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Teachers' knowledge of subject matter in formal definition of limit at senior high school level

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Abstract

Background: Vocational schools in border areas face systemic challenges such as limited infrastructure and poor curriculum-context alignment, hindering effective mathematics instruction and student engagement.
Aims: This study investigates how mathematics learning is organized and implemented in a vocational high school situated in the Entikong border area of West Borneo, Indonesia. The research seeks to identify instructional patterns, contextual challenges, and adaptive strategies used by educators under constrained conditions.
Method: Using a qualitative case study design, data were gathered through direct classroom observation, semi-structured interviews with teachers and school leaders, and review of official teaching documents. Thematic analysis and data triangulation were employed to ensure rigor and credibility.
Results: The analysis highlights a continued reliance on teacher-centered instruction, minimal use of contextual or vocationally integrated methods, and limited student engagement. Assessment practices predominantly measure cognitive outcomes, lacking elements that support student reflection or vocational competencies. Curriculum delivery is often disrupted by time limitations and infrastructural shortfalls.
Conclusion: Improving mathematics instruction in border-based vocational schools necessitates flexible teaching models tailored to the local context. Strengthening professional development, embedding authentic assessments, and enhancing school-community collaboration are crucial steps toward addressing educational disparities in underserved regions.

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INTRODUCTION

Understanding how teaching experience influences PCK in the instruction of limits is essential to improving mathematics education. Teaching mathematics at the secondary level is not only a matter of delivering content, but also guiding students through abstract reasoning. The concept of limits, for instance, asks learners to engage with ideas that do not align easily with everyday logic. Teachers must bridge this gap between abstract mathematical formalism and student understanding, which requires more than procedural competence. They need insight into how students process unfamiliar ideas and where confusion is likely to arise (Prather et al., 2023; Wang, 2025). This is where Pedagogical Content Knowledge, or PCK, becomes a critical element of effective instruction (Agathangelou & Charalambous, 2021; Chan, 2022). It allows educators to anticipate challenges, tailor examples, and build conceptual clarity in ways that traditional subject mastery alone cannot accomplish. Especially in topics that introduce formal definitions early, such as limits, this balance is vital.

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Different teachers approach the same concept in varying ways, often influenced by their teaching experience. An experienced teacher might recognize subtle student hesitations and adjust the lesson mid-stream, while a novice teacher might follow a fixed plan (Jaeger, 2021; Moulds, 2021). Experience may provide cues, but does not guarantee depth of understanding or adaptive pedagogy. Newer teachers might bring creative strategies from recent training but lack familiarity with real classroom responses (Al-khresheh, 2024; Kim et al., 2022). Meanwhile, veterans may rely on past methods that worked but no longer suit changing student profiles. This tension raises the question: how much does experience really shape the way PCK is enacted in the classroom? A comparison between teachers with differing lengths of service can offer valuable insight. This study sets out to explore that comparison in the specific context of teaching mathematical limits.

The idea of a limit challenges students to think in ways that are unfamiliar (Chew & and Cerbin, 2021; Li et al., 2021). It involves understanding what happens as a value gets infinitely close to a point, without necessarily reaching it. Such reasoning can feel unnatural, especially for students who are used to clear, finite answers. The ε–δ definition adds another layer, introducing formal logic that can overwhelm even strong students (Eidin & Shwartz, 2023; B. Zhao et al., 2024). Teachers must decide how and when to introduce these ideas, and what representations to use, whether graphs, numeric tables, or real-life analogies. These decisions influence how students construct meaning from the content. They are also shaped by the teacher's background knowledge, teaching style, and understanding of how students learn. Studying this instructional process reveals how PCK is lived out in everyday teaching.

Efforts to improve mathematics education often highlight the importance of teacher quality (Ekmekci & Serrano, 2022; Llinares, 2021). But defining what makes a teacher "effective" is complex. It is not just about test scores or curriculum coverage, but about how well teachers help students make sense of difficult ideas. Topics like limits serve as a test case for this. If a teacher can successfully guide students through the logic and language of limits, it reflects something deeper than routine instruction. It reflects a kind of pedagogical fluency that integrates content knowledge with real-time awareness of student thinking. This study takes up that challenge by examining how two teachers (one novice and one experienced) approach the same topic with different strategies and outcomes. In doing so, it aims to identify which instructional choices are tied to experience and which are not.

Classrooms are unpredictable spaces. What works for one group of students may not work for another. Teachers have to respond in the moment to questions, confusion, or even silence (Hanna, 2021; Ho et al., 2023). These reactions demand not only subject knowledge, but also a feel for how students learn. In the case of limits, this means knowing which misconceptions are common and which explanations can untangle them (Wild, 2023). For instance, some students might think a function must equal its limit at a point. Others might confuse proximity with exactness. A teacher's ability to recognize and respond to such misunderstandings is part of their pedagogical toolkit. This research explores how that toolkit differs between novice and experienced teachers when faced with the same content challenge.

There is a growing recognition that teaching is not just about what the teacher knows, but about how that knowledge is put into action (Rich et al., 2021; Santos & Castro, 2021). Two teachers might understand the definition of a limit equally well, but use it very differently in practice. One might rely on symbolic explanations; another might use visual examples. One might guide students through questions; another might lecture. These differences do not only reflect personal style—they reveal un derlying beliefs about learning and teaching. When examined closely, such choices reveal the operation of PCK (Handulle & and Vassenden, 2021; Y. Zhao et al., 2022). This study looks closely at those decisions to understand what experience adds (or doesn't add) to the process of instruction.

Another reason this study focuses on PCK is that it connects three crucial elements of teaching: the subject, the students, and the methods (Bagiyan et al., 2021; Bragg et al., 2021). Many frameworks focus only on content or pedagogy in isolation, but PCK requires their integration. This makes it especially useful for analyzing how teachers explain difficult topics like limits. A teacher might know the formal definition, but still struggle to explain it if they don't understand how students will interpret it. Conversely, a teacher might connect well with students but offer inaccurate explanations. PCK helps frame these dilemmas in useful ways. By comparing two teachers, this research offers a detailed picture of how that integration plays out in real instruction (Hui et al., 2022; Pellas et al., 2021).

Finally, this study is timely because both novice and experienced teachers often teach side by side in the same schools (Beck & Nunnaley, 2021; Jederud et al., 2022). Yet little is known about how their practices actually differ when it comes to teaching advanced mathematical topics. Understanding these differences could inform mentoring practices and professional development design. It could also help curriculum planners design materials that support teachers at different career stages. Beyond these practical implications, the study also contributes to a deeper theoretical understanding of teacher expertise (Lei & Medwell, 2021; McPhail, 2021). Rather than assuming that more experience equals better teaching, it asks what kind of knowledge makes a difference. That makes the findings potentially useful for both research and practice. It also aligns with broader educational goals of improving instruction in complex topics that matter.

A variety of researchers have highlighted the complexities involved in mathematics instruction, especially when students face abstract material such as limits. Alemany-Arrebola et al. (2025) noted that cultural settings shape how learners perceive mathematical challenges, while Susada (2025) focused on psychological aspects that affect students' capacity to engage with formal content. Huang et al. (2025) discussed how different problem-solving tools can shape students' understanding, and González-Pérez et al. (2025) emphasized the connection between physical engagement and sustained attention in the classroom. Kim et al. (2022) proposed that mobile platforms using AI could strengthen student interaction with difficult topics. Insights from Búrigo (2025) showed how gender-sensitive strategies in teaching math benefit learners through structured explanation. Aba-Oli et al. (2025) synthesized multiple studies showing that interventions targeting higher-level thinking depend heavily on teachers' ability to adapt instruction. Tong et al. (2025) argued that solid math fundamentals support learning in other subjects, while Diaz Lema et al. (2025) linked student performance to consistency in instructional practice. Lastly, Shvartsberg (2025) reflected on long-term changes in teaching styles and their influence on learning, reinforcing the need to explore how experience shapes instructional decisions today.

Improving how complex mathematical ideas are taught requires more than just strong content knowledge, it also depends on how well that knowledge is transformed into lessons students can understand. Among such topics, the concept of limits is particularly demanding, both in terms of logic and language. It presents a challenge not only to students but also to teachers tasked with explaining its abstract nature in meaningful ways. Teachers must anticipate where students will struggle, decide on suitable representations, and choose when and how to introduce formal definitions. These decisions reflect the operation of Pedagogical Content Knowledge (PCK), which connects knowledge of content, instruction, and student thinking. Teaching experience is often assumed to deepen this kind of knowledge, but how it shapes actual classroom decisions remains unclear. Understanding these dynamics can offer insight into the development of instructional expertise. This study is grounded in the belief that by observing how PCK plays out in practice, especially in the teaching of limits, we can support more effective teacher development.

Despite increasing interest in the concept of PCK, there is still limited research that examines how teaching experience influences the way it is used in real classrooms. Many existing studies focus separately on content knowledge or teaching methods without investigating how they are connected during instruction. Comparisons between novice and experienced teachers often rely on general performance indicators, leaving out close analysis of how specific mathematical topics are taught. Furthermore, few studies examine how each dimension of PCK "such as content knowledge, pedagogy, and knowledge of students" interacts within a particular teaching context. Little attention has been paid to how teachers interpret and respond to student thinking in the moment, especially when teaching abstract content like limits. As a result, important differences in how teachers apply their knowledge may go unnoticed. By investigating how two teachers of different experience levels handle the same topic, this research offers a more focused and detailed view. It aims to fill a critical gap in understanding how teaching practice develops over time.

This research aims to explore how teachers at different career stages use Pedagogical Content Knowledge (PCK) when teaching the mathematical concept of limits. It focuses on identifying how subject knowledge, teaching strategies, and understanding of student thinking are combined and applied in practice. Through classroom observation and follow-up interviews, the study examines how teachers make decisions, respond to student ideas, and present abstract concepts. It compares the approaches of one experienced teacher and one novice teacher to see how their professional backgrounds influence instructional choices. Attention is given to how they introduce formal definitions, deal with student errors, and select tasks and examples. The goal is to understand which aspects of PCK are strengthened through experience and which require intentional development. Insights from this study may be useful for designing teacher training programs that focus not only on knowledge acquisition but also on its effective use in the classroom. Ultimately, the study aims to support efforts to improve the teaching and learning of advanced mathematical ideas in secondary education.

METHOD

Research Design

This research adopted a qualitative exploratory design aimed at capturing the complex nature of pedagogical decision-making in mathematics instruction. Specifically, the study sought to investigate how Pedagogical Content Knowledge (PCK) manifests in the classroom practice of teachers with differing levels of professional experience. By focusing on the teaching of limits "a concept known for its abstract nature and logical precision" the study explored how teachers translate formal mathematical ideas into instructional strategies accessible to students. The design was interpretive in nature, emphasizing the subjective reasoning behind teachers' actions and how their knowledge domains (content, pedagogy, and student understanding) interact during instruction. Rather than seeking statistical generalizations, this study emphasized in-depth, context-rich analysis through a case-based approach. The design also allowed for flexibility, enabling the researcher to trace how teaching experience may influence instructional adaptation, strategy selection, and responsiveness to student thinking. This approach provided a robust framework for analyzing the enactment of PCK in a real classroom environment, where theory and practice intersect.

Participants

The study involved two secondary school mathematics teachers selected through purposive sampling from an initial group of ten observed educators. The selected participants represented two contrasting stages in their professional careers: one experienced teacher with over fifteen years of service and one novice teacher with seven years of experience. Both participants held equivalent academic qualifications in mathematics education and taught the same grade level, subject, and curriculum within the same school. This ensured alignment in content coverage and student demographics, thereby reducing confounding contextual variables. Gender similarity and voluntary participation were additional considerations to foster trust and open communication during interviews and classroom visits. The classification of teaching experience was aligned with national professional standards, which categorize teacher ranks based on years of service and functional status. By selecting participants with distinct career trajectories but similar contextual conditions, the study aimed to isolate how experience itself contributes to variations in PCK application. This selection strategy supported the comparative purpose of the study while maintaining analytical depth.

Instrument

Multiple data collection instruments were employed to support triangulation and ensure the credibility of findings. The primary research tool was the researcher as observer and interpreter, complemented by four validated instruments: (1) the Limit Problem-Solving Task (LPST), used to assess teachers' conceptual and procedural understanding of limits; (2) Student Work Results (SWR), which included student responses containing common misconceptions in limit problems; (3) a structured Teacher Activity Observation Sheet (TAOS) to systematically document classroom interactions; and (4) semi-structured interview guides that explored teachers' planning processes, instructional choices, and diagnostic reasoning regarding student thinking. All instruments were reviewed and validated by two independent experts in mathematics education and qualitative research methodology. A pilot phase was conducted to refine instrument clarity and functionality. These instruments were used consistently for both participants to maintain procedural parity. Together, they offered a multi-faceted lens for analyzing how each component of PCK was demonstrated, adapted, or limited in actual teaching practice.

Data Analysis

The analysis process involved a systematic yet flexible approach grounded in thematic analysis. Data from classroom videos, teacher interviews, and task-based assessments were transcribed and coded using an iterative process that combined inductive theme generation with deductive mapping against the established PCK framework. Open coding was used initially to identify meaningful instructional patterns, which were then refined into categories corresponding to Knowledge of Subject Matter (KSM), Knowledge of Pedagogy (KP), and Knowledge of Students (KS). Triangulation was performed by cross-referencing data across sources to strengthen interpretive accuracy. Member checks with the teachers and peer discussions with academic colleagues enhanced the confirmability and credibility of the findings. The analysis prioritized not only what teachers did, but also how they reasoned through their choices, especially in response to student difficulties and misconceptions. Ultimately, this approach allowed for a richly contextualized understanding of how experience may shape or constrain the practical use of PCK in teaching a cognitively demanding mathematical concept.

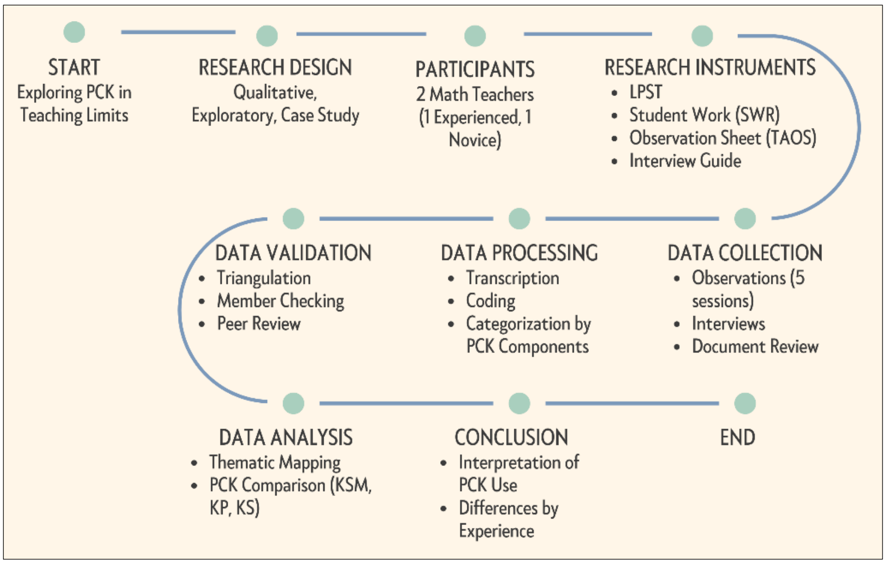

Figure 1. Exploring PCK in the Teaching of Mathematical Limits

The flowchart outlines the step-by-step procedure of this study, starting from research design to conclusion. It shows how participants were selected, data were collected using four instruments, and then processed through transcription and coding. Validation was ensured through triangulation and member checking. The final analysis focused on comparing PCK application between novice and experienced teachers in teaching mathematical limits.

RESULTS AND DISCUSSION

Results

Teachers must possess a variety of knowledge and skills to plan and implement instructional strategies that promote students' understanding and learning. However, in addition to strong content knowledge, pedagogical understanding, familiarity with curriculum, and knowledge of students, teachers must also be capable of effectively applying this knowledge in classroom instruction. In this study, Pedagogical Content Knowledge (PCK) is analyzed in terms of three main components: Knowledge of Subject Matter (KSM), Knowledge of Pedagogy (KP), and Knowledge of Students (KS) in teaching the topic of limits of algebraic functions. The results for each component are presented below.

Knowledge of Subject Matter (KSM) of the Experienced Teacher

According to Kilic (2011), subject matter knowledge involves understanding mathematical facts and concepts and their relationships. This includes the ability to connect mathematical concepts and to explain the rationale behind specific mathematical procedures. In this study, subject matter knowledge was categorized into three types: factual knowledge, conceptual knowledge, and procedural knowledge, particularly in the context of limits of algebraic functions.

The experienced teacher's knowledge of algebraic function limits, in terms of factual knowledge, is demonstrated by their understanding of the meaning of specific facts and elements related to limits in accordance with their actual definitions. This is indicated by the teacher's comprehension of delta (δ) and epsilon (ε) in line with the notation $\forall\varepsilon > 0, \exists\ \delta > 0$, which means that for every epsilon taken arbitrarily, there exists a corresponding delta. Both ε (epsilon) and δ (delta) are greater than 0 because they represent distances, and therefore, they are always positive values. This indicates that the experienced teacher interprets the sentence with multiple quantifiers by first interpreting the universal quantifier (∀), followed by the existential quantifier (∃).

The experienced teacher understands the meaning of the notation $\lim_{x\to a} f(x) = L$ as: when x approaches a then $f(x)$ approaches L but $x \neq a$, They also understand that the notation $|x - a|$ represents the distance from x ke a, $|x - a| = |a - x|$. In addition, the experienced teacher understands that $|f(x) - L|$ represents the distance from$f(x)$ to L which is less than epsilon (ε) and that $|f(x) - L| = |L - f(x)|$. This shows that the experienced teacher does not distinguish between the meanings of $|x - a|$ and $|a - x|$, $|f(x) - L|$ and $|L - f(x)|$, because even when the terms are reversed, the meaning remains the same that is, they both indicate the distance between the points x and a, and between $f(x)$ and L.

The concept of the absolute value of a real number x refers to distance. As a result, absolute value can be used as a measure of the distance between two numbers (points) on the real number line. In the context of the limit of a function at a point, the inequality $0 < |x - a| < \delta$ implies that the distance from x to the point a is never equal to zero, which is equivalent to stating that $x \neq a$. The inequality $|f(x) - L| < \varepsilon$ means that the distance between the function value and the limit value is always less than the chosen ε. In addition, this inequality also implicitly allows for the possibility that $f(x) = L$.

In relation to mathematical content knowledge, Black (2008) classifies it into two categories: conceptual knowledge and procedural knowledge, including mathematical processes for using mathematics. The experienced teacher's Knowledge of Subject Matter (KSM) regarding limits of algebraic functions, specifically conceptual knowledge, is demonstrated by their understanding of concepts and the relationships between concepts in the context of limits. This conceptual understanding includes comprehension of the definition of the limit concept, relationships between concepts, the conditions and uses of formulas, and the categorization of examples and non-examples.

The experienced teacher's understanding of the definition of the limit concept is indicated by their grasp of the formal definition of a limit and their ability to represent it graphically. When representing the formal definition of a limit graphically, the experienced teacher begins by assuming that $f(x)$ is a linear function, with the assumption that as x approaches a, $f(x)$ approaches L, where L is the value of the function $f(x)$, and $f(x)$ does not necessarily have to be defined at $x = a$. First, x approaches a, but prior to that, the distance from x to a, denoted by δ, is determined based on a given ε. When point a is approached by points within the δ -neighborhood, the function values approach L.

The experienced teacher's interpretation of the graph indicates a connection between the formal definition of a limit and the intuitive meaning of limit notation. Through this connection, the teacher understands that the point a is approached first, in accordance with the intuitive meaning of limit notation. Before approaching the point a, the value of δ depending on ε is first determined. With this connection, the experienced teacher does not arbitrarily select points x to approach a, but chooses points within the interval $(a - \delta, a + \delta)$. When points x within this interval are chosen, the point L can be approached by $f(x)$, where $f(x)$ lies within the interval $(L - \varepsilon, L + \varepsilon)$.

The experienced teacher's understanding of the relationships between concepts is indicated by their comprehension of the logic of multiple quantifiers in the formal definition of a limit, specifically that delta depends on epsilon. This relationship between ε (epsilon) and δ (delta) is interpreted through the meaning $\forall\varepsilon > 0, \exists\delta > 0$. The experienced teacher understands $\forall\varepsilon > 0, \exists\delta > 0$ as "for every ε > 0, there exists a δ > 0. This interpretation forms the basis for the teacher's conclusion that δ depends on ε, since in the definition, ε is chosen arbitrarily, whereas δ can only be selected once ε has been specified.

With respect to conceptual knowledge, the experienced teacher could define the formal concept of limit, link concepts together (e.g., the dependency of δ on ε), categorize examples and non-examples of functions with and without limits, and relate formal definitions to graphical representations. They used logical reasoning to explain the implications in the definition of a limit, understood the order of quantifiers, and interpreted the logical structure involved in such definitions.

Regarding procedural knowledge, the experienced teacher understood how to prove a given limit using the ε-δ definition. They demonstrated the ability to derive appropriate values for δ for any given ε, and to logically explain and justify each step in the process. In solving limit problems, the experienced teacher correctly applied substitution and, when necessary, used algebraic techniques such as factoring or multiplying by conjugates to simplify expressions before reevaluating the limit.

Knowledge of Subject Matter (KSM) of the Novice Teacher

The novice teacher also demonstrated understanding in all three knowledge categories, although with notable differences from the experienced teacher, particularly in conceptual depth. In terms of factual knowledge, the novice teacher correctly interpreted the meaning of ε and δ, and their positive values as representing distances. They also interpreted limit notation correctly and did not distinguish between $| x - a |$ and $| a - x |$, recognizing their equivalence.

In conceptual knowledge, the novice teacher could define the formal limit concept and represent it graphically. They understood the intuition behind approaching a point and how this translates into formal definitions. However, a key difference was observed in the understanding of quantifiers: the novice teacher interpreted the expressions $\forall\epsilon > 0, \exists\delta > 0$ and $\exists\delta > 0, \forall\epsilon > 0$ as equivalent, which indicates a misunderstanding of the importance of quantifier order in mathematical logic. Although the novice teacher correctly understood logical implications and their truth values, their interpretation of quantified logic was less rigorous than that of the experienced teacher.

The novice teacher's conceptual knowledge related to implication logic is demonstrated by their understanding of the formal definition of a limit as an implication that involves sufficient and necessary conditions. The sufficient condition is $0 < |x - a| < \delta$, and the necessary condition is $0 < |f(x) - L| < \varepsilon$. he novice teacher understands that an implication is true when the sufficient condition is true and the necessary condition is also true, or when the sufficient condition is false and the necessary condition is true, or even when both the sufficient and necessary conditions are false. This indicates that the novice teacher has an accurate understanding of the logic of implication in the formal definition of a limit.

The novice teacher also demonstrated competence in categorizing examples and non-examples of functions with limits. They understood that a function has a limit at a point if the left-hand and right-hand limits exist and are equal, regardless of whether the function is defined at that point or is continuous there. In procedural knowledge, the novice teacher could correctly implement the ε-δ definition to verify a limit, though with slightly less precision than the experienced teacher. They also solved problems using intuitive approaches and systems of equations when determining unknown variables. The use of basic algebraic techniques to manipulate rational functions was also evident.

Discussion

This study aimed to explore how teaching experience influences the enactment of Pedagogical Content Knowledge (PCK) in the teaching of mathematical limits. The findings revealed that while both the novice and experienced teachers possessed formal knowledge of limits, only the experienced teacher demonstrated the ability to integrate that knowledge effectively with pedagogical strategies and student understanding. This confirms that PCK is not merely about knowing content, but how that knowledge is used in context to facilitate learning. One of the clearest distinctions was found in how each teacher approached the ε–δ definition of limits. The experienced teacher articulated the formal logic with precision, using the dependency between ε and δ to help students visualize what it means for a value to "approach" a point. This aligns with Alemany-Arrebola et al. (2025), who emphasized that experienced educators tend to link formalism with intuitive meaning, enabling students to grasp abstract mathematical concepts through guided reasoning. By contrast, the novice teacher's explanation of ε–δ was limited to symbolic manipulation, without connecting the definition to its conceptual purpose.

Although the novice could apply the steps correctly, their lack of emphasis on logical structure made it harder for students to internalize the reasoning behind the procedure. Susada (2025) found that less experienced teachers often default to formulaic teaching methods that can hinder deep understanding, particularly in topics requiring abstract thinking. Representation played a central role in how the teachers communicated concepts. The experienced teacher used diagrams, tables, and real-world analogies alongside symbolic notation to help students interpret limits visually and contextually. This reflects González-Pérez et al. (2025), who noted that skilled use of multiple representations allows students to shift between different ways of thinking and supports conceptual understanding across varied learners. In contrast, the novice teacher relied almost exclusively on algebraic expressions. The absence of visual scaffolds and contextual explanations limited student engagement and likely reinforced the idea that mathematics is only about following steps. Lee (2025) argues that representational flexibility is a core dimension of PCK that emerges through experience and reflection, and its absence in novice practice often results in narrower learning experiences.

Pedagogically, the experienced teacher created opportunities for student exploration and discussion. Questions were used not just to check answers, but to elicit reasoning and promote reflection. Students were encouraged to explain their thought processes, identify patterns, and even critique each other's strategies. This aligns with Búrigo (2025), who advocates for dialogic pedagogy in mathematics classrooms to support deeper reasoning and equity of participation. The novice teacher, though organized and structured, delivered content with limited interactivity. Lessons were teacher-led, and student responses were often limited to single-word answers or brief confirmations. This rigid approach to classroom talks reflects what Aba-Oli et al. (2025) described as procedural pedagogy, where instruction is dominated by teacher exposition with minimal cognitive engagement from students. Student misconceptions also provided an important lens through which the two teachers differed. The experienced teacher recognized four distinct types of misconceptions and addressed them using tailored interventions, such as rephrasing questions or using alternate representations. Tong et al. (2025) noted that recognizing the diversity of student thinking is a key trait of expert teachers and is crucial in making abstract mathematics accessible. On the other hand, the novice teacher identified only two error types and often attributed mistakes to memorization issues rather than conceptual gaps. The responses typically involved repeating the same explanation, which did not always help students revise their thinking. Huang et al. (2025) warned that without a deep understanding of how students construct knowledge, teachers may misinterpret errors as careless mistakes instead of opportunities to revisit fundamental ideas. Assessment was another area where differences emerged.

The experienced teacher used formative assessment informally throughout the lesson to gauge understanding and adjust the pace accordingly. Students were asked to justify their reasoning or apply concepts in novel contexts, reinforcing metacognitive skills. Diaz Lema et al. (2025) described this practice as pedagogical coherence, where content, method, and student insight are consistently aligned throughout instruction. In contrast, the novice teacher assessed understanding through accuracy on practice problems. Feedback was minimal and mainly corrective. While such methods can be effective for procedural fluency, they often fall short in revealing how well students grasp underlying concepts. This reinforces Shvartsberg's (2025) findings, which suggest that novice teachers are more likely to equate correctness with comprehension and overlook deeper misconceptions. Classroom flexibility also distinguished the two. The experienced teacher modified the sequence of content based on student responses, paused to clarify misunderstandings, and adjusted groupings to support collaboration. These real-time decisions reflect instructional agility, which Alemany-Arrebola et al. (2025) linked to long-term exposure to varied teaching contexts and continuous pedagogical refinement.

The novice teacher followed a predetermined plan with little deviation, even when student confusion became evident. There was hesitation to pause or reframe content differently, likely due to limited experience in managing spontaneous instructional shifts. As Búrigo (2025) emphasized, such rigidity can lead to missed learning opportunities, especially in topics that benefit from dynamic exploration. Both teachers demonstrated commitment and planning, but only one succeeded in converting planning into responsive and student-centered learning. The experienced teacher's ability to integrate subject matter knowledge with adaptive pedagogy and student thinking exemplifies what Shvartsberg (2025) described as expert PCK in action—a fluid, reflective, and intentional practice. This study affirms that while teacher education may provide foundational knowledge, the ability to enact that knowledge meaningfully develops over time through reflection, classroom trial, and dialog with peers. As González-Pérez et al. (2025) argued, professional development should prioritize not only content mastery but also pedagogical fluency and understanding of student cognition In summary, teaching experience influenced how well PCK was integrated in the teaching of mathematical limits. The experienced teacher was able to anticipate challenges, connect representations, and respond flexibly to student needs, while the novice was more rigid and procedural. These findings echo the perspectives of Diaz Lema et al. (2025), who advocates for ongoing, experience-based refinement of PCK as essential to quality teaching in mathematics education.

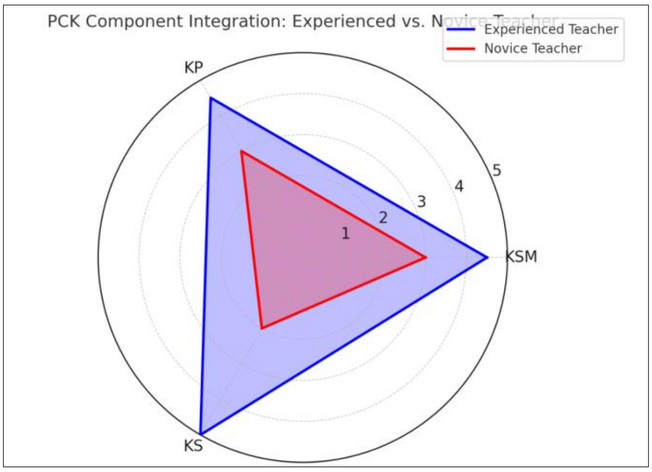

Figure 2. PCK Component Integration: Experienced vs. Novice Teacher

This radar chart shows that the experienced teacher demonstrated higher levels of integration across Knowledge of Subject Matter (KSM), Pedagogy (KP), and Students (KS), compared to the novice teacher. The gap is especially notable in how student misconceptions were diagnosed and addressed.

Implications

The insights generated from this study underscore the vital role of experience in shaping how teachers apply Pedagogical Content Knowledge in real classroom scenarios, particularly when introducing abstract topics like limits. Beyond mastering content, effective instruction hinges on a teacher's ability to interpret student thinking and respond pedagogically in real time. This highlights the need for teacher education programs to emphasize more than theoretical knowledge. Structured mentorship, reflective dialogue, and situated learning opportunities should be prioritized to help novice educators build responsive teaching habits. In particular, the evidence suggests that training should focus on helping teachers use varied representations, recognize misconceptions, and design cognitively engaging tasks. Such skills are not only pedagogical tools—they are enablers of equity, enabling all students to access and engage with challenging mathematical ideas. In this way, teacher growth contributes directly to more inclusive and effective learning environments.

Limitations

While the study offers valuable findings, certain limitations must be acknowledged. The investigation centered on only two teachers, which limits the extent to which conclusions can be generalized. Their teaching contexts, including student profiles and institutional cultures, may have influenced their approaches in ways that were not fully documented. Additionally, the research relied on a snapshot of classroom interactions rather than longitudinal observation, which might not capture the full scope of each teacher's instructional repertoire. The interpretations, though grounded in multiple data sources, still carry the potential for researcher bias during analysis. Moreover, the study focused on teacher practice without directly linking it to measurable changes in student learning. As such, while the enactment of PCK was clearly differentiated, its concrete impact on student outcomes remains inferred rather than empirically tested.

Suggestions

Building on the current findings, future research could broaden the participant pool to include teachers from diverse schools and backgrounds, allowing patterns of PCK development to be compared across settings. Investigations that track changes in teaching over time (particularly in relation to professional development or mentorship) would deepen our understanding of how PCK evolves. Furthermore, research should incorporate data from students to better assess how different teaching approaches affect learning. Studies could also explore how teachers' beliefs and institutional structures interact with PCK enactment. From a practical standpoint, teacher training institutions and school leaders are encouraged to provide structured opportunities for lesson study, peer observation, and collaborative planning. These practices can help novice teachers make sense of their experiences and transition toward more adaptive, reflective instruction. Such investments in teacher growth ultimately contribute to more coherent and responsive mathematics education.

CONCLUSION

This study highlights how teaching experience shapes the practical integration of Pedagogical Content Knowledge (PCK) in mathematics instruction, particularly when dealing with abstract topics like limits. The experienced teacher demonstrated greater flexibility, deeper conceptual framing, and stronger responsiveness to student thinking, in contrast to the novice teacher who tended to rely on procedural explanations and fixed instructional plans. These findings suggest that effective teaching requires more than subject expertise, it involves understanding how students think, recognizing misconceptions, and adjusting strategies in real time. As such, PCK should be viewed not as a fixed skill set, but as a dynamic, evolving competence that is cultivated through sustained reflection, classroom practice, and professional learning. Supporting novice teachers in this developmental journey is essential to fostering meaningful, inclusive, and conceptually rich learning experiences in mathematics.

AUTHOR CONTRIBUTIONS STATEMENT

Ma'rufi conceptualized the research, designed the study framework, led classroom observations, and developed the analytical framework of Pedagogical Content Knowledge (PCK). Served as the primary author in drafting the manuscript.

Muhammad Ilyas collected field data, including classroom observations and interviews, and carried out initial transcriptions. Contributed to qualitative data analysis and the preparation of the methodology section.

Salwah assisted in the validation of research instruments (LPST, SWR, TAOS, and interview guides), conducted data triangulation, and contributed to the writing of the results and discussion sections.

Nur Wahidin Ashari provided academic supervision and conceptual input on the theoretical framework of PCK, contributed to the review of recent international literature, and revised the manuscript to strengthen scholarly quality and alignment with previous research.

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