Right Angle (BED = 90°)"]
   L --> M["2. Alternate Interior Angles (FCD & BCD)"]
   M --> N["3. Angles at Vertex B (CBD & DBG)"]
   end

   N --> O[Visual Styling: <br/>Fill Colors & Labels]
   O --> P([End])
```

Figure 2. Logical Flowchart for Creating a Visualization of Problem 1

In GeoGebra, a drawing can be created not only manually but also using commands such as points, lines, circles, functions, or other geometric shapes. These commands are essentially similar to coding, as they contain logical steps that the program must execute. The coding associated with the visualization in Figure 1 is shown in Figure 3.

```
# 1. Define Primary Points
A = (0.4, -1.2)
B = (5.54, -1.12)
C = (3.41, 2.17)

# 2. Create the Triangle
poly1 = Polygon(A, B, C)

# 3. Create Supporting Lines
LineAB = Line(A, B)
LineBC = Line(B, C)
LineParallelC = Line(C, LineAB)

# 4. Construct Point D and E (Perpendicular)
D = Point(LineParallelC, 0.8)
LinePerpD = PerpendicularLine(D, LineAB)
E = Intersect(LinePerpD, LineAB)

# 5. Auxiliary Points for Labels (F, G, I, K, L, J)
F = Point(LineBC, 0.8)
I = Point(LineBC, -0.3)
G = Point(LineAB, 1.2)
K = Point(LineParallelC, -0.4)
L = Point(LineParallelC, 1.2)

# 6. Additional Segments
g = Segment(B, D)
h = Segment(C, D)
segDE = Segment(D, E)

# 7. Measurements (Angles)
Angle(F, C, D)
Angle(B, C, D)
Angle(C, B, D)
Angle(D, B, G)
Angle(B, E, D)
```

Figure 3. The Coding Associated with the Visualization in Figure 1

It is important to note, however, that the present study did not require students to write GeoGebra commands or engage in formal programming. Students used the available GeoGebra tools to construct points, lines, circles, intersections, and other geometric objects through the software interface. The reference to commands as being similar to coding is therefore intended only as a theoretical reinforcement: whether a construction is produced through typed commands or through toolbar-based actions, GeoGebra still operates by executing an ordered set of logical instructions. Each constructed object depends on previously defined elements and relations, so the diagram is not merely a visual sketch but a dynamic representation governed by mathematical and computational logic. In this sense, GeoGebra supports geometric reasoning because students' actions in constructing a diagram are implicitly connected to sequences of dependency, relation, and constraint, even when they do not explicitly write the corresponding commands.

Once students are able to create a visualization using GeoGebra as shown in Figure 1, they need to complete the next step, namely the proof process. However, the ability to prove still requires conceptual understanding, logical-deductive reasoning, choice of method, creativity, the use of mathematical techniques, and the ability to check evidence (Mone et al., 2025). Exploring and testing conjectures through a dynamic geometric environment provides opportunities to explore, propose, and test conjectures, and helps them construct mathematical reasoning and geometric proofs (Rizos & Gkrekas, 2023), but does not automatically generate formal evidence (Rosyidi & Rahmadhani, 2024). Therefore, non-significant results are more appropriately understood as insufficient statistical evidence to state a difference, rather than as evidence that the two strategies have no effect at all, as reminded by Edelsbrunner & Thurn (2024), while still considering sample size, initial ability, implementation quality, treatment duration, and effect size. Another possibility is the methodological aspect. The average advantage of the group treated with GeoGebra-assisted element detection and relation detection strategies compared to the group treated with GeoGebra-assisted conventional strategies was small (1.05 points), and the available sample size may not have been large enough to detect such a small difference. Thus, the current findings support a conservative conclusion: element and relation detection appear pedagogically promising, but the added value beyond conventional instruction supported by GeoGebra may be gradual rather than dramatic in a short intervention.

This interpretation is consistent with other recent research in mathematics teacher education and may extend recent work in mathematics teacher education by showing how the affordances of dynamic geometry software can be conceived of not only in terms of more robust visual exploration but also as part of a larger proof-oriented instructional sequence. For example, Segal et al. (2021) mentioned that dynamic geometry environments can improve prospective teachers' content-specific knowledge. Hovik & Nolan (2024) mentioned that designing exploration tasks based on GeoGebra can change the way prospective teachers think about the structure of tasks and activities in the classroom. Öztürk & Güven (2025) stated that the proof-writing skills can be significantly improved when the dynamic geometry software is embedded into a socially mediated learning environment. This extension of previous findings explicates the instructional connections between reading geometric situations, constructing and manipulating diagrams, generating conjectures, and translating those conjectures into formal arguments. In this sense, the contribution of this study is not only to confirm that GeoGebra supports geometry learning, but also that its pedagogical value is more evident when the diagrammatic exploration is intentionally linked to the discipline of proof construction. Thus, effective instruction in proof design is a coordinated design, where reading, construction, exploration, conjecture-making, and formal justification are integrated aspects of the same process, not distinct phases of instruction (Dasari et al., 2025).

In practice, this research suggests that university plane geometry courses should explicitly teach students how to analyze problems without visualizations before asking them to prove them. This could be operationalized by a simple sequence: detection of geometric elements, detection of relations, construction of dynamic diagrams, testing of invariants, formulation of conjectures, and finally formalization of the proof (Ariawan et al., 2025). Such a sequence may be particularly valuable in mathematics teacher education, as prospective teachers need not only to prove but also to design learning experiences that support proofs for their own students.

LIMITATIONS

This study has several limitations. First, it was conducted in a single institution with intact classes and a relatively small sample. Although this design preserved the natural instructional setting, it limits causal interpretation and reduces the generalizability of the findings. In addition, the analysis did not employ ANCOVA or another baseline-adjusted procedure, so possible initial differences in students' prior proof ability could not be fully controlled. Second, instructional and implementation factors may have influenced the results. Possible instructor effects cannot be ruled out, particularly in relation to how GeoGebra activities were introduced, guided, and connected to proof construction. Treatment contamination may also have occurred if students from different classes exchanged information outside the formal learning sessions. Moreover, students' prior proficiency with GeoGebra was not directly measured, even though such familiarity may have shaped their ability to construct diagrams and explore geometric relations effectively. Third, the intervention was implemented over a relatively short period. Consequently, the study could not determine whether the observed benefits would be sustained over time or transferred to more complex proof tasks. Future research should involve larger and more diverse samples, measure students' initial GeoGebra proficiency, use baseline-adjusted analyses, and combine experimental comparisons with process-oriented investigation to clarify how students move from diagram construction and exploration toward formal proof writing.

CONCLUSION

This study suggests that GeoGebra-supported instruction may offer potential benefits for helping students prove plane geometry problems presented without visual representations. Although the element detection and relation detection conditions produced the highest average scores, they did not differ significantly from the conventional Polya problem-solving condition when both were supported by GeoGebra. Thus, the results should not be interpreted as demonstrating the empirical superiority of element and relation detection over GeoGebra-supported conventional instruction. Instead, they suggest that GeoGebra can serve as a valuable representational tool across instructional approaches, while the specific added value of element and relation detection requires further empirical examination.

Therefore, the implications of this study should be interpreted cautiously. First, the findings indicate that dynamic geometry software may serve as a useful instructional support for teaching proofs in non-visual geometry tasks. Second, structured pre-proof routines that guide students to identify elements and relations appear to have pedagogical value, particularly in helping students organize their understanding of geometry problems presented without visualization. These routines may also serve as a reusable teaching framework in similar instructional contexts. Overall, element detection and relation detection strategies offer a theoretically grounded and empirically promising approach for helping students prove plane geometry problems presented without visual representations, particularly when integrated with the representational and exploratory features of GeoGebra. However, the empirical support should be understood cautiously: these strategies performed better than the non-GeoGebra problem-solving conditions, but not significantly better than conventional Polya-based instruction supported by GeoGebra. The findings, therefore, point to their potential value within GeoGebra-supported proof instruction rather than demonstrating their superiority over other GeoGebra-supported approaches.

AUTHOR CONTRIBUTIONS

Conceptualization, IPWA, IMA; methodology, IPWA, IMA; validation, SS, IMS, DGHD, AA; formal analysis, IPWA, IMA, IMS, DGHD; investigation, SS, AA and RYS; resources, IPWA, IMA, IMS, SS; data curation, AA and RYS.; writing original draft preparation, IPWA, RYS, SS; writing, review, and editing, IMA, DGHD, RYS; visualization, IPWA, SS, DGHD; supervision, AA and DGHD; project administration, IMS and IMA. All authors have read and agreed to the published version of the manuscript.

ACKNOWLEDGMENT

The authors would like to thank the Institute for Research and Community Service (LPPM) of Ganesha University of Education for providing funding for this research through research contract number: 1248/UN48.16/LT/2024 dated June 14, 2024.

REFERENCES

Agusfianuddin, Herman, T., & Turmudi. (2024). Difficulties in mathematical language and representation among elementary school students when solving word problems. *Jurnal Elemen, 10*(3), 567–581. https://doi.org/10.29408/jel.v10i3.25814

Alfianti, A., & Kuswanto, H. (2024). Learning analytics approach to improve multiple representation skills in direct-current circuits. *Jurnal Ilmiah Pendidikan Fisika Al-Biruni, 13*(1), 1–14. https://doi.org/10.24042/jipfalbiruni.v13i1.14514

Anwar, L., Goedhart, M. J., & Mali, A. (2023). Learning trajectory of geometry proof construction: Studying the emerging understanding of the structure of Euclidean proof. *Eurasia Journal of Mathematics, Science and Technology Education, 19*(5), 1–24. https://doi.org/10.29333/ejmste/13160

Anwar, L., Mali, A., & Goedhart, M. J. (2021). The effect of proof format on reading comprehension of geometry proof: The case of Indonesian prospective mathematics teachers. *Eurasia Journal of Mathematics, Science and Technology Education, 17*(4), 1–15. https://doi.org/10.29333/ejmste/10782

Ariawan, I. P. W. (2022). The influence of formative assessment methods on learning outcomes of plane geometry by controlling critical thinking skills. *Al-Ishlah: Jurnal Pendidikan, 14*(3), 4125–4132. https://doi.org/10.35445/alishlah.v14i3.1266

Ariawan, I. P. W., Ardana, I. M., Divayana, D. G. H., & Sugiarta, I. M. (2024). Scoring rubric design to measure the ability to prove plane geometry problems not accompanied by image visualization. *Jurnal Elemen, 10*(1), 70–88. https://doi.org/10.29408/jel.v10i1.22550

Ariawan, I. P. W., Suyasa, P. W. A., Adiarta, A., Sukawijana, I. K. G., Santiyadnya, N., & Divayana, D. G. H. (2025). User interface design of digital test based on backward chaining as a measuring tool for students' critical thinking. *International Journal of Advanced Computer Science and Applications, 16*(1), 584–590. https://doi.org/10.14569/IJACSA.2025.0160156

Awi, A., Sutamrin, S., Ihsan, H., Nasir, N., Naufal, M. A., & Ikram, M. (2025). Spatial geometry learning in higher education through flipped classroom with metacognitive scaffolding: A mixed-methods study. *Online Learning in Educational Research (OLER), 5*(2), 583–600. https://doi.org/10.58524/oler.v5i2.817

Belay, A. M., Machaba, F., & Makgakga, T. P. (2024). Introducing a supportive framework to address students' misconceptions and difficulties in learning mathematical proof techniques: A case of Debark University in Ethiopia. *Research in Social Sciences and Technology, 9*(1), 63–84. https://doi.org/10.46303/ressat.2024.4

Ben-Motreb, K. S., & Al-Makhalid, K. A. M. (2026). The effectiveness of visualization on cognitive load and problem-solving in gifted and non-gifted students. *Anales de Psicología, 42*(2), 1–12. https://doi.org/10.6018/analesps.654851

Darmanova, Z., Abylkassymova, A., & Nurmukhamedova, Z. (2025). A systematic review of technology use in middle and high school mathematics education: Insights from contextual, methodological, and evaluation characteristics. *Frontiers in Education, 10*, Article 1644284. https://doi.org/10.3389/feduc.2025.1644284

Dasari, D., Hendriyanto, A., Ariawan, I. P. W., Muhaimin, L. H., & Pahmi, S. (2025). Crafting epistemic knowledge in statistics education via didactical engineering integrating the theory of didactical situations and the SECI model. *Discover Sustainability, 6*, Article 928. https://doi.org/10.1007/s43621-025-01838-x

Davor, I., Dawmah, P., & Larbi, E. (2026). An investigation into senior high school students' performance and difficulties in proving Euclidean geometry in mathematics curriculum: Evidence from Ghana. *Contemporary Mathematics and Science Education, 7*(1), 1–13. https://doi.org/10.30935/conmaths/17990

Edelsbrunner, P. A., & Thurn, C. M. (2024). Improving the utility of non-significant results for educational research: A review and recommendations. *Educational Research Review, 42*, Article 100590. https://doi.org/10.1016/j.edurev.2023.100590

Foster, C. (2023). Problem solving in the mathematics curriculum: From domain-general strategies to domain-specific tactics. *The Curriculum Journal, 34*(4), 594–612. https://doi.org/10.1002/curj.213

Gurmu, F., Tuge, C., & Hunde, A. B. (2024). Effects of GeoGebra-assisted instructional methods on students' conceptual understanding of geometry. *Cogent Education, 11*(1), Article 2379745. https://doi.org/10.1080/2331186X.2024.2379745

Hamidah, Kusuma, J. W., Chotimah, S., & Senjayawati, E. (2025). Evaluation of geometry skills: Integrating logic and drawing indicators through GeoGebra-based learning. *Journal of Research and Advances in Mathematics Education, 10*(3), 182–199. https://doi.org/10.23917/jramathedu.v10i3.8802

Hartono, S., Siswono, T. Y. E., & Ekawati, R. (2024). From informal to formal proof in geometry: A preliminary study of scaffolding-based interventions for improving preservice teachers' level of proof. *Mathematics Teaching Research Journal, 16*(2), 48–62.

Hartono, S., Siswono, T. Y. E., Ekawati, R., & Bonyah, E. (2025). Using Moore's theory to explain preservice teachers' difficulties in proving the triangle sum theorem. *Jurnal Ilmiah Ilmu Terapan Universitas Jambi, 9*(1), 104–118. https://doi.org/10.22437/jiituj.v9i1.36349

Hein, K., & Prediger, S. (2024). Scaffolds for seeing, using, and articulating logical structures in proofs: Design research study with high school students. *Journal of Mathematical Behavior, 74*, Article 101123. https://doi.org/10.1016/j.jmathb.2023.101123

Hlongwana, P., Mudaly, V., & Zulu, M. W. (2025). Enhancing geometry problem-solving through visualization for multilingual learners. *Eurasia Journal of Mathematics, Science and Technology Education, 21*(7), 1–18. https://doi.org/10.29333/ejmste/16564

Hoerudin, C. W. (2022). Indonesian language learning using the discovery learning model based on higher order thinking skills (HOTS) on students' analytical thinking ability. *Munaddhomah: Jurnal Manajemen Pendidikan Islam, 4*(1), 108–118. https://doi.org/10.31538/munaddhomah.v4i1.370

Hovik, E. K., & Nolan, K. (2024). Pre-service teachers' understandings of exploratory task design in mathematics: From GeoGebra task design to the 8th grade student. *Cogent Education, 11*(1), 1–20. https://doi.org/10.1080/2331186X.2024.2421088

Ikhsan, M., Rochaminah, S., & Mastura, A. (2024). The development of Geo-Math application by integrating GeoGebra applets to improve students' spatial ability. *Jurnal Ilmiah Peuradeun, 12*(3), 1129–1154. https://doi.org/10.26811/peuradeun.v12i3.1492

Iwani Muslim, N. E., Zakaria, M. I., & Yin Fang, C. (2023). A systematic review of GeoGebra in mathematics education. *International Journal of Academic Research in Progressive Education and Development, 12*(3), 1192–1203. https://doi.org/10.6007/IJARPED/v12-i3/19133

Jablonski, S., & Ludwig, M. (2023). Teaching and learning of geometry: A literature review on current developments in theory and practice. *Education Sciences, 13*(7), Article 682. https://doi.org/10.3390/educsci13070682

Juman, Z. A. M., Mathavan, M., Ambegedara, A. S., & Udagedara, I. G. K. (2022). Difficulties in learning geometry component in mathematics and active-based learning methods to overcome the difficulties. *Shanlax International Journal of Education, 10*(2), 41–58. https://doi.org/10.34293/education.v10i2.4299

Kovács, Z., Recio, T., & Vélez, M. P. (2024). Showing proofs, assessing difficulty with GeoGebra discovery. In *Proceedings of the 14th International Conference on Automated Deduction in Geometry (ADG 2023)* (Vol. 398, pp. 43–52). https://doi.org/10.4204/EPTCS.398.8

Laugwitz, R., Brereton, N., & Cheng, M. (2025). Undergraduate students' perception of mathematical proof. *Teaching Mathematics and Its Applications: An International Journal of the IMA*, 1–25. https://doi.org/10.1093/teamat/hraf017

Lessing, M., & Ogbonnaya, U. I. (2026). Reasoning and proof in mathematics education: A systematic literature review. *Discover Education, 5*, Article 31. https://doi.org/10.1007/s44217-025-01016-1

Lestyanto, L. M., Rahmadani, D., Asmianto, D. E. T., & Cahyowati. (2022). The factors influencing students' proof comprehension on mathematical induction. In *Proceedings of the 2nd National Conference on Mathematics Education 2021 (NaCoME 2021)* (Vol. 656, pp. 135–139). https://doi.org/10.2991/assehr.k.220403.018

Liburd, K. K. D., & Jen, H. (2021). Investigating the effectiveness of using a technological approach on students' achievement in mathematics: Case study of a high school in a Caribbean country. *Sustainability, 13*(10), Article 5586. https://doi.org/10.3390/su13105586

Malik, M., & Mursalin. (2018). Role of self-efficacy toward students' achievement in mathematical multiple representation ability (MMRA). *Jurnal Ilmiah Peuradeun, 6*(1), 31–40. https://doi.org/10.26811/peuradeun.v6i1.174

Martínez-Zarzuelo, A., Nolla, Á., Recio, T., Tolmos, P., & Ariño-Morera, B. (2025). An experience with pre-service teachers using GeoGebra discovery automated reasoning tools for outdoor mathematics. *Education Sciences, 15*(6), Article 782. https://doi.org/10.3390/educsci15060782

Menouer, B., Faouzi, L., Rachid, A., Benslimane, Y., & Nachit, B. (2025). Exploring the challenges of geometric reasoning in middle school: An analysis of teachers' perceptions and pedagogical perspectives. *Educational Process: International Journal, 17*, Article e2025387. https://doi.org/10.22521/edupij.2025.17.387

Mishra, P., Pandey, C. M., Singh, U., Gupta, A., Sahu, C., & Keshri, A. (2019). Descriptive statistics and normality tests for statistical data. *Annals of Cardiac Anaesthesia, 22*(1), 67–72. https://doi.org/10.4103/aca.ACA_157_18

Mone, F., Simarmata, J. E., & Nahak, S. (2025). Ability to construct GeoGebra-assisted proofs in view of gender differences. *Jurnal Riset Pendidikan Matematika, 12*(1), 67–73. https://doi.org/10.21831/jrpm.v12i1.83952

Moralı, H. S., & Filiz, A. (2023). Incorrect theorems and proofs: An analysis of pre-service mathematics teachers' proof evaluation skills. *Journal of Pedagogical Research, 7*(3), 248–262. https://doi.org/10.33902/JPR.202318840

Muzaini, M., Arriah, F., Sulong, A., Kaharuddin, A., & Jumiarti. (2026). Mapping students' cognitive load profiles in solving geometric integral problems. *European Journal of STEM Education, 11*(1), Article 18288. https://doi.org/10.20897/ejsteme/18288

Öztürk, T., & Güven, B. (2025). Enhancing pre-service mathematics teachers' proofwriting skills: The effect of a social learning environment enriched with dynamic geometry software. *Journal on Efficiency and Responsibility in Education and Science, 18*(3), 197–218. https://doi.org/10.7160/eriesj.2025.180305

Pradana, K. C., Noer, S. H., & Sutiarso, S. (2025). Enhancing critical thinking in mathematics through Android-based multimedia and PjBL-STEM. *Online Learning in Educational Research (OLER), 5*(1), 81–93. https://doi.org/10.58524/oler.v5i1.534

Pratiwi, N. P. D. A., Ariawan, I. P. W., & Sariyasa. (2025). Pengaruh ilustrasi gambar pada soal geometri bidang terhadap kemampuan pemecahan masalah matematis siswa SMP. *Circle: Jurnal Pendidikan Matematika, 5*(2), 126–142. https://doi.org/10.28918/circle.v5i2.10008

Putra, Z. H., Afrillia, Y. M., Dahnilsyah, & Tjoe, H. (2023). Prospective elementary teachers' informal mathematical proof using GeoGebra: The case of 3D shapes. *Journal on Mathematics Education, 14*(3), 449–468. https://doi.org/10.22342/jme.v14i3.pp449-468

Rahmawati, D. U., Wilujeng, I., Jumadi, J., Kuswanto, H., Sulaeman, N. F., & Astuti, D. P. (2020). Problem based learning e-handout: Improving students' mathematical representation and self efficacy. *Jurnal Ilmiah Pendidikan Fisika Al-Biruni, 9*(1), 41–50. https://doi.org/10.24042/jipfalbiruni.v9i1.4607

Ramírez-Uclés, R., & Ruiz-Hidalgo, J. F. (2022). Reasoning, representing, and generalizing in geometric proof problems among 8th-grade talented students. *Mathematics, 10*(5), Article 789. https://doi.org/10.3390/math10050789

Razali, N. M., & Wah, Y. B. (2011). Power comparisons of Shapiro-Wilk, Kolmogorov-Smirnov, Lilliefors and Anderson-Darling tests. *Journal of Statistical Modeling and Analytics, 2*(1), 21–33.

Ridwan, & Haeruddin. (2025). Analyzing students' errors and misconceptions in proof construction in abstract algebra course. *Primatika: Jurnal Pendidikan Matematika, 14*(2), 383–398. https://doi.org/10.30872/primatika.v14i2.6217

Rizal, S., Nahar, S., & Al Farabi, M. (2023). Islamic values: Integration in learning mathematics and science at MAN 2 level 2022/2023 academic year. *Munaddhomah: Jurnal Manajemen Pendidikan Islam, 4*(3), 732–745. https://doi.org/10.31538/munaddhomah.v4i3.653

Rizos, I., & Gkrekas, N. (2023). Is there room for conjectures in mathematics? The role of dynamic geometry environments. *European Journal of Science and Mathematics Education, 11*(4), 589–598. https://doi.org/10.30935/scimath/13204

Rohmah, L. A., Anggoro, B. S., & Gunawan, W. (2023). PDEODE strategy assisted by GeoGebra: Improving students' critical thinking and mathematical analysis. *Online Learning in Educational Research (OLER), 3*(1), 15–22. https://doi.org/10.58524/oler.v3i1.203

Rosyidi, A. H., & Rahmadhani, Y. N. (2024). Students' construction of conjectures assisted by GeoGebra for graphing linear equations: Cases of female students. *JMPM: Jurnal Matematika dan Pendidikan Matematika, 9*(2), 88–104. https://doi.org/10.26594/jmpm.v9i2.4858

Sari, P. M., Herlina, K., & Abdurrahman. (2022). Online learning with multi-representation worksheets for oral and written communication skills on light reflecting material. *Online Learning in Educational Research, 2*(1), 49–56. https://doi.org/10.58524/oler.v2i1.122

Schoenherr, J., & Schukajlow, S. (2024). Characterizing external visualization in mathematics education research: A scoping review. *ZDM–Mathematics Education, 56*(1), 73–85. https://doi.org/10.1007/s11858-023-01494-3

Segal, R., Oxman, V., & Stupel, M. (2021). Using dynamic geometry software to enhance specialized content knowledge: Pre-service mathematics teachers' perceptions. *International Electronic Journal of Mathematics Education, 16*(3), Article em065. https://doi.org/10.29333/iejme/11065

Shimizu, Y. (2022). Relation between mathematical proof problem solving, math anxiety, self-efficacy, learning engagement, and backward reasoning. *Journal of Education and Learning, 11*(6), 62–75. https://doi.org/10.5539/jel.v11n6p62

Suparman, Marasabessy, R., & Helsa, Y. (2024). Fostering spatial visualization in GeoGebra-assisted geometry lessons: A systematic review and meta-analysis. *Eurasia Journal of Mathematics, Science and Technology Education, 20*(9), 1–18. https://doi.org/10.29333/ejmste/15170

Tapo, M. M., & Rudhito, M. A. (2025). Mathematical proof difficulties among mathematics education students: A cognitive perspective. *International Journal of Advanced Education and Research, 10*(3), 12–20.

van Nooijen, C. C. A., de Koning, B. B., Bramer, W. M., Isahakyan, A., Asoodar, M., Kok, E., van Merriënboer, J. J. G., & Paas, F. (2024). A cognitive load theory approach to understanding expert scaffolding of visual problem-solving tasks. *Educational Psychology Review, 36*, Article 12. https://doi.org/10.1007/s10648-024-09848-3

Winer, M. L., & Battista, M. T. (2022). Investigating students' proof reasoning: Analyzing students' oral proof explanations and their written proofs in high school geometry. *International Electronic Journal of Mathematics Education, 17*(2), Article em067. https://doi.org/10.29333/iejme/11713

Yoon, H., Hwang, J., Lee, K., Roh, K. H., & Kwon, O. N. (2024). Students' use of generative artificial intelligence for proving mathematical statements. *ZDM–Mathematics Education, 56*(7), 1531–1551. https://doi.org/10.1007/s11858-024-01629-0

Zengin, D., & Broutin, M. S. T. (2025). An analysis of gifted students' proof and reasoning process based on the Toulmin model. *International Electronic Journal of Mathematics Education, 20*(4), Article em081. https://doi.org/10.29333/iejme/17050