Online Learning in Educational Research

Volume 6, Issue 2, 97-108.
e_ISSN: 2798-2351
https://www.journal.foundae.com/index.php/oler/index
DOI: 10.58524/oler.v6i2.1143

Linking Adaptive Reasoning to Newman's Error Patterns in Digital Geometry Learning

Ramon Muhandaz*
Universitas Negeri Yogyakarta, INDONESIA

Sugiman
Universitas Negeri Yogyakarta, INDONESIA

Elly Arliani
Universitas Negeri Yogyakarta, INDONESIA

Risnawati
Universitas Islam Negeri Sultan Syarif Kasim Riau, INDONESIA

M. Fikri Hamdani
Universitas Islam Negeri Sultan Syarif Kasim Riau, INDONESIA

Article Info

Article history:

Received: March 15, 2026
Revised: May 23, 2026
Accepted: June 18, 2026

Keywords:

Adaptive reasoning;
Digital geometry learning;
Error diagnosis;
Newman's error analysis;
Pythagorean theorem.

Abstract

Adaptive reasoning is a fundamental component of mathematical proficiency, yet limited research has examined how students' reasoning relates to specific stages of problem-solving failure in digital geometry learning. Existing studies on Newman's Error Analysis (NEA) primarily classify error types, whereas research on adaptive reasoning tends to evaluate reasoning quality without explaining its relationship to problem-solving errors. This study investigated students' adaptive reasoning profiles, identified error patterns using Newman's Error Analysis, and examined the relationship between reasoning ability and error occurrence in solving Pythagorean theorem problems within a digital geometry learning environment. An embedded mixed-methods design was employed involving 30 eighth-grade students from a junior secondary school in Indonesia. Students learned through an interactive digital flipbook integrated with GeoGebra before completing adaptive reasoning tasks and participating in semi-structured interviews. The findings revealed that students demonstrated moderate adaptive reasoning, with transformation errors emerging as the most frequent problem-solving difficulty, followed by comprehension and process-skill errors. Students with lower adaptive reasoning experienced substantially more difficulties in transforming contextual information into mathematical representations and constructing coherent solution strategies. This study extends current research by demonstrating a systematic relationship between adaptive reasoning and Newman's error stages, providing diagnostic insights for designing reasoning-oriented scaffolding in technology-enhanced geometry instruction.

To cite this article: Muhandaz, R., Sugiman, S., Arliani, E., Risnawati, R., & Hamdani, M. F. (2026). Linking adaptive reasoning to Newman's error patterns in digital geometry learning. *Online Learning in Educational Research, 6*(2), 97-108. https://doi.org/10.58524/oler.v6i2.1143

INTRODUCTION

Over the past two decades, mathematical reasoning has become a central focus of mathematics education reform worldwide. The Programme for International Student Assessment (PISA) emphasizes that mathematical literacy extends beyond computational skills to include the ability to formulate, apply, and interpret mathematics in various real-life contexts (OECD, 2023). In this regard, reasoning plays a crucial role in enabling students to justify solutions, evaluate arguments, and construct meaningful mathematical understanding. Similarly, the National Council of Teachers of Mathematics (2014) identifies reasoning and proof as essential practices in mathematics instruction. These perspectives highlight that reasoning, argumentation, and reflection are fundamental

* **Corresponding Author**
Ramon Muhandaz, Universitas Negeri Yogyakarta, INDONESIA ✉ ramonmuhandaz.2025@student.uny.ac.id

competencies in twenty-first-century mathematics learning. Within the framework of mathematical proficiency, adaptive reasoning is recognized as one of the five key strands, alongside conceptual understanding, procedural fluency, strategic competence, and productive disposition (Kilpatrick et al., 2001). Adaptive reasoning refers to the ability to think logically about mathematical relationships, explain why procedures work, and evaluate the validity of solutions. Amirah et al. (2023) reported that technology-supported learning environments contributed positively to students' adaptive reasoning development. From the perspective of reasoning-process frameworks, mathematical reasoning is not merely the production of correct answers but a dynamic cognitive process involving conjecturing, representing, validating, and reflecting on mathematical ideas (Jeannotte & Kieran, 2017). However, empirical evidence indicates that many students are able to execute procedures but struggle to provide coherent justifications and construct logically valid arguments (Jeannotte & Kieran, 2017; Weber, 2001). This suggests that students' reasoning abilities remain a critical challenge in mathematics education.

Studies on mathematical modeling further demonstrate that successful problem solving requires students to transform contextual situations into mathematical representations and interpret the resulting solutions meaningfully. In geometry learning, these processes become more complex because students must coordinate visual, symbolic, and verbal representations simultaneously. According to representation theory, mathematical understanding develops through the ability to translate and connect multiple representations meaningfully (Duval, 2006). Difficulties in coordinating these representations may hinder reasoning processes and lead to problem-solving errors (Malik & Muslain, 2018; Rahmawati et al., 2020). From a cognitive perspective, such difficulties can also be explained through Cognitive Load Theory (Sweller et al., 2011). When students are required to process diagrams, formulas, contextual information, and symbolic representations simultaneously, their working memory may become overloaded, particularly if conceptual understanding is weak. This condition may interfere with reasoning processes such as interpreting information, selecting appropriate mathematical strategies, and constructing logical arguments (Mayer & Fiorella, 2021).

From a global perspective, students' reasoning ability is still relatively limited, particularly in tasks requiring higher-order thinking and conceptual justification. Studies aligned with the PISA framework demonstrate that reasoning competencies involve not only applying procedures but also generalizing, explaining, and evaluating mathematical arguments (Özaydın & Arslan, 2022). Nevertheless, many students tend to rely on imitative reasoning rather than developing deeper conceptual understanding (Lithner, 2017). Similar findings have also been reported in Indonesian mathematics education contexts. Students often experience difficulties in interpreting mathematical objects and symbols in the Pythagorean theorem (Rudi et al., 2020), transforming contextual information into mathematical models (Jupri & Drijvers, 2016), and completing problem-solving stages involving transformation, process skills, and encoding in Pythagorean theorem tasks.

Geometry constitutes a crucial domain for developing mathematical reasoning because it requires the integration of visual, symbolic, and logical representations. Dynamic geometry environments are particularly relevant to this process because they allow students to manipulate geometric objects, observe invariant relationships, formulate conjectures, and move from visual exploration toward mathematical justification (Granberg & Olsson, 2015; Kaplar et al., 2021; Žakelj & Klančar, 2022). Therefore, geometry learning supported by dynamic visualization requires students not only to observe diagrams but also to explain the relationships represented in those diagrams (Baccaglini-Frank & Mariotti, 2010; Laborde, 2000; Leung et al., 2013). In these environments, digital tools become meaningful when they support conjecturing, testing, and justification rather than merely displaying visual objects. Studies have shown that dragging, manipulation, and carefully designed tasks can help students notice invariant relationships and develop mathematical reasoning when guided by appropriate instructional prompts (Baccaglini-Frank & Mariotti, 2010; Komatsu & Jones, 2018). The use of interactive technologies such as GeoGebra and digital flipbook-based media can further enhance visualization and support conceptual understanding (Ardimas et al., 2021; Drijvers et al., 2016; Drijvers & Sinclair, 2023; Engelbrecht & Borba, 2023; Ikhsan et al., 2024; Nuralam et al., 2024). Nevertheless, students frequently encounter difficulties in interpreting diagrams and constructing mathematical models, which may hinder mathematical reasoning (Musa et al., 2025; Nuralam et al., 2024).

In the context of the Pythagorean theorem, students frequently make errors such as misidentifying the hypotenuse, incorrectly applying formulas, and failing to model contextual situations mathematically. These difficulties suggest challenges in coordinating visual, symbolic, and contextual representations, which are essential for successful geometry problem solving (Arcavi, 2003; Žakelj & Klančar, 2022). Furthermore, multimodal reasoning studies suggest that students must coordinate multiple forms of representation to successfully solve geometry problems (Cho et al., 2025).

To diagnose such difficulties, Newman's Error Analysis (NEA) provides a systematic framework for identifying students' errors across five stages: reading, comprehension, transformation, process skills, and encoding. Previous studies indicate that transformation and comprehension errors are among the most dominant in mathematical problem solving (Ghefira & Jupri, 2025; Jupri & Drijvers, 2016). Similar evidence from Indonesian word-problem contexts also shows that students' errors frequently occur at the comprehension, transformation, process skill, and encoding stages, particularly when students are required to translate verbal information into mathematical expressions (Wardhani & Argaswari, 2022). However, most NEA studies primarily focus on identifying and classifying error types without examining the underlying reasoning processes that may contribute to those errors. As a result, NEA findings often remain descriptive and provide limited explanation regarding why certain error patterns emerge during mathematical problem solving. Therefore, analyzing errors through NEA provides important insights into students' reasoning processes.

On the other hand, studies on adaptive reasoning generally emphasize the measurement or categorization of reasoning ability, such as students' capacity to justify solutions, construct arguments, or evaluate mathematical relationships (Jeannotte & Kieran, 2017; Weber, 2001). Although these studies provide important insights into reasoning quality, they rarely explain how different levels of reasoning are associated with specific stages of problem-solving failure. In other words, research on adaptive reasoning and research using Newman's Error Analysis have largely developed in parallel. Review and empirical studies in mathematics education indicate that reasoning research tends to emphasize cognitive performance and argument quality, whereas NEA-based studies focus mainly on procedural error identification (Jeannotte & Kieran, 2017; Jupri & Drijvers, 2016). Consequently, the theoretical relationship between reasoning quality and error stages remains insufficiently explored.

Therefore, there is a need for a more comprehensive approach that connects students' adaptive reasoning with their error patterns within a digital geometry learning environment. This study proposes a cognitive linkage perspective between adaptive reasoning and Newman's Error Analysis stages, assuming that reasoning quality may influence how students interpret information, transform representations, execute procedures, and formulate conclusions during problem solving. Based on this research gap, this study addresses the following research questions:

1. How are students' adaptive reasoning profiles manifested in digital geometry learning on the topic of the Pythagorean theorem?
2. What types of errors do students exhibit at each stage of Newman's Error Analysis when solving Pythagorean theorem problems?
3. How is students' adaptive reasoning related to their error patterns across the stages of the problem-solving process?

METHOD

The research approach adopted here involved the use of embedded mixed methods, whereby both quantitative descriptive and qualitative case analyses were used. Quantitative data from the adaptive reasoning tests and Newman's error analysis were used to determine the students' reasoning profile and error patterns, while qualitative data from written responses and interviews helped to clarify the process behind the reasoning pattern. The integration of the two data sets took place at the interpretation stage of analysis (Creswell & Creswell, 2018; Tashakkori & Teddlie, 1998). In terms of participants, the current study focused on 30 students in Grade 8 from one junior secondary school in Pekanbaru, Indonesia. Students in the sample engaged in digital geometry learning sessions in their own classroom context. The size of the sample was chosen taking into account the characteristics of the class where the experiment was conducted, since the research itself was carried out in the classroom environment (Ary et al., 2018).

From the total participants, three students were purposively selected for in-depth qualitative analysis, representing high, medium, and low adaptive reasoning score categories. The selection process was based primarily on students' test results and was further supported by mathematics teacher recommendations regarding students' communication ability and willingness to participate in follow-up interviews. This combination of score-based categorization and teacher input was intended to ensure that the selected participants could provide rich and representative explanations of their reasoning processes and error patterns, consistent with purposive case selection principles in integrated research designs (Yin, 2018). The study was conducted during two 90-minute instructional sessions on the Pythagorean theorem using an interactive digital flipbook integrated with GeoGebra. The digital learning environment served as the instructional context rather than an experimental intervention (Prasasti & Anas, 2023). Students explored contextual geometry problems, manipulated geometric representations dynamically, and completed reasoning-oriented tasks requiring them to formulate mathematical relationships, justify procedures, and interpret solutions. The teacher facilitated discussions and provided scaffolding questions related to geometric reasoning and problem solving (Awi et al., 2025). An example of the digital visualization used in the learning activity is presented in Figure 1.

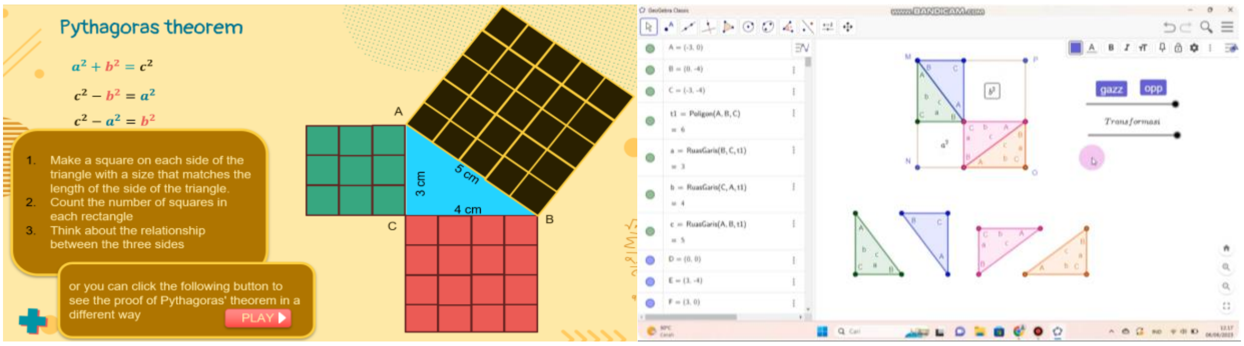

Figure 1. Digital Learning Media Used in the Study: Interactive Visualization of the Pythagorean Theorem Integrated with GeoGebra.

The research instruments consisted of five essay-type questions designed to measure students' adaptive reasoning, including logical justification, representation connection, mathematical argument construction, and solution evaluation. The indicators were adapted from the concept of adaptive reasoning as logical thought, reflection, explanation, and justification, and from mathematical reasoning processes involving representation, argumentation, and validation (Jeannotte & Kieran, 2017; Kilpatrick et al., 2001). Essay-based assessments were used because written responses provide evidence of students' reasoning processes, including how they connect representations, justify procedures, and evaluate their solutions (Lehmann & Friend, 2025; Maoto et al., 2018). A representative item required students to solve a contextual Pythagorean theorem problem and justify the reasoning underlying their solution. Each item was scored using an analytic rubric adapted from the adaptive reasoning indicators proposed by Kilpatrick et al. (2001) and further refined based on recent studies on mathematical reasoning (Özaydın & Arslan, 2022). The rubric employed a scale ranging from 0 to 4, where: (0) no meaningful response, (1) incorrect reasoning with irrelevant explanation, (2) partially correct procedure with limited justification, (3) mostly correct reasoning with adequate explanation, and (4) complete and logically coherent reasoning supported by accurate mathematical representation and justification. The maximum total score across all items was 20. In addition, an interview guideline was employed to obtain qualitative data aimed at deepening the interpretation of the quantitative findings and providing further insight into students' reasoning processes and error patterns. Figure 2 illustrates the overall research procedure employed in this study.

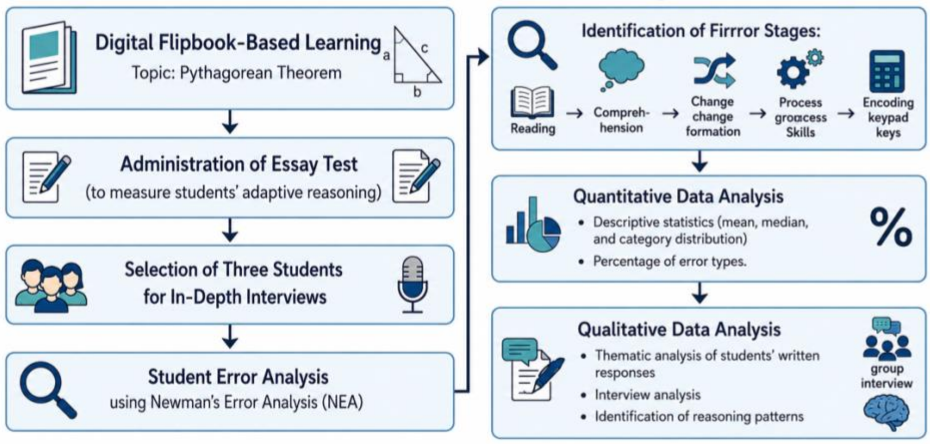

Figure 2. Research Procedure Flow Diagram

Initially, students were exposed to a learning activity involving the use of an interactive electronic flip book based on the subject matter of the Pythagorean Theorem. Following the completion of the instruction activity, students were then subjected to a test involving an essay-type item as the main measurement tool for assessing their ability in adaptive reasoning. Three randomly chosen students underwent individual interviews to further investigate their cognitive processes while answering the problem. The coding framework involved the classification of the mistakes according to the different categories set by Newman's Error Analysis (NEA): reading, comprehension, transformation, process skills, and encoding. The coding of students' responses was done independently by two researchers in mathematics education based on the previously defined criteria (Landis & Koch, 1977).

Quantitative data were analyzed using descriptive statistics, including mean scores, median values, and the distribution of ability categories. In addition, percentage analysis was conducted to identify the dominant types of errors based on the stages of Newman's framework. This descriptive approach aligns with recommendations in educational research aimed at systematically mapping students' abilities without examining causal relationships (Gall et al., 2007). Meanwhile, qualitative data from students' written responses and interviews were analyzed using thematic analysis to identify recurring patterns in students' reasoning and to relate these patterns to the types of errors that emerged during problem solving (Braun & Clarke, 2006; Nowell et al., 2017).

Content validity was established through expert judgment involving two mathematics education specialists and one educational evaluation expert. Revisions were made based on their feedback regarding item clarity, representation accuracy, and alignment with adaptive reasoning indicators. Instrument reliability was supported by a Cronbach's alpha coefficient of 0.82, indicating high internal consistency. Because the assessment involved open-ended responses, inter-rater reliability was also examined using Cohen's Kappa, yielding a value of 0.81, which indicated substantial agreement (Landis & Koch, 1977).

Ethical approval was obtained from the researchers' institutional ethics committee. Permission was granted by the participating school, parental consent was secured, and all data were anonymized.

RESULTS AND DISCUSSION

Students' Adaptive Reasoning Profile

The results of the descriptive statistical analysis indicate that students' adaptive reasoning ability in digital geometry learning falls within the moderate category, with a mean score of 13.47 (out of 20), a median of 14, and a standard deviation of 3.21. This finding suggests that, although students demonstrate an adequate level of reasoning, there is considerable variation in their performance. The descriptive statistics of students' performance are presented in Table 1.

Table 1. Descriptive Statistics of Adaptive Reasoning Scores (N = 30)

Statistic	Value
Mean	13.47
Median	14
Standard Deviation	3.21
Minimum Score	7
Maximum Score	19
Range	12

The proximity between the mean and median values indicates a relatively symmetrical distribution of scores. However, the standard deviation of 3.21 and the score range of 12 suggest considerable variation in students' reasoning abilities. This variation suggests that while some students have developed relatively coherent reasoning structures, others still encounter conceptual barriers when constructing and evaluating mathematical arguments during problem solving. Further analysis based on adaptive reasoning indicators provides additional insight into students' reasoning performance, as presented in Table 2.

Table 2. Mean Score by Adaptive Reasoning Indicator

Indicator	Mean
Logical justification	2.85
Argument coherence	2.73
Representation connection	2.61
Solution evaluation	2.48

The results indicate that the solution evaluation indicator obtained the lowest average score (2.48). This finding suggests that many students tend to terminate the problem-solving process after obtaining a numerical answer without conducting reflective verification or evaluating the plausibility of the solution. Such behavior indicates that students' reasoning processes remain primarily procedural rather than reflective. Adaptive reasoning is widely recognized as a core component of mathematical proficiency involving the ability to justify procedures, construct logical arguments, and evaluate mathematical conclusions (Kilpatrick et al., 2001). In geometry learning, adaptive reasoning requires students to coordinate visual representations, symbolic relationships, and contextual interpretation simultaneously. The moderate reasoning profile identified in this study indicates that students were generally able to apply procedures appropriately, but many still experienced difficulty when required to explain, justify, and evaluate their solutions critically.

The low performance in solution evaluation may indicate that students focused primarily on procedural completion rather than reflective reasoning. This finding is consistent with previous studies reporting that students often emphasize obtaining correct answers while giving limited attention to verification and justification processes in mathematical reasoning (Putri & Warmi, 2024). In geometry problem solving, reflective evaluation requires students to reconnect symbolic results with diagrams and contextual meaning, which may increase cognitive demands during problem solving.

From the perspective of representation theory, successful mathematical reasoning depends on students' ability to coordinate and translate multiple forms of representation flexibly (Duval, 2006). Students who struggle to relate symbolic equations to geometric representations may experience difficulty evaluating whether their solutions are conceptually reasonable. Furthermore, Cognitive Load Theory suggests that processing multiple sources of information simultaneously may overload working memory capacity, particularly for students with weaker conceptual understanding (Sweller et al., 2011). As a result, students may rely on procedural shortcuts instead of engaging in reflective verification processes.

The digital flipbook and GeoGebra integration appeared to support students in visualizing geometric relationships dynamically; however, visualization alone was insufficient to ensure deeper reasoning development. This result is consistent with studies on dynamic geometry environments showing that visual manipulation supports reasoning only when students are guided to identify invariant relationships, formulate conjectures, and justify why a mathematical relationship remains valid under different configurations (Baccaglini-Frank & Mariotti, 2010; Komatsu & Jones, 2018; Leung et al., 2013; Rohmah et al., 2023). Similar findings are reported in recent GeoGebra-based ethnomathematics studies showing that dynamic visualization can support students in connecting geometric representations and symbolic reasoning more meaningfully during geometry problem solving (Sartika et al., 2024). This finding aligns with recent studies emphasizing that digital transformation in ethnomathematics learning should not merely focus on technological interaction but also on supporting meaningful conceptual reasoning and representation processes (Tanujaya et al., 2025). Students with stronger adaptive reasoning profiles tended to use visual representations as a basis for constructing symbolic relationships and evaluating solutions, whereas students with weaker reasoning ability often focused primarily on surface-level visual features without developing coherent mathematical justification. Therefore, reasoning-oriented instructional scaffolding involving explanation, justification, and verification activities remains essential in digital geometry learning contexts.

Distribution of Errors Based on Newman's Error Analysis

To further examine students' reasoning processes, an error analysis was conducted using Newman's Error Analysis (NEA). From a total of 150 responses (30 students × 5 items), multiple errors were allowed within a single response because students could experience difficulties across more than one Newman stage simultaneously. Therefore, the reported percentages represent the proportion of identified error occurrences rather than mutually exclusive response classifications. The distribution of error types is presented in Figure 3.

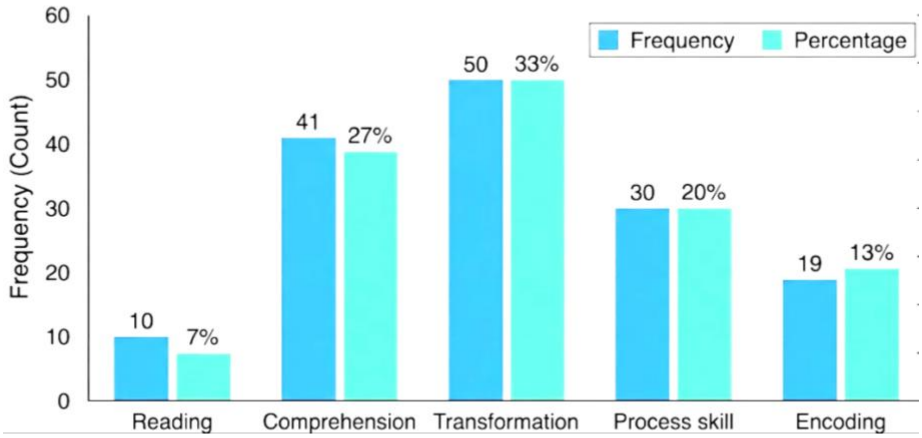

Figure 3. Distribution of Newman's Errors

The findings indicate that transformation errors were the most dominant category, with 50 identified occurrences (33%), followed by comprehension errors with 41 occurrences (27%), process skill errors with 30 occurrences (20%), encoding errors with 19 occurrences (13%), and reading errors with 10 occurrences (7%). These results show that students' main difficulties did not occur at the reading stage, but at the transformation stage, where they were required to convert contextual and visual information into appropriate mathematical representations. The dominance of transformation errors suggests that many students were able to read the problem statement but still struggled to construct accurate mathematical models from the given geometric situation. In solving Pythagorean theorem problems, students had to coordinate contextual information, diagrams, symbolic relationships, and numerical procedures. Therefore, the high percentage of transformation errors indicates that students' difficulties were closely related to representation coordination rather than merely computational procedures. This finding is consistent with previous studies indicating that transformation errors are among the most common difficulties encountered in geometry problem solving (Jupri & Drijvers, 2016; Marbun et al., 2025; Muthia Ghefira & Jupri, 2025). However, the present study extends previous findings by situating these difficulties within a digital geometry learning environment integrating GeoGebra and interactive visual representations.

From the perspective of representation theory, mathematical reasoning depends on students' ability to move flexibly between visual and symbolic representations (Duval, 2006). Students who fail to coordinate these representations appropriately may produce incorrect mathematical models even when they recognize the procedural steps required to solve the problem. This phenomenon may also be interpreted through Cognitive Load Theory (Sweller et al., 2011). Geometry problem solving requires students to process multiple sources of information concurrently, including diagrams, contextual statements, formulas, and symbolic operations. For students with weaker conceptual understanding, these simultaneous processing demands may overload working memory capacity and lead students to rely on procedural shortcuts rather than constructing coherent mathematical relationships.

The digital flipbook and GeoGebra environment appeared to support visual exploration of geometric relationships; however, the findings suggest that visual interaction alone did not automatically reduce transformation errors. Students with stronger reasoning ability tended to use visual representations to verify mathematical relationships, whereas students with weaker reasoning ability often focused only on manipulating visual objects without establishing meaningful symbolic connections. Previous studies have shown that dynamic mathematics software such as GeoGebra can support students' reasoning and representational understanding when students are encouraged to explain and justify mathematical relationships during exploration activities (Drijvers et al., 2010; Drijvers & Sinclair, 2023).

It was also observed that there exists a consistent relationship between the level of adaptive reasoning of students and error categories made by students during the different phases described in Newman's model. It is evident from the results that students who had poor levels of reasoning were faced with problems during the comprehension and transformation stages, whereas those with high levels of reasoning exhibited systematic reasoning and few errors in concepts.

Selected Case Analysis (Embedded Findings)

To deepen the interpretation of the quantitative findings, three students were purposively selected for in-depth qualitative analysis, representing high, moderate, and low adaptive reasoning categories. The selection process was based primarily on adaptive reasoning scores and was further supported by teacher recommendations regarding students' communication ability during interviews. The scoring distribution of the selected students is presented in Table 3.

Table 3. Individual Scoring Sheet

Student Code	Q1	Q2	Q3	Q4	Q5	Total	Percentage	Level
S1	4	3	4	4	3	18	90%	High
S2	3	2	3	2	2	12	60%	Moderate
S3	1	1	0	1	0	3	15%	Low

The student classified in the high reasoning category demonstrated the ability to integrate visual, symbolic, and verbal representations effectively when solving geometry problems. During the interview, the student explained: "The triangle sides already form a right triangle, so I used the Pythagorean theorem to determine the missing side length. After calculating the result, I checked whether the answer was reasonable based on the diagram." This response indicates that the student not only applied the formula correctly but also engaged in verification and representation-based reasoning. No major errors were identified across the Newman stages. The student demonstrated systematic reasoning by identifying relevant geometric relationships, selecting appropriate procedures, and evaluating the final solution.

Moderate-Ability Student

The student in the moderate reasoning category generally understood the problem context and selected appropriate formulas. However, the explanations provided were often incomplete and lacked formal justification. During the interview, the student stated: "I knew I should use the

Pythagorean formula because the picture showed a right triangle, but I was not sure how to explain why." This response suggests that the student recognized procedural patterns but experienced difficulty articulating explicit mathematical justification. Minor transformation errors were also identified because the student did not consistently explain the relationships among the triangle sides before applying the formula.

Low-Ability Student

The student categorized in the low reasoning group exhibited frequent transformation and process skill errors. The written responses showed that the student directly substituted numerical values into formulas without carefully identifying the hypotenuse or interpreting the geometric relationships presented in the diagram. During the interview, the student explained, "I just used the numbers from the question because I thought all triangle problems use the same formula." This finding indicates limited conceptual understanding and weak coordination between visual and symbolic representations. The student appeared to rely primarily on memorized procedures rather than constructing mathematical models based on the problem context.

The results of the embedded case analysis provide an illustration of how various adaptive reasoning profiles correspond to various types of problem-solving difficulties that occur due to the varying degree of students' reasoning skills. In particular, it was found that reasoning skills allow better coordination among visual, symbolic, and contextual representations, whereas weak reasoning makes students apply procedural imitation without developing mathematical explanations for their actions. In contrast to other studies focusing on either adaptive reasoning or Newman's Error Analysis, the current research tries to establish connections between reasoning skills and the points at which problem-solving difficulties occurred during students' problem-solving within the digital learning environment. It seems reasonable to conclude that students of various adaptive reasoning profiles show various types of dominant problem-solving errors related to the comprehension and transformation stages. This analytical approach can be viewed as a means of expanding existing theoretical knowledge about both mathematical reasoning and students' errors in problem solving. In pedagogical terms, the above findings point to the importance of providing mathematics learners with appropriate reasoning-based scaffolding when working with visual representations and symbolic equations in digital learning environments.

LIMITATION

Several limitations in this study need to be considered when analyzing the results. First, the sample used in this research was quite small since it included 30 students enrolled at one particular junior secondary school. This might impact the possibility of generalizing the obtained results to wider contexts of education. Nevertheless, despite the fact that generalization was not among the main purposes of the study, further investigations involving larger and more diversified samples to verify the consistency of the findings. Second, as has already been mentioned above, this was a descriptive quantitative study with an embedded qualitative approach, which means that there were no chances for making causal conclusions based on this study. The researchers established only a structural relationship, but not the causal mechanism behind it. Finally, it should be noted that this study concentrated on one specific topic, namely, the Pythagorean theorem, studied by students using a digital geometry learning environment. Thus, the results cannot reflect the patterns of adaptive reasoning and errors in students who work on other topics or under different conditions.

CONCLUSION

In using a digital geometry problem-solving task on the Pythagorean theorem, students exhibited different levels of adaptive reasoning. In applying Newman's Error Analysis in this particular study, it can be inferred that transformation errors and process skill errors were the predominant types of errors, which signify the difficulty of the students in transforming context-based and visualization information into an appropriate mathematical approach. It is further observed that there is a consistency in the level of difficulty of transformation based on the quality of adaptive reasoning, with students possessing poor adaptive reasoning having more difficulty in transformation and comprehension.

AUTHOR CONTRIBUTIONS

The contributions made by the authors are as follows. The conceptualization of this paper is credited to RM. Methodology was formulated by SS and EA. Investigation of this paper was done by RM, MFH, and RR. Data curation was done by SS, EA, and RR. Formal analysis was performed by RM and MFH. Writing of the original draft was done by RM. Editing and reviewing of writing was initially done by RM and further edited by other authors. Visualization of the results was done by RM. Supervision was done by SS and EA.

ACKNOWLEDGEMENT

The authors would like to express their sincere gratitude to the school, teachers, and students who participated in this study for their cooperation and valuable contributions. Appreciation is also extended to the experts in mathematics education and educational evaluation who provided constructive feedback during the instrument validation process.

REFERENCES

Amirah, I., Anggoro, B. S., & Gunawan, W. (2023). The implementation of accelerated learning model assisted by Autograph on students' adaptive reasoning ability and environmental literacy. *Online Learning in Educational Research (OLER), 3*(2), 117–126. https://doi.org/10.58524/oler.v3i2.282

Arcavi, A. (2003). The role of visual representations in the learning of mathematics. *Educational Studies in Mathematics, 52*(3), 215–241. https://doi.org/10.1023/A:1024312321077

Ardimas, A., Basith, A., Priyasmika, R., Annovasho, J., & Risdianto, N. S. (2021). Dynamic electrical subject for senior high school: Developing interactive tutorial method animation. *Online Learning in Educational Research, 1(*2), 103–111. https://doi.org/10.58524/oler.v1i2.72

Ary, D., Jacobs, L. C., Sorensen, C. K., & Walker, D. A. (2018). *Introduction to research in education* (10th ed.). Cengage Learning.

Awi, A., Sutamrin, S., Ihsan, H., Nasir, N., Naufal, M. A., & Ikram, M. (2025). Spatial geometry learning in higher education through flipped classroom with metacognitive scaffolding: A mixed-methods study. *Online Learning in Educational Research, 5*(2), 583–600. https://doi.org/10.58524/oler.v5i2.817

Baccaglini-Frank, A., & Mariotti, M. A. (2010). Generating conjectures in dynamic geometry: The maintaining dragging model. *International Journal of Computers for Mathematical Learning, 15*(3), 225–253. https://doi.org/10.1007/s10758-010-9169-3

Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology, 3*(2), 77–101. https://doi.org/10.1191/1478088706qp063oa

Cho, S., Qin, Z., Liu, Y., Choi, Y., Lee, S., & Kim, D. (2025). *Plane geometry problem solving with multi-modal reasoning: A survey* [Preprint]. arXiv. https://doi.org/10.18653/v1/2026.findings-eacl.7

Creswell, J. W., & Creswell, J. D. (2018). *Research design: Qualitative, quantitative, and mixed methods approaches* (5th ed.). SAGE Publications.

Drijvers, P., Ball, L., Barzel, B., Heid, M. K., Cao, Y., & Maschietto, M. (2016). Uses of technology in lower secondary mathematics education. In P. Drijvers (Ed.), *Uses of technology in lower secondary mathematics education* (pp. 1–34). Springer. https://doi.org/10.1007/978-3-319-33666-4_1

Drijvers, P., Doorman, M., Boon, P., Reed, H., & Gravemeijer, K. (2010). The teacher and the tool: Instrumental orchestrations in the technology-rich mathematics classroom. *Educational Studies in Mathematics, 75*(2), 213–234. https://doi.org/10.1007/s10649-010-9254-5

Drijvers, P., & Sinclair, N. (2023). The role of digital technologies in mathematics education: Purposes and perspectives. *ZDM–Mathematics Education, 56*(2), 239–248. https://doi.org/10.1007/s11858-023-01535-x

Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational Studies in Mathematics, 61*(1–2), 103–131. https://doi.org/10.1007/s10649-006-0400-z

Engelbrecht, J., & Borba, M. C. (2023). Recent developments in using digital technology in mathematics education. *ZDM–Mathematics Education, 56*(2), 281–292. https://doi.org/10.1007/s11858-023-01530-2

Gall, M. D., Gall, J. P., & Borg, W. R. (2007). *Educational research: An introduction* (8th ed.). Pearson Education.

Ghefira, A. M., & Jupri, A. (2025). Newman's error analysis on junior high school students' mathematical problem-solving ability in the Pythagorean theorem. *Mathline: Jurnal Matematika dan Pendidikan Matematika, 10*(4), 913–928. https://doi.org/10.31943/mathline.v10i4.1033

Granberg, C., & Olsson, J. (2015). ICT-supported problem solving and collaborative creative reasoning: Exploring linear functions using dynamic mathematics software. *The Journal of Mathematical Behavior, 37*, 48–62. https://doi.org/10.1016/j.jmathb.2014.11.001

Ikhsan, M., Rochaminah, S., & Mastura, A. (2024). The development of Geo-Math application by integrating GeoGebra applets to improve students' spatial ability. *Jurnal Ilmiah Peuradeun, 12*(3), 1129–1154. https://doi.org/10.26811/peuradeun.v12i3.1492

Jeannotte, D., & Kieran, C. (2017). A conceptual model of mathematical reasoning for school mathematics. *Educational Studies in Mathematics, 96*(1), 1–16. https://doi.org/10.1007/s10649-017-9761-8

Jupri, A., & Drijvers, P. (2016). Student difficulties in mathematizing word problems in algebra. *Eurasia Journal of Mathematics, Science and Technology Education, 12*(9), 2481–2502. https://doi.org/10.12973/eurasia.2016.1299a

Kaplar, M., Radović, S., Veljković, K., Simić-Muller, K., & Marić, M. (2021). The influence of interactive learning materials on solving tasks that require different types of mathematical reasoning. *International Journal of Science and Mathematics Education, 20*(2), 411–433. https://doi.org/10.1007/s10763-021-10151-8

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics*. National Academy Press.

Komatsu, K., & Jones, K. (2018). Task design principles for heuristic refutation in dynamic geometry environments. *International Journal of Science and Mathematics Education, 17*(4), 801–824. https://doi.org/10.1007/s10763-018-9892-0

Laborde, C. (2000). Dynamic geometry environments as a source of rich learning contexts for the complex activity of proving. *Educational Studies in Mathematics, 44*(1–2), 151–161. https://doi.org/10.1023/A:1012793121648

Landis, J. R., & Koch, G. G. (1977). The measurement of observer agreement for categorical data. *Biometrics, 33*(1), 159–174. https://doi.org/10.2307/2529310

Lehmann, T. H., & Friend, L. (2025). Mathematical argumentative writing in K–12 education: A systematic literature review. *International Journal of Science and Mathematics Education, 23*(8), 3679–3705. https://doi.org/10.1007/s10763-025-10590-7

Leung, A., Baccaglini-Frank, A., & Mariotti, M. A. (2013). Discernment of invariants in dynamic geometry environments. *Educational Studies in Mathematics, 84*(3), 439–460. https://doi.org/10.1007/s10649-013-9492-4

Lithner, J. (2017). Principles for designing mathematical tasks that enhance imitative and creative reasoning. *ZDM–Mathematics Education, 49*(6), 937–949. https://doi.org/10.1007/s11858-017-0867-3

Malik, M., & Mursalin. (2018). Role of self-efficacy toward students' achievement in mathematical multiple representation ability (MMRA). *Jurnal Ilmiah Peuradeun, 6(*1), 31–40. https://doi.org/10.26811/peuradeun.v6i1.174

Maoto, S., Masha, K., & Mokwana, L. (2018). Teachers' learning and assessing of mathematical processes with emphasis on representations, reasoning and proof. *Pythagoras, 39*(1), Article a373. https://doi.org/10.4102/pythagoras.v39i1.373

Marbun, N., Siburian, R., Nasution, N., Fatiha, S., Girsang, R. L., Sitorus, A., & Fauzi, M. A. (2025). Students' errors in solving plane geometry problems: A Newman's error analysis approach. *Jurnal Absis: Jurnal Pendidikan Matematika dan Matematika, 8*(2), 369–381. https://doi.org/10.31000/prima.v8i2.11199

Mayer, R. E., & Fiorella, L. (2021). Introduction to multimedia learning. In R. E. Mayer & L. Fiorella (Eds.), *The Cambridge handbook of multimedia learning* (3rd ed., pp. 3–16). Cambridge University Press. https://doi.org/10.1017/9781108894333.003

Musa, L. A. D., Munir, N. P., Ikram, M., Garcia, J. G., & Rodriguez-Nieto, C. A. (2025). Students' problem-solving skills in HOTS geometry tasks: A case study of spatial ability. *Al-Jabar: Jurnal Pendidikan Matematika, 16*(2), 597–614. https://doi.org/10.24042/ajpm.v16i2.28482

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. National Council of Teachers of Mathematics.

Nowell, L. S., Norris, J. M., White, D. E., & Moules, N. J. (2017). Thematic analysis: Striving to meet the trustworthiness criteria. *International Journal of Qualitative Methods, 16*(1), 1–13. https://doi.org/10.1177/1609406917733847

Nuralam, H., Jupri, A., & Alifulloh, W. (2024). Exploring high school students' mathematical reasoning in geometry using GeoGebra-assisted problem-based learning. *JTP: Jurnal Teknologi Pendidikan, 26*(3), 1132–1147. https://doi.org/10.21009/jtp.v26i3.49887

OECD. (2023). *PISA 2022 results (Volume I): The state of learning and equity in education*. OECD Publishing. https://doi.org/10.1787/53f23881-en

Özaydın, Z., & Arslan, Ç. (2022). Assessment of mathematical reasoning competence in accordance with PISA 2021 mathematics framework. *Journal of Theoretical Educational Sciences, 15*(3), 453–474. https://doi.org/10.30831/akukeg.1027601

Prasasti, R. D., & Anas, N. (2023). Pengembangan media digital berbasis flipbook untuk meningkatkan kemampuan berpikir kritis pada peserta didik. *Munaddhomah: Jurnal Manajemen Pendidikan Islam, 4*(3), 694–705. https://doi.org/10.31538/munaddhomah.v4i3.589

Putri, D. A., & Warmi, A. (2024). An analysis of adaptive reasoning ability in middle school students mathematics learning. *Jurnal Pendidikan Matematika (JPM), 10*(2), 117–125. https://doi.org/10.33474/jpm.v10i2.21981

Rahmawati, D. U., Wilujeng, I., Jumadi, J., Kuswanto, H., Sulaeman, N. F., & Astuti, D. P. (2020). Problem based learning e-handout: Improving students' mathematical representation and self efficacy. *Jurnal Ilmiah Pendidikan Fisika Al-Biruni, 9*(1). https://doi.org/10.24042/jipfalbiruni.v9i1.4607

Rohmah, L. A., Anggoro, B. S., & Gunawan, W. (2023). PDEODE strategy assisted by GeoGebra: improving students' critical thinking and mathematical analysis. *Online Learning in Educational Research, 3*(1), 15–22. https://doi.org/10.58524/oler.v3i1.203

Rudi, R., Suryadi, D., & Rosjanuardi, R. (2020). Identifying students' difficulties in understanding and applying Pythagorean theorem with an onto-semiotic approach. *MaPan: Jurnal Matematika dan Pembelajaran, 8*(1), 1–18. https://doi.org/10.24252/mapan.2020v8n1a1

Sartika, D., Saputra, E., & Herizal. (2024). Batak ulos ethnomathematics exploration based on GeoGebra software. *Pasundan Journal of Mathematics Education: Jurnal Pendidikan Matematika, 14*(1), 56–69. https://doi.org/10.23969/pjme.v14i1.11914

Sweller, J., Ayres, P., & Kalyuga, S. (2011). *Cognitive load theory*. Springer. https://doi.org/10.1007/978-1-4419-8126-4

Tanujaya, B., Lisnani, L., & Fadhil, Z. N. (2025). Digital transformation of ethnomathematics: Preserving culture through technology in mathematics education. *Journal of Emerging Technologies in Ethnomathematics, 1*(2), 38–50.

Tashakkori, A., & Teddlie, C. (1998). *Mixed methodology: Combining qualitative and quantitative approaches*. Sage.

Wardhani, T. A. W., & Argaswari, D. P. A. D. (2022). High school students' error in solving word problem of trigonometry based on Newman error hierarchical model. *Infinity Journal, 11*(1), 87–102. https://doi.org/10.22460/infinity.v11i1.p87-102

Weber, K. (2001). Student difficulty in constructing proofs: The need for strategic knowledge. *Educational Studies in Mathematics, 48*(1), 101–119. https://doi.org/10.1023/A:1015535614355

Yin, R. K. (2018). *Case study research and applications: Design and methods* (6th ed.). Sage.

Žakelj, A., & Klančar, A. (2022). The role of visual representations in geometry learning. *European Journal of Educational Research, 11*(3), 1393–1411. https://doi.org/10.12973/eu-jer.11.3.1393