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Comparison between concept-based and procedure-based in circle theorems

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Abstract

Background: The debate between concept-based instruction and procedure-based approaches to improving students' performance, understanding, and skill development in Circle Theorems cannot be over-emphasized. The researchers employed the non-equivalent quasi-experimental design to investigate the effectiveness and appropriateness of the two methods, using circle theorems.
Aim: This study aims to compare the effectiveness of concept-based and procedure-based instructional approaches in the teaching of Circle Theorems among senior high school students.
Method: A quasi-experimental design was adopted involving 70 students selected from two purposively sampled schools. One school was assigned as the experimental group (concept-based instruction) and the other as the control group (procedure-based instruction). Geometry achievement tests were administered as pre-tests and post-tests. Data were analyzed using paired sample t-tests, independent sample t-tests, and effect size calculations with a significance level set at 5%.
Result: The findings showed statistically significant differences between the experimental and control groups. Students taught using concept-based instruction performed significantly better than those taught through procedure-based instruction. High effect sizes further supported the superiority of the concept-based approach.
Conclusion: Concept-based instruction enhances students' understanding and performance in Circle Theorems. It is recommended that mathematics educators adopt teaching methods that promote conceptual understanding and active knowledge construction over algorithmic procedures.

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INTRODUCTION

Procedural knowledge is the step-by-step, sequential understanding of symbols, arithmetic operations, and procedures used to solve mathematical tasks. In contrast, conceptual knowledge involves the meaning and understanding of mathematical concepts, procedures, rules, and algorithms (Engelbrecht et al., 2017). The instructional strategies adopted by teachers greatly influence students' comprehension and development in mathematics. Bashiru and Nyarko (2019) emphasized that meaningful learning strongly depends on the teacher's selection of instructional activities and strategies.

Ntow and Hissan (2021) argue that concept-based training is becoming increasingly relevant due to the demands of a rapidly evolving technological society, where learners are expected to possess not only procedural fluency but also a deep understanding of mathematical concepts. This instructional approach shifts away from rote memorization and instead emphasizes active learning processes that enable students to make sense of mathematical ideas. Concept-based instruction is fundamentally grounded in pedagogies such as inquiry-based learning, problem-based learning, problem-solving activities, and self-construction of knowledge, all of which are designed to foster learners' critical thinking and independence (Biney & Ali, 2023; Mensah-Wonkyi & Adu, 2016). These

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methods encourage students to explore, question, and discover underlying principles through guided tasks and contextual situations, making the learning process more meaningful and enduring. Furthermore, several empirical studies have affirmed that concept-based instruction not only promotes deep comprehension of mathematical content but also plays a crucial role in reducing mathematics anxiety among students—an emotional barrier that often hinders achievement (Joung & Kim, 2022; Khoule et al., 2017). By engaging students cognitively and emotionally, concept-based learning creates a supportive environment where learners feel empowered to take ownership of their mathematical understanding.

Although conceptual instruction may demand more time, the conventional procedural approach often leads to students' struggles in understanding mathematical concepts. Studies have shown that teachers often fail to introduce the concept of Circle Theorems adequately (Abreh et al., 2018; Bashiru & Nyarko, 2019; Hakim & Yasmadi, 2021; Setiawan & Sunardi, 2023; Tay & Wonkyi, 2018). Findings from international assessments such as TIMSS indicate persistently low mathematics achievement (Fletcher, 2018). The procedural method—typically lecture-based and teacher-centered—has been linked to students' poor performance in standardized mathematics exams, including the West African Senior Secondary Certificate Examination (WASSCE) (Abreh et al., 2018).

In particular, Circle Theorems remain difficult for most senior high school students, who find them abstract and mechanical (Boson-Amedenu, 2017). The WAEC Chief Examiner's report highlighted that many students failed to attempt Circle Theorem questions, and those who did often demonstrated poor conceptual understanding and application (WAEC, 2021). This is concerning given that Circle Theorems are interconnected with many mathematics topics—such as quadratic equations, movement geometry, constructions, plane geometry, and solid geometry (Abreh et al., 2018; Ali, 2022). These theorems are also essential in practical contexts, including calculating pi, areas of circular shapes, equations of circles, volumes, conic sections, and digital geometry tools (Erebakyere & Agye, 2022).

Conceptual knowledge is characterized by a network of interconnected ideas where the relationships between facts are as critical as the facts themselves (Rittle-Johnson & Siegler, 2022). Hakim and Yasmadi (2021) noted that conceptual knowledge provides the structure and meaning behind mathematical methods. Geary et al. (2018) explained that students demonstrate conceptual understanding when they can classify, create, and recognize mathematical instances. This knowledge can be developed through inquiry and exploration (Ali, 2023; Hechter et al., 2022). Visualization and experiential learning through diagrams, algorithms, and planning—help bridge abstract concepts and real understanding (Hurrell, 2021; Hussein & Csíkos, 2023).

On the other hand, procedural knowledge relies on symbols, rules, and algorithms (Hurrell, 2021; Rittle-Johnson & Siegler, 2022). Learners follow a set of steps often without understanding the underlying principles. This can limit reasoning and result in mechanical application (Yu at al., 2018). Hakim et al. (2021) found that many students rely on instrumental understanding, often generating their own incorrect rules and explanations. Similarly, Asilo-Ebisa and Lomibao (2024) discovered that students struggle to apply knowledge in new contexts if they only rely on memorized procedures.

Although extensive literature exists, many prior studies lack methodological rigor and fail to provide a comprehensive understanding of Circle Theorems and related misconceptions. Therefore, this study aims to bridge theoretical, methodological, and practical gaps by comparing the effectiveness of conceptual-based and procedural-based instructional strategies in improving students' understanding and performance in Circle Theorems.

METHOD

Research Design

A quasi-experimental non-equivalent (pre-test post-test) design was used. This study followed a quasi-experimental design and is known for its ability to withstand maturation, history, and pre-testing (Gopalan et al., 2020). The research used quantitative methods by taking the population of two senior high school students. Gopalan et al. (2020) encourage the use of this design when dealing with intact classes. Concept-based method of teaching was applied to the Experimental Group (EG) while the Control Group (CG) received procedural method instruction. Consequently, the study used two categorical independent variables and one dependent variable. The independent variables were the two instructional learning approaches used in teaching the Circle Theorems. The dependent variable was students' performance in the Circle Theorems.

Research questions sought to determine whether or not there were substantial improvements in the performance of students taught circle theorem with the concept-based method and those taught with the traditional method were answered using the pre-test and post-test scores obtained from students in both groups. The students threw more light on the perception of the concept-based method of instruction (Asilo-Ebisa & Lomibao, 2024).

Table 1. Design of treatment

Groups	Pre-test	Treatment	Post-test
Experimental	Pretest 1	Concept-based	Post Test 1
Control	Pretest 2	Procedural-based	Post Test 2

Table 1, the concept-based teaching approach was applied to the treatment group while the CG received regular classroom instruction. Accordingly, two categorical independent variables and one dependent variable were used. The independent variables (concept-based and procedural-based methods) cause or influence an outcome. The dependent variable or outcome variable of this study was improving students' performance through concept-based instruction that will enhance student understanding and skill development in the circle theorem. The pre-test and post-test were conducted to compare students' entry-level achievement and treatment effect respectively.

The study was conducted in a classroom, intact classes were used to avoid disrupting the school programme. The sample size for each group was dependent on the number of students in a particular class at the time the interventions were carried out. Only one teacher was recruited to participate in the study. The teacher was allocated to the CG to play the role of preserving traditional learning conditions in the traditional teaching method. Comparatively, learners in the experimental and control schools were taught by the researchers who employed the concept-based instruction approach (CBI).

Participant

Out of the 78 participants who agreed to take part in the study, 70(89.7%) of them were considered to have participated fully in the study, that is, they wrote both the pre-test and the post-test. Full participation in the study meant that the participant (learner):

- was able to attend all teaching sessions
- was able to participate in teaching tasks
- participated in both achievement tests at pre- and post-stages
- was in either CG or EG groups
- was subjected to the lesson observations that also characterized this research.

Almost all 70(89.7%) learners met the four requirements; hence they were chosen fully as participants in the study. To monitor the attendance of participants (learners) in both the EG and CG, the researchers kept records of the participants' daily attendance. Using these monitoring tools, it was possible to track down participants who did not participate in all research activities of the study. For instance, Data from participants who did not participate fully in the study from both the CG and the EG were discarded and not analyzed. The data analysis that is presented in this report covered only that of the 70 learners: EG (35); and CG (35) who participated fully in the study. In total, 8(10.3%) participants did not participate fully in the study.

The researchers employed the simple random sampling (lottery) technique. Out of 500, a sample of 70 participants; consisted of 35 participants from School 'A' and 35 participants from School 'B' for the CG and EG groups respectively. This sample size was made up of 23 males and 12 females for the CG) and (22 males and 13 females for the EG). It is worthy of note that larger samples lead to smaller sampling errors. This means that sample values will be closer to the true population values (Biney & Ali, 2023). Most importantly, the goal was to select a sample that is likely to be "information-rich" concerning the anticipated outcomes of the study. The 78 students were selected to ensure that the mean of the sample was representative of the population mean. The sample helped the researcher discover, gain insight, and understand the problem of the students in the circle theorem (Ntow & Hissan, 2021).

Instrument

The data collection instruments were self-designed pre-test and post-tests based on the properties of Circle Theorems. The research instrument consisted of geometry achievement test (GAT) items. The geometry achievement test items were constructed mainly on the properties of the circle theorem and based on the learning objectives. Some of the items were constructed by the researchers to ensure that the items were within the acceptable content and context. The test contained 25 questions of which 20 were objectives and 5 were subjective. In selecting the questions, each item selected had to pass through: expert criticism, item difficulty, and item discrimination analyses. Both the pre-and post-intervention test items were similar in terms of item type and difficulty levels to ensure an accurate comparison (Bashiru & Nyarko, 2019).

The essence of the pre-intervention test was to find out whether the two groups were similar in geometry abilities before the treatment, and this provided a baseline for the learning of the topic (circle theorem). Conversely, the post-test was aimed at finding out the performance of students after the treatment. Both the pre-and post-intervention test items were similar to ensure an accurate comparison. The reliability of the test instrument was established using Pearson Product Moment Correlation. To check the inter-rater reliability of the test since the test was open-ended, the students' answers were rated by two different scorers who have several years of experience marking national examinations and following moderation. The result of reliability was 0.996 between scorer 1 and scorer 2. The results revealed a significant relationship between the scores of the two raters, hence the test was reliable.

Data Analysis

The study began with the administration of a pre-test which was an achievement test to both groups (experimental and control) and also administered a post-test after the interventions. The test lasted for two hours and a double period of Mathematics was used for this purpose. The researchers administered the test in the experimental school, while the teacher administered the test in the control school. To ensure that conditions remained similar for both groups, the researchers met with the teacher before the test. In addition, the teacher was requested to start and end the test on time and to encourage learners to be on time for the test. The teacher was asked to invigilate honestly and credibly and to remain at the invigilation station during the test. The teacher was also reminded not to provide any assistance to learners while they were writing the test. These precautions ensured that test conditions were fairly similar in the two schools.

In addition, a semi-structured interview guide was administered to some students in the EG to explore their experiences in the concept-based instruction learning environment. This was done to provide a deeper understanding of data collected through the intervention test. Ten (10) students who took part in the conceptual lesson (experimental group) were randomly selected and interviewed individually by the second researcher after the post-intervention test. The data were analyzed using descriptive and associated inferential statistics. In particular, the quantitative data collected from the achievement tests were analyzed using the paired sample t-test, and independent sample t-test analyses.

Treatments of the study

The EG was taught using concept-based instruction whereas the CG received the procedure-based instruction. In the CG, definitions and theorems, rules, and algorithms were presented as a series of steps consistent with our conceptual framework. The main focus of teaching in the CG was on solving as many problems as possible to reach a correct answer using mainly teacher demonstrations of solved questions and practice questions for students. In the traditional approach, methods such as demonstration and illustration using examples, questions and answers method, and lecture approach were used. For example, in introducing the concept of the circle theorem, instruction begins by presenting definitions, theorems, and algorithms on the various angle properties of the circle theorem, and working through examples on how to calculate the angle properties of the circle.

In the control group, students were given nine circle theorems. They were helped to state the theorem correctly, draw diagrams of representations, and perform computations. They then proved each of the theorems based on the lessons and undertook written tasks. This was to help the students recall, remember, and use them to solve circle problems. In the experimental group, students were also given the same theorems. They engaged in hands-on activities by constructing, measuring, and investigating angles formed and possible relationships using mathematical set instruments such as the protractor, compass rule, etc. This was done to help students discover the various properties and theorems of circle concepts by themselves such as alternative segment theorem, angles in a cyclic quadrilateral tangent and radius theorem, and so on. The idea was to conceptualize the concept of cyclic quadrilateral by realizing that angles in opposite segments are supplementary. The concept-based instruction lesson in the EG focused on verifying and justifying each step of the procedure, hands-on activities investigations in an experiential way.

Circle Theorems

The following were the nine theorems for both the experimental and control groups.

Property 1 : *The angles a chord or arc subtends at the circumference in the same segment of a circle are equal.*

Property 2 : *The angle a chord subtends at the Centre of a circle is twice the angle it subtends at the circle's circumference.*

Property 3 : *The sum of the angles a chord or an arc subtends at the circumference of the opposite segment of a circle is equal to* $180°$.

Property 4 : *The angle the diameter of a circle subtends at the circumference is equal to* $90°$.

Property 5
a. *Equal chords or arcs subtend the same angles at the circumference of a circle.*
b. *Equal chords or arcs subtend the same angles at the center.*

Property 6 *Angle between the tangent to a circle and the radius of the circle is equal to* 90°.
Property 7 *: The opposite of a cyclic quadrilateral is supplementary (they add up to* 180°*).*
Property 8 *: Two tangents to a circle from the same point are equal in equal.*
Property 9 *: The angle between a chord and a tangent through one of the endpoints of the chord is equal to the angle in the alternate segment.*

Validity and reliability

Reliability refers to the uniformity, evenness, and consistency of a measure concerning time and even across varied researchers (Abubakar et al., 2018). A measure or research procedure is described as reliable if when conducted by different researchers yields the same outcome. Findings in this research were compared to already existing findings from previous researchers to check their conformity or otherwise.

Validity is defined as the restructuring of test items to ensure their measurement does not deviate from the sole objective of a study (Abubakar et al., 2018). The data gathering tool used (GAT) was drafted and presented to the Head of the Mathematics Department in both schools and the research supervisor for thorough checks before field data gathering took place.

Internal threats to validity

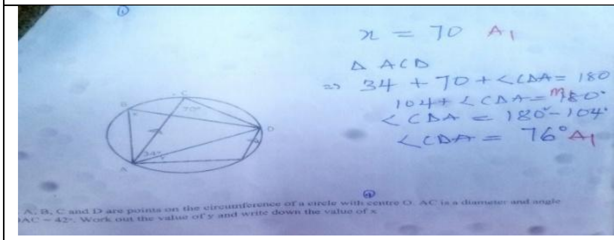
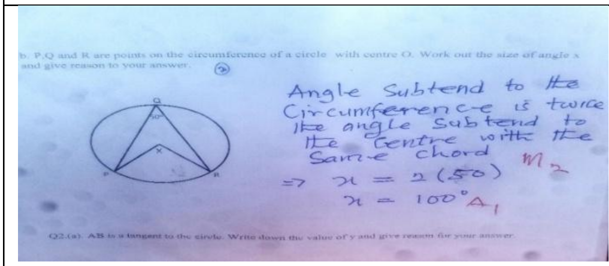
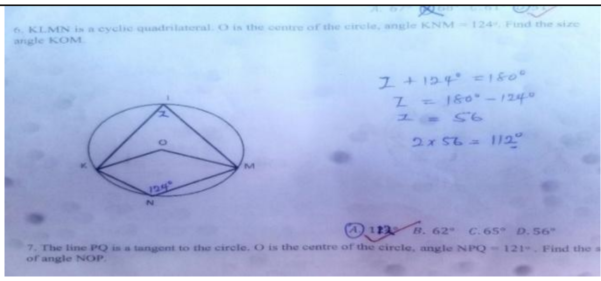
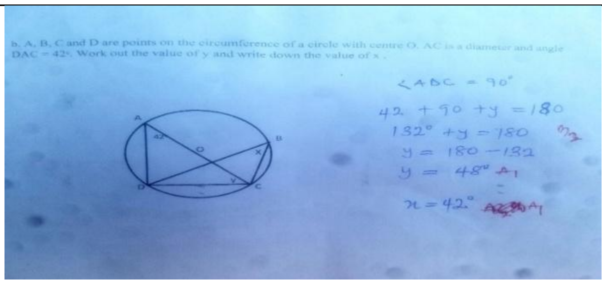
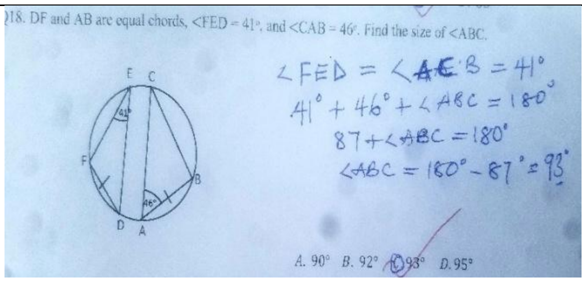
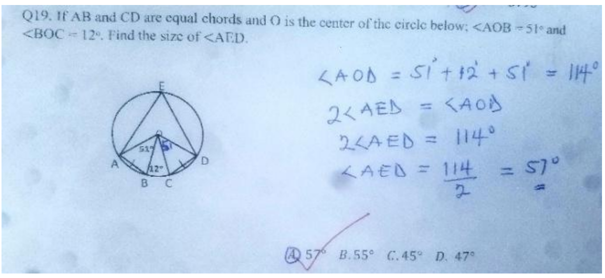
Internal validity refers to the extent to which the researcher would be certain that the findings of the research were solely due to the comparative effects of the concept-based instruction method, which characterized instruction in the experimental group, and also due to the traditional teaching method employed in the CG. Possible threats to internal validity such as diffusion of intervention or contamination, experimenter or researcher effects were controlled. Contamination can occur when learners in different groups talk to each other or borrow each other's study tools (Ugras, 2019).
In addition, contamination could have threatened the internal validity of the study when the CG interacted with the EG that was exposed to the intervention. Howe et al. (2019) argue that contamination can reduce the "*statistical significance and precision of effect estimate*" needed to make a statistical conclusion that the observed difference between two groups is due to only the intervention.

Researcher effect which refers to how the deliberate or unintended effect of the researcher can influence the learners' responses in the post-test was controlled. To do this, the lessons were carefully planned and the researcher ensured that the instructions were strictly limited to the lesson plans. This was to ensure that the researcher was not tempted to teach any aspects of the questions in the achievement tests. Again, the question papers were collected from the learners immediately after writing the pre-test. This was done to prevent learners from discussing the questions in the pre-test and possibly asking the researcher for an explanation of certain questions in the test during the lesson.

RESULTS AND DISCUSSION

This section presents the results of a quasi-experimental study. The results compared the Concept-Based Instruction (CBI) and Procedural-Based Instruction (PBI) in the circle theorem. The use of independent t-tests and paired sampled t-tests as inferential statistics procedures to analyze the quantitative data. The qualitative methods of analysis are used to analyze data obtained from the lesson observations.

Table 2. Students' tasks and their Errors in Circle Theorems

Task	Theorem	Error
	1. The angles a chord or arc subtends at the circumference in the same segment of a circle are equal	Some learners identified that the angles formed on the same side of a chord were equal.
	2. The angle a chord subtends at the Centre of a circle is twice the angle it subtends at the circumference of the circle	Some learners were considering the same segment angle theorem instead of the inscribed angle theorem.
	3. The sum of the angles a chord or an arc subtends at the circumference of an opposite segment of a circle is equal to 180°.	Here some learners were using theorem 2 instead of finding the sum of the angles in opposite segments to the circumference of a circle.
	4. The angle the diameter of a circle subtends at the circumference is equal to 90°.	Few learners assume that chords that almost form diameter when subtends to the circumference form an angle of 90 degrees.
	5. a. Equal chords or arcs subtend the same angles at the circumference of a circle.	Some learners do not understand the mathematical language or sign used herein identifying the same or equal chords.
	b. Equal chords or arcs subtend the same angles at the circle's center.	

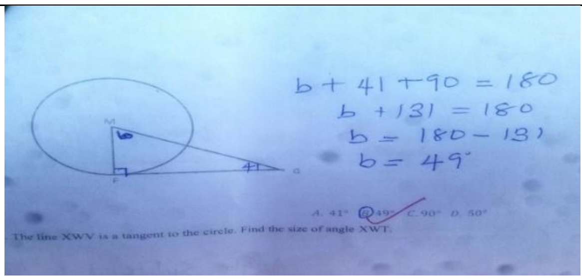
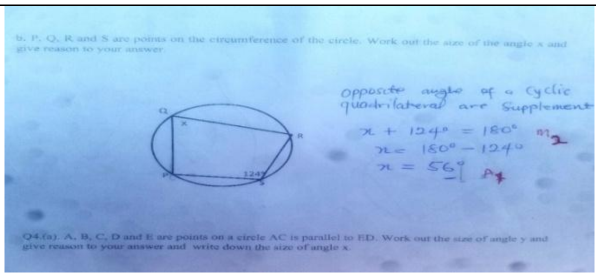
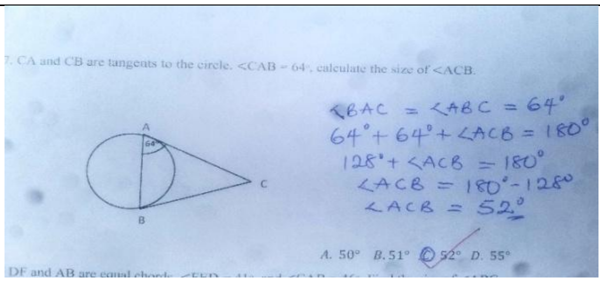
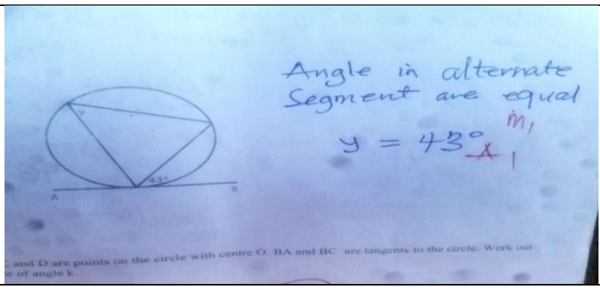
	6. The angle between the tangent to a circle and the radius of the circle is equal to 90°.	Some learners couldn't identify the angle at which the radius and the tangent meet.
	7. The opposite of a cyclic quadrilateral is supplementary (they add up to 180°)	Some learners assumed that a quadrilateral lying inside a circle is a cyclic quadrilateral which it is not.
	8. Two tangents to a circle from the same point are equal in equal.	Learners were using theorem 6 not knowing that it only applies to the point where the radius or diameter and tangent meet.
	9. The angle between a chord and a tangent through one of the endpoints of the chord is equal to the angle in the alternate segment.	Some learners were finding angles on either side of the chord thinking that they were equal in alternate segments.

Table 2, the nine errors emanated from the outcomes of procedural-based instruction. The students perceived the Theorems as independent of the conception behind them. Theorem 8 was a typical example. As they perceived two tangents from the same point as equal, they never conceived that it must be accompanied by the radius and diameter. After having taken through the conceptual-based instruction, the students significantly improved in performance, understanding, and skill development in the Circle Theorem. They detected the errors and improved their performance. The results in Tables 3 to 8 support this assertion.

Table 3. Descriptive statistics for the experimental group

Test	Mean	N	S.D	SEM	Correlation	P-value	Effect size
Pre-test	25.80	35	7.31	1.24	0.63	0.000	2.20
Post-test	44.63	35	9.68	1.64			2.20

Table 3 shows that the pre-test mean was 25.8 with a standard deviation of 7.31. The post-test mean was 44.63 and the standard deviation was 9.68. A paired sample performed indicated that 35 learners took the pre-test and post-test. The correlation between the pre-test and post-test scores was 0.63 with an associated probability of 0.000 and an effect size of 2.20. This result suggests that the correlation was significant. Therefore, it is reasonable to conclude that there was a moderate linear relationship between the pre-test and post-test scores.

Table Error! No text of specified style in document.. Independent t-test for the experimental group

Test	Mean	N	S.D	SEM	T	df	P-value	Effect size
Pre-test Post-test	35	18.8	2.37	0,4	-14.63	34	0.000	7.93

The result of the paired sample test in Table 4 indicated that the pair differences between the pre-test and post-test scores was a mean is 18.8. This means that the use of the concept-based teaching method potentially improves learners' understanding and skills in the circle theorem. The results show that there was a gain of 18.8 points in the mean scores as a result of using concept-based instruction in the experimental group.

Given these observations, it is, therefore, reasonable to conclude that there were statistically significant improvements in learners' performance, understanding, and skills development in circle theorem, from 25.8 ± 7.31 to 44.63 ± 9.68 ($p < 0.05$), following the implementation of a concept-based instruction in the experimental group. However, with a high effect size of 7.93, the improvement of learners' tendency in performance amounted to 18 ± 2.37. Given that $p < 0.05$ it was reasonable to reject hypothesis one. Therefore, it can be concluded that concept-based instruction is effective in improving the performance, understanding, and skills development of learners in the circle theorem.

Table 5. Descriptive statistics for the Control Group

Test	Mean	N	SD	SEM	Correlation	P-value	Effect Size
Pre-test	24.14	35	6.97	1.18	0.41	0.376	0.91
Post-test	30.46	35	6.79	1.15			0.91

Table 5 shows that the pre-test mean was 24.14 with Standard Deviation 6.97. The post-test mean was 30.46 (SD=6.79). A paired sample conducted showed that 35 learners took the pre-test and post-test. The correlation between the pre-test and post-test scores was 0.41 with an associated p-value of 0.376 and a high effect size of 0.91. This result indicated that the correlation was positive but weak significant. Given these observations, the study therefore concludes that even though there was a positive weak linear relationship between the pre-test and post-test scores for the CG, indicating a little or low improvement in the scores as a result of the traditional teaching method employed in the CG.

Table 6. Independent t-test for the Control Group

Test	N	Mean	SD	SEM	T	df	P-value	Effect Size
Pre-test – Post-test	35	6.32	0.18	-0.03	-4.98	34	0.109	35.

The findings of the paired sample t-test in Table 6 indicate that the pair difference between the pre-test and post-test mean scores was 6.32. This result suggests little or low improvement in the marks obtained by learners who were taught using the traditional teaching method. Concerning the mean scores of the pre-test and post-test and the t-value from 24.14 ± 6.97 to 30.46 ± 6.79 ($p = 0.109 > \propto = 0.05$), the study concluded that no statistically significant improvement in learners in the CG was taught with the traditional teaching method. Since the probability value of 0.109 is more than 5% then this means that the traditional teaching method has no significance in improving learners' performance, understanding, and skill development in the circle theorems.

A comparison was made between the mean gains of 18.83 with an associated p-value of 0.000 of the EG to 6.32 with a p-value of 0.109 of the CG. Therefore, it is reasonable to conclude that concept-based instruction was more effective in improving learners' performance, understanding, and skill development in the circle theorem than the traditional teaching method and this was statistically significant.

Table 7. Paired samples t-test for Experimental and Control Groups

Groups	Mean	S.D	t	df	p-value	Effect Size
E.G	25.80	7.31	0.971	68	0.335	0.23
C.G	24.14	6.97				

An independent sample t-test was conducted to determine whether or not the mean difference observed was statistically significant. This is reported in Table 7. With an effect size of 0.23, the p-value of (0.334) was greater than 5% hence there is no significant difference between the two groups [t (68) = 0.971; p = 0.335>∝ = 0.05]. This result shows that the learners in the CG and EG groups were similar in geometric abilities before the intervention hence, any difference in post-test scores could be attributed to the instructional effect.

Table 8. Paired samples t-test for Experimental and Control Groups

Groups	Mean	SD	t	df	p-value	Effect Size
E.G	44.63	9.68	7.07	68	0.000	1.69
C.G	30.46	6.79				

The data presented in Table 8 is the post-test score results of both groups; the EG obtained an average score of (M= 44.63; S.D = 9.68) while the CG obtained a mean score of (M = 30.46; S.D = 6.79). The results indicate that both the EG and CG groups differ in post-test mean scores, with a (14.17) mean point difference and effect size of 1.69. This means that the EG scored better, on average, than the CG in the post-test, and since p=0.000 < 0.05 we reject hypothesis two, this means that there was a statistically significant difference between concept-based instruction and procedural (traditional) method of instruction. This is evidence that the intervention activity did improve students' performance understanding and skill development in the circle theorem and can, therefore, be suggested that concept-based instruction may have had more effect on learners studying circle theorem compared to the traditional method.

The findings reported evidence of improving students' performance through concept-based instruction that will enhance understanding and skill development in circle theorem by comparing students' performance on circle theorems when exposed to two different teaching methods. The study found that the use of concept-based instruction in teaching circle theorems had a significant influence on students' performance in geometry as compared to the traditional method (Asilo-Ebisa & Lomibao, 2024). One of the reasons for the observed difference is that concept-based instruction helps students to understand Circle Theorems experientially (Joung & Kim, 2022). This motivates and brings students' level of reasoning to the expected level of the topic as advocated by Bashiru & Nyarko (2019).

Moreover, the most glaring elements in the EG were explicit connections between algebraic and geometric representations, between prior learning and current lessons, and between different problems (Ali, 2022). There were multiple students' inputs and students were asked to justify their reasoning. This conceptual emphasis exemplified the conceptual teaching indicators espoused by Hussein & Yusuf (2022). The explicit connections in the algebraic, numerical, symbolic equations and geometric representations contributed to students' remarkable performance in the experimental group. In contrast, the lessons in the CG were focused primarily on the step-by-step methods of finding answers to each problem encountered (Ntow & Hissan, 2021). The lessons of the CG did not encourage students to use their procedures in solving questions. This is consistent with the findings of Hurrell (2021). However, the concept-based instructional methods made lessons in the EG more practical and allowed students to explore and verify concepts experientially. This approach to mathematics instruction helps students understand mathematical concepts the reasons behind using each procedure or formula and how they relate to each other.

In the t-test analyses, most students could not recognize the required circle theorem properties to answer pre-test questions that required informal arguments. However, in the post-achievement test, the students provided meaningful arguments for theorems in answering the post-test. This was made possible by investigating constantly, verifying, and justifying the procedures involved. The possible reasons were that students taught with the concept-based method in the EG were exposed to the concepts of discussion, group work, problem-solving, activity-based, and guided discovery (Hussein & Csíkos, 2023). The conceptual-based instruction provided an opportunity for learners in the EG to understand mathematical concepts and techniques involving meaningful definitions as well as helping them know the reasons behind executing every step of the procedure. Consequently, this improved students' conceptual understanding of the topic and would create sustainability in the learning (Borji et al., 2021). Notwithstanding, the result of the study is consistent with other contemporary studies on conceptual learning (Khoule et al., 2017).

CONCLUSION

The findings of this study indicate that while some students performed adequately under procedural-based instruction, concept-based instruction significantly minimized errors and improved students' understanding and skill development in Circle Theorems. These results highlight the importance of developing conceptual understanding rather than focusing solely on skills, procedures, and algorithms. This study contributes to bridging the empirical gap and advocates for a pedagogical shift toward concept-based teaching in mathematics. Such a shift serves as a valuable resource for practicing teachers, teacher educators, and researchers aiming to improve mathematics instruction. To ensure the efficacy of concept-based teaching, active commitment is required from learners, teachers, and educational stakeholders. Effective pedagogy should empower learners through autonomy, encourage teachers to serve as facilitators, and provide opportunities for students to discover and construct their own mathematical knowledge. This learner-centered approach promotes deeper understanding and prepares students for long-term retention and application of mathematical concepts.

AUTHOR CONTRIBUTIONS STATEMENT

FGP designed the study, conducted the intervention, and analyzed the data. RA contributed to the literature review, structured the manuscript, and refined the academic writing. Both authors interpreted the results and approved the final version of the manuscript.

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