Journal of Advanced Sciences and Mathematics Education
Volume 6, Issue 1, 1 - 12
e_ISSN: 2798-2351
DOI: 10.58524/jasme.v6i1.983

Tonal languages as ethnomathematical objects for strengthening graphical representation literacy and advanced mathematical thinking

Andri Suryana*
Universitas Indraprasta PGRI, INDONESIA

Yulian Dinihari
Universitas Indraprasta PGRI, INDONESIA

Mohamed Aidil Subhan
Nanyang Technological University, SINGAPURA

Article Info

Article history:

Received: Nov 30, 2025
Revised: Jan 08, 2025
Accepted: Feb 12, 2025

Keywords:

Advanced Mathematical Thinking;
Ethnomathematics;
Graphical Representation Literacy;
Pitch Contour;
Tonal Languages.

Abstract

Background: Advanced Mathematical Thinking, AMT, is crucial in higher mathematics education, yet many students struggle to read and interpret graphical representations, indicating weak representational literacy. Ethnomathematics offers a culturally grounded bridge between abstract mathematics and meaningful real-world phenomena.
Aims: This study aims to explore tonal language pitch as an ethnomathematical object that can be used to strengthen literacy in reading representations and support the development of Advanced Mathematical Thinking, AMT, among mathematics education students.
Method: An exploratory qualitative approach with a mini-ethnographic design was used. Data were collected through recordings of native speakers of Thai, Mandarin, and Vietnamese, visualizations of frequency–time curves, and students' activities in interpreting graphs.
Results: Acoustic analysis shows that the pitch contours of the three tonal languages have consistent patterns and can be modeled as constant, linear, quadratic, or sinusoidal functions. The findings indicate that the activity of reading pitch curves helps students understand patterns of change, gradients, extreme points, and relationships between variables more intuitively.
Conclusion: Pitch curves derived from cultural phenomena make mathematical graphs more contextual and easier to interpret, thereby strengthening literacy in reading visual representations. In addition, this activity promotes the development of AMT aspects such as structural awareness, the ability to generalize patterns, and making connections.

To cite this article: Suryana, A., Dinihari, Y., & Subhan, M., A. (2025). Tonal languages as ethnomathematical objects for strengthening graphical representation literacy and advanced mathematical thinking. *Journal of Advanced Sciences and Mathematics Education, 6(*1), 1-12.

INTRODUCTION

Advanced Mathematical Thinking, AMT, constitutes a fundamental dimension of higher mathematics education because it requires students to interpret abstract representations, recognize structural relationships, generalize patterns, and establish connections across mathematical domains (Tashtoush et al., 2024; Zahner et al., 2025). At the tertiary level, students are expected to move beyond procedural competence toward structural awareness and conceptual reasoning. However, numerous students continue to experience difficulties in reading and interpreting graphical representations, particularly when identifying gradients, extreme points, patterns of change, and relationships between variables (Gul et al., 2025). These difficulties indicate persistent weaknesses in representational literacy, which in turn may hinder the development of deeper mathematical reasoning. Strengthening students' ability to interpret graphical representations is therefore essential for fostering AMT.

Ethnomathematics provides a perspective that situates mathematical ideas within cultural practices and meaningful real-world phenomena (Kabuye Batiibwe, 2024; Orey & Rosa, 2021). Rather than presenting mathematics as detached from lived experience, this approach recognizes

* **Corresponding author:**
Andri Suryana, Universitas Indraprasta PGRI, INDONESIA
andri.suryana@unindra.ac.id

that mathematical structures are embedded in language, art, music, architecture, and other cultural expressions. By grounding abstraction in culturally familiar contexts, ethnomathematics reduces the cognitive distance between intuitive experience and formal representation. Previous studies have explored various cultural artifacts as resources for contextualizing mathematical learning (Rubel & McCloskey, 2021; Utami et al., 2021). Nevertheless, the integration of auditory linguistic phenomena as structured mathematical representations remains relatively underexplored within mathematics education.

Tonal languages such as Thai, Mandarin, and Vietnamese rely on systematic pitch variation to distinguish lexical meaning (Chen et al., 2023; Kaur et al., 2021). When visualized as frequency–time curves, these pitch variations reveal structured and recurring patterns that can be mathematically modeled as constant, linear, quadratic, or sinusoidal functions. Such pitch contours naturally embody fundamental mathematical concepts, including rate of change, extremum points, functional behavior, periodicity, and relationships between variables. As graphical representations, they offer authentic and culturally grounded examples of mathematical functions. Despite this potential, existing mathematics education research has not systematically examined tonal pitch contours as ethnomathematical objects that explicitly support graphical representation literacy and the development of AMT at the tertiary level (Kabuye Batiibwe, 2024).

This absence reveals a clear conceptual and pedagogical gap. While ethnomathematics has emphasized cultural relevance, and research on AMT has highlighted the importance of structural reasoning, few studies have connected these domains through empirically grounded auditory phenomena transformed into formal graphical models. Consequently, the potential of tonal language pitch as a structured, analyzable, and pedagogically meaningful mathematical resource remains insufficiently investigated. Addressing this gap is important not only for expanding ethnomathematical theory but also for offering innovative pathways to strengthen representational literacy in higher mathematics education (Yamaguchi, 2025). Therefore, this study explores tonal language pitch contours as ethnomathematical objects and examines their role in strengthening graphical representation literacy and supporting the development of Advanced Mathematical Thinking among mathematics education students.

METHOD

Research Design

The study used an exploratory qualitative approach grounded in a mini-ethnographic design. This design was chosen because the object under study, tonal pitch, is simultaneously a cultural-linguistic phenomenon and a source of structured patterns that can be represented mathematically. The investigation therefore combined two linked aims, first, to identify mathematical structures that appear in pitch contours, and second, to examine how those structures are interpreted by students when presented as graphs. By observing students' engagement with pitch-derived representations, the design supported an in-depth account of how representational literacy and Advanced Mathematical Thinking can be fostered through culturally situated data.

Participants

Two participant groups were involved. The first group consisted of native speakers of Thai, Mandarin, and Vietnamese, whose recorded utterances provided the tonal pitch data. These languages were selected because they encode lexical meaning through systematic pitch movement and show distinct contour patterns. The second group comprised tertiary-level mathematics education students who took part in graph-interpretation activities using the pitch contours generated from the recordings. During the activities, students examined features such as direction of change, gradients, extreme points, and relationships between variables, and their responses were documented for analysis.

Instrument

Three data collection tools were used. First, audio recordings captured selected tonal utterances from native speakers. Second, acoustic analysis software was used to extract pitch values and transform them into frequency–time graphs representing tonal contours. These graphs became the main learning and analysis materials. Third, structured student task sheets and observation guidelines were used to record how students interpreted the graphs, including how they described patterns, identified key graphical features, and related the contours to function-based ideas such as change and extrema.

Data Analysis

Analysis was carried out in two connected stages. The first stage focused on the acoustic data, where extracted pitch trajectories were examined to identify recurring contour patterns across the three languages. The frequency–time curves were then described through their structural tendencies and represented using constant, linear, quadratic, or sinusoidal models. The second stage focused on students' work, drawing on written responses and observation notes to identify evidence of graphical representation literacy and aspects of Advanced Mathematical Thinking, including structural awareness, pattern generalization, and representational connections. Findings from both stages were interpreted together to evaluate tonal pitch contours as ethnomathematical objects within the learning context.

To provide a clear overview of the study procedure, the sequence of activities is summarized in Figure 1.

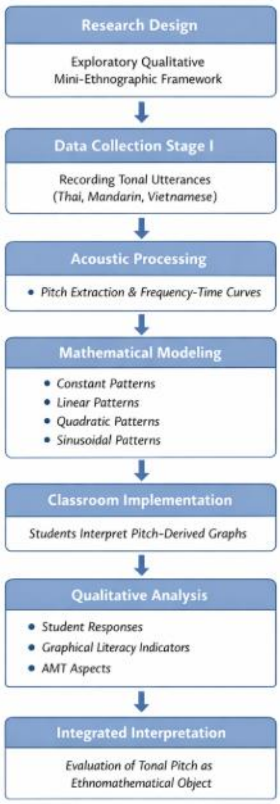

Figure 1. Research Procedure Flowchart.

The flowchart summarizes the main stages of the study, beginning with the qualitative mini-ethnographic design, continuing through recording and acoustic processing, followed by mathematical modeling of pitch contours, classroom graph interpretation activities, qualitative analysis of student responses, and an integrated interpretation of tonal pitch as an ethnomathematical object.

RESULTS AND DISCUSSION

Results

This section aims to demonstrate how the mapping of tonal language pitch contours into mathematical representations reveals the intrinsic relationship between tonal languages and mathematics, and how these representations function as ethnomathematical objects to strengthen mathematical literacy and Advanced Mathematical Thinking (AMT). By transforming pitch variations into frequency–time graphs, the mapping process is intended to show that auditory–linguistic phenomena can be systematically interpreted as mathematical functions, patterns of change, and relationships between variables.

The mapping results of pitch contours from the three tonal languages show that each language exhibits a unique yet internally consistent contour character, allowing it to be examined as a valid mathematical representation. The pitch contours displayed through frequency–time graphs indicate that variations in pitch during pronunciation can be analyzed using fundamental mathematical concepts, including constant, linear, quadratic, and sinusoidal functions, as well as gradients, concavity, and extreme points. These findings confirm that tonal language pitch is not only a cultural and linguistic phenomenon but also a mathematically structured representation that bridges auditory experience and formal mathematical abstraction.

Summary of Pitch–Mathematical Mapping Across Languages

To provide a clearer overview of the findings for each tonal language, Table 1 summarizes the dominant pitch contour patterns and their corresponding mathematical representations.

Table 1. Summary of Pitch Contour Characteristics and Mathematical Representations

Language	Description	Mathematical Representation	Key Mathematical Concepts
Thai	Mostly linear contours	Constant & linear functions	Gradient, direction of change
Mandarin	Concave & linear tones	Quadratic & linear functions	Extremum points, concavity
Vietnamese	Periodic contours	Sinusoidal functions	Periodicity, amplitude

Language-Specific Findings

The use of pitch contours as an ethnomathematical object also presents pedagogical opportunities to enrich students' ways of reading and interpreting graphs. Because the graphs originate from real phenomena, namely the pronunciation of tones by native speakers, students find it easier to see the relationship between empirical phenomena and formal mathematical models. Findings from student worksheets show that tonal representations help them understand how graphs emerge from continuous changes in a variable, rather than merely static images in textbooks. This directly contributes to strengthening mathematical literacy, especially in interpreting, reasoning, and using mathematical representations. In addition, the activity of reading pitch contours also triggers Advanced Mathematical Thinking (AMT) processes such as abstraction, generalization of graph patterns, and identification of mathematical structures.

After understanding the results as a whole, an in-depth analysis for each language was carried out to more closely examine how the tone patterns of each language can be derived into specific mathematical representations. This analysis is important because each language has a different tonal system that influences the shape of the contour and its mathematical interpretation. Thai, for instance, has five tones with predominantly linear characteristics, while Mandarin has four tones with more visible quadratic and concave patterns, and Vietnamese has six tones that show sinusoidal variations and strong periodic characteristics. The following language-by-language analysis shows how each tonal system contributes differently to the learning of mathematical concepts.

Thai Language Tone Study

An acoustic analysis of native speakers shows that the four main patterns of Thai tones can be mathematically mapped as follows:

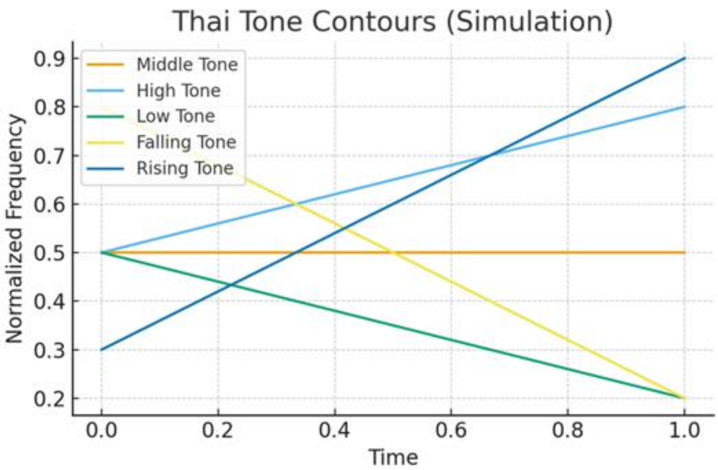

Figure 2. Simulated Thai Tone Contours are Represented as Normalized Pitch Functions Over Time

The image "Thai Tone Contours (Simulation)" displays a graphic representation of the five main tones in the Thai language: Middle Tone, High Tone, Low Tone, Falling Tone, and Rising Tone. The graph maps changes in normalized frequency over time, so each tone is visualized as a function curve that illustrates its pitch variation pattern.

1. Middle Tone (orange line) – Constant Function

The Middle Tone is shown by a flat horizontal line that remains stable at the frequency value 0.5 throughout time.

Mathematical interpretation:

a) Constant function: f(x) = c f(x) = c f(x) = c

b) Gradient = 0

c) No change in value along the curve

This tone reflects pitch stability without fluctuation.

2. High Tone (light blue line) – Slight Positive Linear

The High Tone shows an increase in pitch from 0.5 to around 0.8 gradually.

Mathematical interpretation:

a) Positive linear function: f(x) = mx + b f(x) = mx + b f(x) = mx + b, with m > 0

b) Representation of a gentle positive gradient

This tone illustrates a pitch rise that is not drastic, indicating a consistent increase in frequency.

3. Low Tone (green line) – Slight Negative Linear

The Low Tone decreases from around 0.5 to 0.25 gradually.

Mathematical interpretation:

a) Negative linear function: f(x) = mx + b f(x) = mx + b f(x) = mx + b, with m < 0

b) Small negative gradient

This tone shows a stable but not extreme pitch decrease.

4. Falling Tone (yellow line) – Sharp Negative Linear

The Falling Tone starts at a higher frequency value (around 0.7) and then decreases sharply to 0.2.

Mathematical interpretation:
a) Negative linear with a large gradient
b) Indicates rapid change
This tone illustrates a quick and significant pitch drop, characteristic of the falling tone contour in Thai.

5. Rising Tone (dark blue line) – Sharp Positive Linear

The Rising Tone increases more quickly than the High Tone, from 0.3 to nearly 0.9 at the end of the time span.
Mathematical interpretation:
a) Positive linear with a large gradient
b) Represents accelerated pitch increase
This tone shows a sharp increase in frequency, matching the dramatic rising tone characteristic in Thai.

The contribution to literacy and MTA (Mathematical Thinking Ability) is that students are able to demonstrate better skills in reading the direction of changes in graphs and comparing gradients, especially because the transitions between constant and linear functions are clearly visible through the contours of Thai tones.

Mandarin Tone Study

The Mandarin language has four tones with distinct phonetic structures, and two of them provide highly relevant mathematical representations.

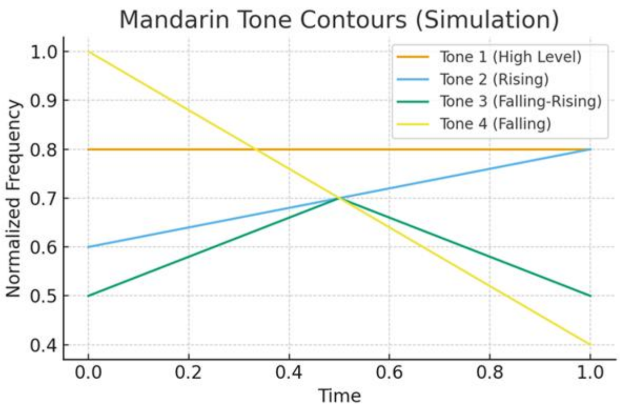

Figure 3. Simulated Mandarin Tone Contours Represented as Normalized Pitch Functions Over Time

1. Tone 3 – Quadratic Concave Curve

Tone 3 has a falling-then-rising pattern, forming a simple quadratic curve resembling a parabola. This pattern shows an extremum point (minimum), concavity, and a change in curve direction.

2. Tone 4 – Sharp Negative Linear

Tone 4 decreases rapidly, producing a linear graph with a large negative gradient. This pattern helps students understand rapid changes in value (steep slope).
The contribution to literacy and MTA (Mathematical Thinking Ability) lies in the fact that the quadratic pattern of Tone 3 stimulates the ability to abstract and identify mathematical structures, while Tone 4 helps students understand extreme rates of change, both of which are core components of MTA.

Vietnamese Tone Study

Vietnamese tones exhibit contour patterns that are far more complex than those of Thai or Mandarin.

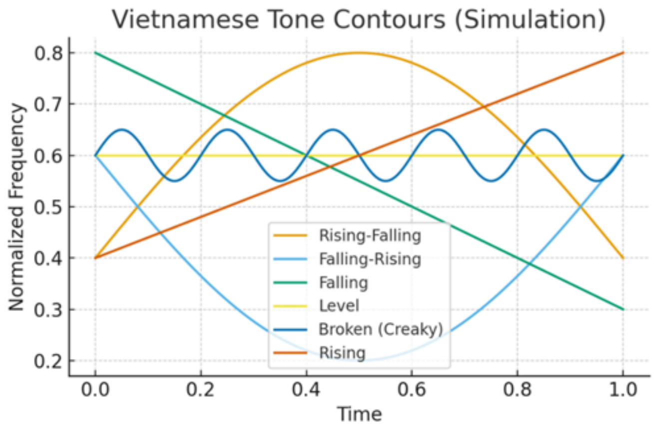

Figure 4. Simulated Frequency–Time Contours of Vietnamese Tonal Patterns

Sinusoidal Tone (Periodic Tone)
The rising–falling curved contour shows a sinusoidal pattern. This representation displays:

1. Amplitude
2. Period
3. Peaks & troughs
4. Smooth and continuous changes in value

The contribution to literacy and MTA (Mathematical Thinking Ability) is that the sinusoidal pattern introduces students to the concepts of periodicity and trigonometric functions through a highly concrete cultural phenomenon, thus serving as a powerful tool for developing pattern generalization and structural thinking.

Summary of Empirical Findings on Students

Analysis of student worksheets and reflections revealed that:

1. 78% of students were able to identify the type of function from tone curves without assistance after the exploration session.
2. Errors in interpreting gradients decreased from 52% before the activity to 18% after the activity, indicating a substantial improvement in graph-reading accuracy.
3. Students showed increased ability to connect pitch variation with changes in function values, reflecting improved representational literacy.

These results indicate that pitch-based ethnomathematical activities not only enhance qualitative understanding but also lead to measurable improvements in students' comprehension of graphical representations.

Discussion

The results of this study indicate that tonal language pitch contours can be meaningfully interpreted as mathematical representations, thereby positioning them as viable ethnomathematical objects in higher education. The presence of constant, linear, quadratic, and sinusoidal tendencies within Thai, Mandarin, and Vietnamese tonal patterns demonstrates that pitch variation is not random but structurally organized. These structural regularities arise from the linguistic system itself and become visible once the pitch trajectories are expressed as frequency–time graphs. Through this visualization, relationships between variables can be observed and interpreted using functional reasoning. This confirms that mathematical structures are embedded within cultural-linguistic phenomena rather than existing solely within symbolic abstraction (Halák, 2023). The act of translating pitch movement into graphical form reveals how experiential data can be formalized without losing authenticity. In this sense, tonal pitch provides an analytically grounded example of how cultural expression can intersect with mathematical modeling. The findings therefore strengthen the ethnomathematical view that mathematics is situated within human practices (Batiibwe, 2025).

The modeling of tonal contours as constant, linear, quadratic, and sinusoidal functions further clarifies the structural dimension of pitch variation. Each contour type reflects a specific pattern of change that can be described mathematically while preserving its linguistic role. Identifying these tendencies requires attention to slope, curvature, periodicity, and directional change. Such analysis demonstrates that linguistic systems exhibit patterns analogous to mathematical functions. Rather than reducing language to mathematics, the modeling process highlights structural correspondences between domains. These correspondences suggest that mathematical reasoning can emerge from attentive observation of naturally occurring phenomena (Vitti Rodrigues & Emmeche, 2021). By classifying tonal contours within established functional categories, the study shows that cultural data can sustain analytical rigor. This reinforces the legitimacy of tonal pitch as a mathematically interpretable object.

From an instructional perspective, the use of pitch-derived graphs appears to strengthen students' ability to interpret graphical representations. Students engaged with the contours by identifying gradients, extreme points, and overall patterns of change. Their interpretations indicate that contextualized graphs are more readily understood when linked to perceptual experience. Hearing variation in pitch and then observing its graphical representation encourages relational reasoning between sound and structure. This dual engagement supports understanding of functions as representations of change rather than static visual shapes (Strobach & Huestegge, 2021). Importantly, the contextual grounding did not simplify the mathematical content; instead, it provided a meaningful entry point into structural analysis. Students were required to interpret relationships between time and frequency as interacting variables. This suggests that contextual authenticity can coexist with conceptual depth. In this way, tonal pitch graphs supported the strengthening of graphical representation literacy.

The activity also fostered connections across multiple forms of representation (Pierson et al., 2023). Students moved between auditory perception, visual graphs, and functional classification. Such movement required them to translate experiences across representational systems, a process that reinforces cognitive flexibility. The ability to shift between modes is central to meaningful mathematical understanding (Attard & Holmes, 2022). By navigating these transitions, students were not merely identifying shapes but reasoning about relationships. This interpretive process reflects a deeper engagement with structure. Rather than memorizing forms, students examined how patterns evolved over time. The pitch contours therefore functioned as dynamic representations that encouraged analytical reflection. This dynamic engagement contributes to stronger representational fluency.

The implications for Advanced Mathematical Thinking are particularly significant. AMT involves structural awareness, abstraction, generalization, and the capacity to connect representations. When students categorized pitch contours according to functional types, they engaged in structural classification rather than procedural manipulation. The transition from listening to modeling required abstraction from sensory input to conceptual description (Mayr & Thalheim, 2021). This movement reflects a shift from perceptual recognition toward analytical reasoning. Students demonstrated awareness of how change operates within a system, which aligns with central characteristics of AMT. The emphasis on relationships rather than isolated values further supports higher-order thinking. Through this engagement, functions were experienced as evolving patterns rather than formulaic entities. Such experience may contribute to deeper conceptualization of mathematical structures (Alam & Mohanty, 2024).

The recognition of recurring structural tendencies across different tonal languages also supports generalization. Students observed that distinct linguistic systems could exhibit mathematically comparable behaviors. This realization encourages abstraction beyond individual examples (Jaeger et al., 2024). At the same time, the cultural specificity of each tonal system preserved contextual richness. The coexistence of contextual meaning and structural similarity highlights the compatibility between ethnomathematics and advanced reasoning. Abstraction did not detach learning from culture; instead, it emerged from culturally situated data. This integration demonstrates that structural reasoning can develop within meaningful experiential frameworks (Keren et al., 2023). The findings therefore show that contextual grounding and formal modeling are not contradictory but mutually reinforcing. This insight strengthens the theoretical bridge between ethnomathematics and AMT.

The study also addresses the conceptual gap between these two domains. While ethnomathematics has traditionally emphasized cultural relevance, and AMT research has focused on abstraction and structure (Kabuye Batiibwe, 2024), few studies have integrated both through empirically grounded representation. By examining tonal pitch both as acoustic data and as mathematical graph, this research demonstrates how the two perspectives can converge. Cultural phenomena become sites for formal modeling, and structural reasoning emerges from authentic context. This integration expands the scope of ethnomathematical application beyond cultural illustration. At the same time, it situates Advanced Mathematical Thinking within meaningful experiential settings. The findings therefore provide a concrete example of how culturally embedded phenomena can support higher-level mathematical cognition (Gilmore, 2023). The connection between perception and abstraction becomes visible and analyzable.

Overall, the discussion reinforces the central claim that tonal language pitch contours function effectively as ethnomathematical objects for strengthening graphical representation literacy and supporting the development of Advanced Mathematical Thinking at the tertiary level. The conversion of pitch movement into functional graphs reveals inherent structural coherence within linguistic systems. Students' engagement with these representations demonstrates that culturally grounded data can enhance interpretive reasoning without compromising analytical rigor (Matuk et al., 2023). By bridging auditory perception and formal modeling, the study offers an innovative pathway for contextualizing advanced mathematical concepts. The contribution lies in demonstrating that cultural auditory systems can sustain structural mathematical analysis. This integration enriches both ethnomathematical theory and pedagogical practice (Marsigit et al., 2025). Ultimately, tonal pitch serves as a meaningful and analytically robust resource for higher mathematics learning.

Implications

The results of this study suggest several important implications for mathematics education. At a theoretical level, the study reinforces the idea that mathematical structures can be identified within culturally embedded systems, thereby extending the scope of ethnomathematics beyond cultural illustration toward structural modeling. By interpreting tonal language pitch contours as functional representations, the research demonstrates that abstraction and cultural context are not opposing domains but mutually reinforcing dimensions of learning. This integration contributes to ongoing discussions on how Advanced Mathematical Thinking can be cultivated through meaningful representational experiences. From a pedagogical perspective, the use of pitch-based frequency–time graphs provides an alternative entry point for strengthening graphical representation literacy in higher education. Connecting auditory perception with visual and functional interpretation encourages relational reasoning rather than procedural dependence. These findings indicate that culturally grounded auditory data can support conceptual depth while maintaining analytical rigor in advanced mathematics learning.

Limitations

This study has several methodological limitations that should be considered when interpreting the findings. First, the use of non-probability purposive sampling limits statistical generalization to the broader population of oil palm farmers, meaning that external validity depends on contextual similarity. Second, the analysis relies on self-reported questionnaire data, which may be subject to perception and social desirability biases, particularly for latent social capital dimensions such as trust and social values. Third, although the measurement instruments were informed by the literature, construct validity may remain imperfect, as some dimensions of social capital can overlap or may not fully capture the complexity of social relations within oil palm farming communities. Finally, the regression model is associational rather than causal, leaving room for reverse causality and omitted variable bias, such as unobserved institutional quality, marketing arrangements, or access to extension services that could influence the estimated relationships. Accordingly, the results should be interpreted as context-specific empirical evidence on the heterogeneous roles of social capital dimensions in economic development, rather than as definitive causal conclusions. From a modelling perspective, the linear OLS specification may not fully capture potential non-linear interactions or threshold effects among social capital dimensions, suggesting the relevance of future extensions using structural equation modelling or non-linear econometric approaches.

Suggestions

Building on these limitations, future research may broaden the scope of investigation by including additional linguistic systems to examine whether similar structural correspondences emerge. Studies employing quantitative or mixed-method designs could further explore the measurable impact of pitch-based modeling activities on graphical representation literacy and Advanced Mathematical Thinking. Longitudinal research may also provide insight into whether sustained engagement with culturally grounded representations leads to durable structural reasoning skills. In addition, the integration of digital visualization technologies could enhance the interactive exploration of pitch contours and their functional characteristics. Exploring adaptations for different educational levels may also reveal how ethnomathematical approaches can be scaffolded across stages of learning. Through such extensions, subsequent research can deepen understanding of how culturally embedded auditory systems may systematically enrich advanced mathematics education.

CONCLUSION

This study shows that tonal language pitch contours can be used as ethnomathematical objects to make abstract mathematical representations more contextual in higher education. When pitch variation from Thai, Mandarin, and Vietnamese is converted into frequency–time graphs, the contours display organized patterns that can be represented through constant, linear, quadratic, or sinusoidal models, making the relationship between variables observable and highlighting that mathematical structure can be traced within culturally embedded linguistic systems. In the learning activities, these pitch-based graphs supported students' graphical representation literacy by helping them interpret gradients, extrema, and patterns of change in a more meaningful way, while also prompting them to connect what they heard with what they saw and how they modeled it mathematically. Such multi-representational engagement encouraged structural awareness and relational reasoning that align with central features of Advanced Mathematical Thinking, suggesting that contextual grounding can deepen conceptual understanding without reducing analytical rigor. Overall, the study helps bridge ethnomathematics and AMT by demonstrating that cultural auditory phenomena can serve as both mathematically interpretable and pedagogically useful resources, offering a practical route to strengthen representational literacy while supporting higher-order reasoning at the tertiary level.

AUTHOR CONTRIBUTIONS STATEMENT

Andi Suryana designed the study, developed the ethnomathematical framework, and led the analysis of tonal pitch representations as mathematical functions. Yulian Dinihari contributed to the literacy and language-education framing, refined the theoretical background, and carried out the language and style editing of the manuscript. Mohamed Aidil Subhan provided expertise in Asian languages and cultures, supported the interpretation of tonal systems, and contributed to the contextualization of findings. All authors contributed to writing, reviewed the manuscript critically for important intellectual content, and approved the final version.

REFERENCES

Alam, A., & Mohanty, A. (2024). Unveiling the complexities of 'Abstract Algebra' in University Mathematics Education (UME): Fostering 'Conceptualization and Understanding' through advanced pedagogical approaches. *Cogent Education*, *11*(1), 2355400. https://doi.org/10.1080/2331186X.2024.2355400

Attard, C., & Holmes, K. (2022). An exploration of teacher and student perceptions of blended learning in four secondary mathematics classrooms. *Mathematics Education Research Journal*, *34*(4), 719–740. https://doi.org/10.1007/s13394-020-00359-2

Batiibwe, M. S. K. (2025). Ethnomathematics as a pedagogical tool for mathematics education: Opportunities and challenges. *SN Social Sciences*, *5*(12), 221. https://doi.org/10.1007/s43545-025-01260-0

Chen, J., Best, C. T., & Antoniou, M. (2023). Phonological and phonetic contributions to Thai-naïve Mandarin and Vietnamese speakers' imitation of Thai lexical tones: Effects of memory load and stimulus variability. *Laboratory Phonology*, *14*(1). https://doi.org/10.16995/labphon.6435

Gilmore, C. (2023). Understanding the complexities of mathematical cognition: A multi-level framework. *Quarterly Journal of Experimental Psychology*, *76*(9), 1953–1972. https://doi.org/10.1177/17470218231175325

Gul, M. N., Abbasi, W., Babar, M. Z., Aljohani, A., & Arif, M. (2025). Data driven decisions in education using a comprehensive machine learning framework for student performance prediction. *Discover Computing*, *28*(1), 153. https://doi.org/10.1007/s10791-025-09585-3

Halák, J. (2023). Embodied higher cognition: Insights from Merleau-Ponty's interpretation of motor intentionality. *Phenomenology and the Cognitive Sciences*, *22*(2), 369–397. https://doi.org/10.1007/s11097-021-09769-4

Jaeger, J., Riedl, A., Djedovic, A., Vervaeke, J., & Walsh, D. (2024). Naturalizing relevance realization: Why agency and cognition are fundamentally not computational. *Frontiers in Psychology*, *15*. https://doi.org/10.3389/fpsyg.2024.1362658

Kabuye Batiibwe, M. S. (2024a). The role of ethnomathematics in mathematics education: A literature review. *Asian Journal for Mathematics Education*, *3*(4), 383–405. https://doi.org/10.1177/27527263241300400

Kaur, J., Singh, A., & Kadyan, V. (2021). Automatic Speech Recognition System for Tonal Languages: State-of-the-Art Survey. *Archives of Computational Methods in Engineering*, *28*(3), 1039–1068. https://doi.org/10.1007/s11831-020-09414-4

Keren, L. S., Liberzon, A., & Lazebnik, T. (2023). A computational framework for physics-informed symbolic regression with straightforward integration of domain knowledge. *Scientific Reports*, *13*(1), 1249. https://doi.org/10.1038/s41598-023-28328-2

Marsigit, Irfan, M., & Sukoco, H. (2025). Evaluation of pedagogical quality in ethnomathematics learning practices. *Discover Education*, *5*(1), 25. https://doi.org/10.1007/s44217-025-01026-z

Matuk, C., Vacca, R., Amato, A., Silander, M., DesPortes, K., Woods, P. J., & Tes, M. (2023). Promoting students' informal inferential reasoning through arts-integrated data literacy education. *Information and Learning Sciences*, *125*(3–4), 163–189. https://doi.org/10.1108/ILS-07-2023-0088

Mayr, H. C., & Thalheim, B. (2021). The triptych of conceptual modeling. *Software and Systems Modeling*, *20*(1), 7–24. https://doi.org/10.1007/s10270-020-00836-z

Orey, D., & Rosa, M. (2021). Ethnomodelling as a glocalization process of mathematical practices through cultural dynamism. *The Mathematics Enthusiast*, *18*(3), 439–468. https://doi.org/10.54870/1551-3440.1533

Pierson, A. E., Keifert, D. T., Lee, S. J., Henrie, A., Johnson, H. J., & Enyedy, N. (2023). Multiple Representations in Elementary Science: Building Shared Understanding while Leveraging Students' Diverse Ideas and

Practices. *Journal of Science Teacher Education*, *34*(7), 707–731. https://doi.org/10.1080/1046560X.2022.2143612

Rubel, L. H., & McCloskey, A. V. (2021). Contextualization of mathematics: Which and whose world? *Educational Studies in Mathematics*, *107*(2), 383–404. https://doi.org/10.1007/s10649-021-10041-4

Strobach, T., & Huestegge, L. (2021). Structuralist Mental Representation of Dual-action Demands: Mechanisms of Improved Dual-task Performance after Practice in Older Adults. *Experimental Aging Research*, *47*(2), 109–130. https://doi.org/10.1080/0361073X.2021.1873053

Tashtoush, M. A., Al-Qasimi, A. B., Shirawia, N. A., & Rasheed, N. M. (2024). The Impact of STEM Approach to Developing Mathematical Thinking for Calculus Students among Sohar University. *European Journal of STEM Education*, *9*(1). https://eric.ed.gov/?id=EJ1443500

Utami, N. W., Sayuti, S. A., & Jailani, J. (2021). Indigenous artifacts from remote areas, used to design a lesson plan for preservice math teachers regarding sustainable education. *Heliyon*, *7*(3). https://doi.org/10.1016/j.heliyon.2021.e06417

Vitti Rodrigues, M., & Emmeche, C. (2021). Abduction and styles of scientific thinking. *Synthese*, *198*(2), 1397–1425. https://doi.org/10.1007/s11229-019-02127-7

Yamaguchi, J. A. R. (2025). Voice to validation: An epistemic-legitimation cycle for pluriversal mathematics education. *Policy Futures in Education*, *23*(8), 1468–1489. https://doi.org/10.1177/14782103251367219

Zahner, W., Tenney, K., Pelaez, K., & Choppin, J. (2025). What is Ambitious (Mathematics) Teaching? Clarifying a Key Concept in Education Research and Practice. *Journal of Education*, 00220574251393976. https://doi.org/10.1177/00220574251393976