



High school students' mathematical representation ability: Evaluation of disposition based on mastery learning assessment model (MLAM)

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Abstract

Mathematical representation is the ability of students to explore and use mathematical ideas explicitly. This study aimed to learn about the effectiveness of the Mastery Learning Assessment Model (MLAM) in mathematics learning at the Assyfa Learning Center Foundation Pasuruan, which then described students' abilities in solving mathematical representation problems evaluated from mathematical dispositions. MLAM is based on repeating a similar assessment through an overhaul of mastery learning from previous researchers. This study used a descriptive qualitative method. In the study presented here, 20 subjects were pooled and surveyed using a continuous assessment tool based on high, low, and moderate math ability. The results showed that almost all students could use representational images to solve mathematical problems, create mathematical models, and solve problems related to mathematical expressions. On the other hand, most students could not complete the two indicators of mathematical representation, namely making sketches and mathematical models based on the given situation and data. This result is indicated by the 75% of students who completed this lesson. Based on the Teaching Evaluation (TER) results, most students are also satisfied with this approach. It is expected that the MLAM assessment can improve students' mathematical representation skills.

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INTRODUCTION

The low achievement of Indonesian students is a classic problem in the world of education (Faisal & Martin, 2019; Maghfiroh & Rohayati, 2020; Mangelep & Kaunang, 2018). Compared to other countries, Indonesian students' math results occupy a low position (Gersten et al., 2009; OECD, 2019; Rohana & Ningsih, 2020; Tremblay, 2013). Mathematical representation performance and disposition are two important parameters of mathematics education in Indonesia (Wang et al., 2020). Representation is a fundamental ability for students to develop and develop (Ke & M. Clark, 2020; Scholkopf et al., 2021; Sterner et al., 2020). The ability of representation is a facility that supports students to explore and use mathematical ideas explicitly (Abramovich et al., 2019; Yeh et al., 2019). Mathematical representations are grouped into five categories; real-life experiences, concrete patterns, arithmetic symbols, narrative or verbal, and diagrams or graphs (Hatisaru, 2020, Carbonneau et al. 2013). Of the five categories, the last three represent the level of abstraction and, in mathematical problem solving, have a higher representation (Sterner et al., 2020). A learning approach that plays a role in providing opportunities to help students communicate their mathematical ideas without any limitations is the role of representation (Chung & Wang, 2018; Guan et al., 2021; Lee et al., 2020; Scholkopf et al., 2021). Like the previous understanding, Komala & Sarmini (2020) state that representation is when students look back at a problem in a simple form based on their understanding and communicate the solutions obtained through representations. Exterior representation can be verbal, symbolic, or visual (Royana et al., 2020), while internal representation is how students develop their knowledge to work in their minds, especially mathematical knowledge (Komala & Sarmini, 2020; Pedersen et al., 2021). The indicators of mathematical representation ability, among others; a) in solving a problem, a visual representation will be used; b) presenting data/information from a representation in the form of charts, graphs, or tables

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and solving a problem orally or in writing; c) develop mathematical equations or models from given representations and solve problems involving mathematical expressions; d) drawing patterns, writing mathematical steps in words and using solving mathematical problems using mathematical expressions; and e) generate problem situations from data based on available data.

Mathematics learning is not solely aimed at developing cognitive dimensions (Suwanto, 2010; Wijaya et al., 2020) but learning also needs to involve and develop students' affective dimensions (Kuswanti et al., 2017). The affective dimension is another term for mathematical disposition (Fitrianna et al., 2018; Miatus & Khusna, 2020). Sumarmo (2010) defines disposition as students' strong willingness, awareness, and dedication to learn and perform various mathematical activities. Sarifah et al. (2018) describe several indicators of mathematical disposition, including self-confidence, hope and metacognition, tenacity and seriousness in learning mathematics, persistence in dealing with and solving problems, high curiosity, and the ability to share opinions and information with other people. These indicators imply mathematical disposition as the main factor in determining student success in learning to have mastery of mathematics.

Mastery learning was first introduced by John Carroll more than 80 years ago (Sajadi & Khaghanizadeh, 2016). Carroll emphasized that all students have the potential to learn within different periods to achieve a certain subject matter (Emery et al., 2018; Yudkowsky et al., 2015). Learning mastery is positive, especially in the areas of achievement, attitudes toward learning, and content retention, in the subject matter undertaken by students (Davis, 2019; Guskey, 2014). Mastery learning has the potential to benefit students at various talent levels. Munawaroh (2017) explains that mastery learning is very beneficial for students with low talent whose experience with failure in the subject matter has reduced their motivation for the subject matter. Compared to the traditional approach to teaching that most educators are familiar with, mastery learning has proven successful in various academic fields and courses such as science and technology education (Emery et al., 2018), athletic training education (Dunn et al., 2017), biology (Blumenfeld et al., 2020), chemistry (Susilaningih et al., 2019), and mathematics (Shafie et al., 2010). Under the influence of Ralph Tyler, Benjamin Bloom realized that students should not be compared by their academic achievements but rather that students should be helped to achieve the goals of the curriculum they are following (Darling-Hammond et al., 2020). Bloom outlines specific strategies for using formative assessment to guide teachers in differentiating their instruction, and the differentiation process is labeled as 'mastery learning' (Yeh et al., 2019). Bloom also recommends using classroom assessments as a learning tool and then following up on these assessments with positive feedback and corrective procedures (Blumenfeld et al., 2020). In the mastery learning class, the corrective procedure is adjusted to the specific weaknesses of each student. In contrast, in the non-mastery class, there is no additional opportunity for students to rework their coursework (Hayat, 2018).

Mastery testing has a learning goal through a process known as repetition. Thus, the mastery test uses repetition of up to two times for each subject chapter through an alternative test covering the same objectives and content introduced by Felder et al. (2000). Lin et al. (2008) applied web-based mastery learning to those who failed in previous studies. He makes remedial learning for those who have not mastered it and will repeat the assessment until students master the subject. The only problem is that students cannot move on to the next module until they have mastered the current one. This process is shown in Figure 1.

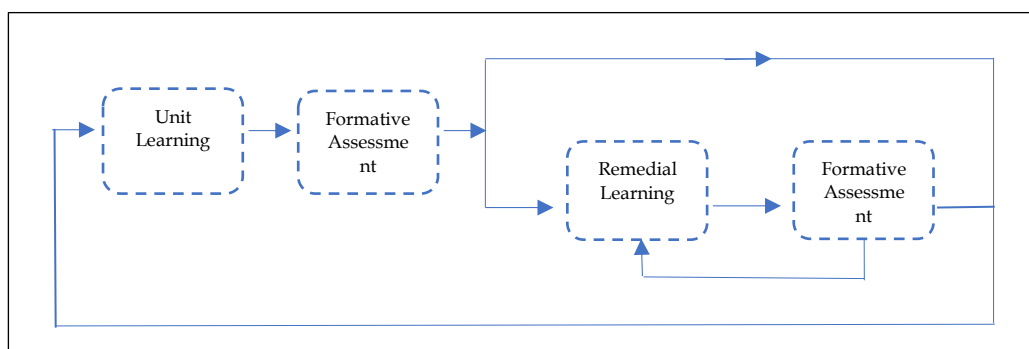


Figure 1. The Entire Learning Process

Research related to representation ability has been carried out (Imaduddin & Haryani, 2019; Ke & Clark, 2020; Khaerunnisa & Santosa, 2020; Scholkopf et al., 2021; Sterner et al., 2020; Chung & Wang, 2018; Guan et al., 2021; Hatisaru, 2020; Lee et al., 2020; Mainali, 2021; Fitrianna et al., 2018; Lengkana et al., 2020). Some of these studies show that using models and media in learning can improve students' mathematical representation abilities. Then research related to mathematical disposition has also been carried out (Hendriana et al., 2013; Herlina, 2013; Miatusun & Khusna, 2020; Nursyam, 2019; Rahlan & Sofyan, 2021; Sumarmo et al., 2012; Irawandi et al., 2021; Rizky & Sritresna, 2021; Sarifah et al., 2018; Susilo et al., 2019). The results of this study indicate that students' mathematical disposition skills are better when using cooperative learning models than conventional learning. Students' mathematical disposition is also influenced by interest, curiosity, and self-confidence. Further research on misconceptions has been carried out (Ati, 2019; Fitri, 2018; Pujayanto et al., 2018; Ramadhan et al., 2017; Sopiany & Rahayu, 2019; Tes et al., 2016; Antika et al., 2018; Fitriani et al., 2017; Gulo, 2019; Harahap et al., 2019; Irawan et al., 2012; Savitri & Subanti, 2016) which shows that students still often experience misconceptions in several stages, such as misconceptions in understanding concepts and in calculating signs. However, research linking students' representational abilities, mathematical dispositions, and misconceptions by using the mastery learning assessment model (MLAM) as an evaluation tool to determine the ability of mathematical representation assessed from mathematical dispositions related to student difficulties has not been found. Therefore, this study aims to explain students' difficulties in solving mathematical representation problems based on their mathematical dispositions using the mastery learning assessment model (MLAM).

METHOD

This study used a descriptive qualitative method. Qualitative descriptive research describes students' difficulties in solving mathematical representation problems. This research was conducted at the Assyfa Learning Center Foundation, Pasuruan, East Java, with 20 high school students in class XII. The instruments used to collect data are questionnaires and tests. Questionnaires determine students' mathematical dispositions in the mathematics learning process. The test determines the students' ability to solve mathematical representation problems. Five questions represent each Indicator of the mathematical representation. The test results were analyzed using qualitative analysis according to the indicators of mathematical representation ability. Meanwhile, to find out the completeness of student learning in the material "Derivatives of Algebraic Functions" using the MLAM assessment.

The students' mathematical disposition refers to the mathematical disposition ability questionnaire instrument from Nuraida (2017) in the form of a Likert scale: strongly agree (SS), agree (S), disagree (S), and strongly disagree (TS). Then the scale is converted into quantitative data with the criteria in Table 1.

Table 1. Students' Mathematical Disposition Criteria

Score	Category
$Score < 58,0$	Low (L)
$58,0 \leq Score \leq 65,0$	Medium (M)
$Score > 65,00$	High (H)
Total	

Based on the questionnaire results, 20 students were grouped into three categories, as shown in Table 2.

Table 2. The Category of the Student's Mathematical Disposition Ability

Total students	Category
4	Low (L)
7	Medium (M)
9	High (H)
Total	20

Furthermore, Figure 2 below shows how MLAM is implemented in this study. At the same time, the process of mastery learning is also introduced to the same class. The course is broken down into n units. The division of this course is based on the topic of the subject. For example, students who score 75% for assessments in certain units are considered to have mastered certain units and can move on to

the next unit. The process described here is different from that described by Lin lin,); students who score less than 75% will repeat the same unit but will be allowed to continue at the same time to the next unit. Students are required to repeat their assessments in specific units until they master the repeated assessment units within the specified period. This process is shown in Figure 2.

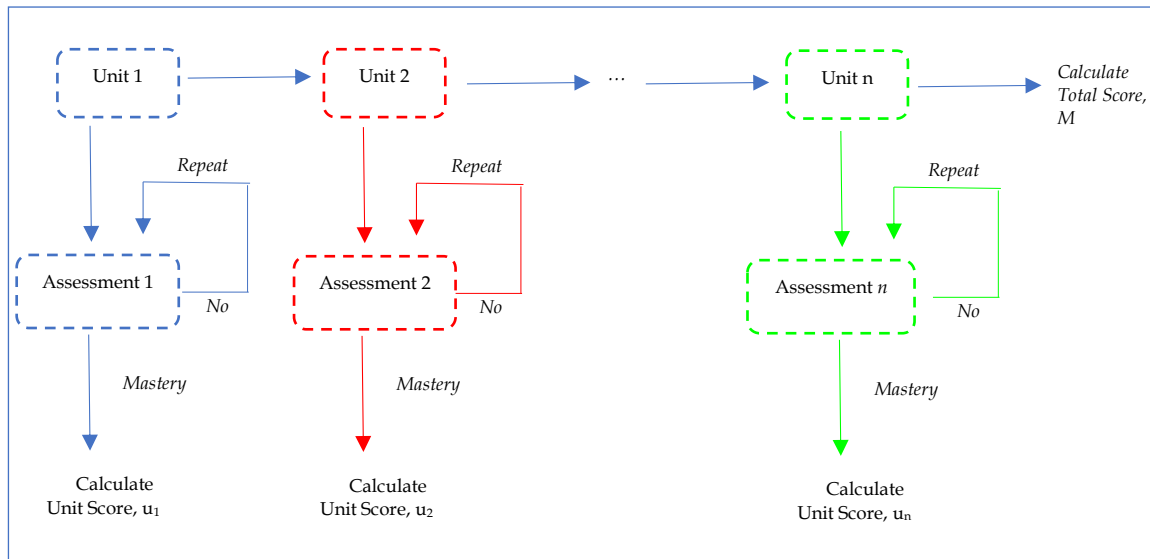


Figure 2. Learning Completeness Assessment Process

The recommended completeness assessment model for calculating the completeness score is shown in equations 2 and 3, respectively, as follows:

Let,

$a_i =$ Score for i unit (%)

$u_i =$ Mastery learning Score for i unit, where $75\% \leq u_i \leq 100\%$, and

$w_i =$ Weight for i units, where $0 < w_i < 1$, $\sum_{i=1}^n w_i = 1$

then,

$$u_i = \begin{cases} 100\%, & i=1 \\ maks(a_i), & i \neq 1 \end{cases} ; i \in Z^+ \quad 1)$$

$$Total\ score\ for\ mastery\ learning\ assessment, M = \sum_{i=1}^n w_i u_i \quad 2)$$

To encourage students not to repeat the assessment, students who achieve proficiency in a particular unit for the first time will receive full marks. Each unit specified will carry a weighting factor. The weight of each unit should be based on hierarchical learning objectives (Highley & Edlin, 2009). Assessment questions are designed to understand concepts but not by memorizing content. Then, each group selected one student to be interviewed.

RESULTS and DISCUSSION

Student Test Results Before and After Using MLAM Assessment

The evaluation results in the first unit in each group with low, medium, and high dispositional abilities through tests containing mathematical representation abilities show completeness data, as shown in Table 3 below.

Table 3. Category Students' Mathematical Disposition Ability

Category	Number of Students	Code	Complete	Not Complete
High (T)	4	T1, T2, T3, T4	T1, T2	T3, T4
Medium (S)	7	S1, S2, S3, S4, S5, S6, S7	S1, S2, S3, S6	S4, S5, S7
Low (L)	9	L1, L2, L3, L4, L5, L6, L7, L8, L9	L1, L2, L2	L3, L4, L5, L6, L7, L8, L9
Total		20	9	11

Based on the data in table 3, the test in the first unit shows that the completeness reaches an average of 45%. This shows that many students still have not completed and that a reassessment is needed. Students are required to repeat their assessments in certain units until they master the repeated assessment units within the specified period. Students who score less than 75% will repeat the same unit but will be allowed to continue at the same time to the next unit. Meanwhile, students who score 75% for assessment in certain units are considered to have mastered certain units and can continue to the next unit. Students who have mastered the first unit will get full marks for the first time, which is considered to have achieved the learning objectives. In contrast, students who have not mastered the first unit will be encouraged with questions with assessment questions that are arranged to understand the concept but not by memorizing its contents. For example, the following is one of three similar assessment questions tested on topics related to counting techniques.

Set 1: First Question (Q1). How many different routes can a bus take from City A to City C if:

- (i) There are three roads from City A to City B.
- (ii) There are four roads from City B to City C.
- (iii) must pass through the city of B.
- (iv) There is no other way to get from City A to City C than via City B.

Set 2: The First Question (Q1). How many ways can the school choose a candidate to represent the school in a competition?

- (i) The school only chooses two representatives.
- (ii) Regardless of the order,

Based on the description and the MLAM method, the results are described in table 4 below!

Table 4. Category Group of Students' Mathematical Disposition Ability with MLAM

Category	Number of Students	Code	Complete	Not Complete
Tinggi (T)	4	T1, T2, T3, T4	T1, T2, T3, T4	-
Sedang (S)	7	S1, S2, S3, S4, S5, S6, S7	S1, S2, S3, S4, S7	S5, S6
Rendah (L)	9	L1, L2, L3, L4, L5, L6, L7, L8, L9	L1, L2, L2, L6, L8, L9	L3, L4, L5, L7
Total		20	15	5

Referring to the MLAM data and assessment results, one subject from each group will be taken as a representative in each category by taking into account the highest score. Furthermore, after testing the mathematical representation problem based on the ability of mathematical disposition, it will be presented with a description as follows.

Students' Mathematical Representation Ability in the First Indicator

First Indicator: in solving a problem, a visual representation will be used in the low disposition group (L1).

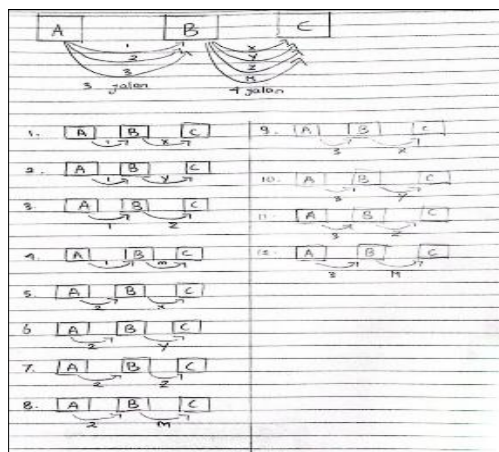


Figure 3. Subject L1's Answer to Solving the First Problem

The answer in Figure 3 by subject L1 has shown the ability of visual representation in the form of a detailed description of the road route on each road that will be traversed from City A to City B and then to City C.

First Indicator: in solving a problem, a visual representation will be used in the moderate disposition group (S1)

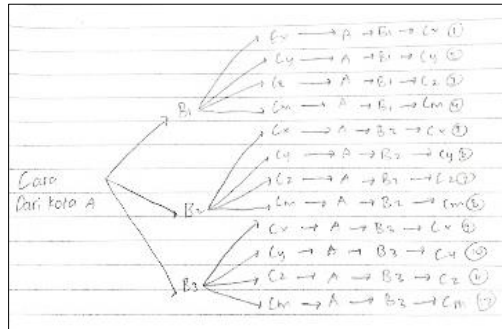


Figure 4. Subject S1 Answers in Solving the First Problem

The answer written by the subject S1 in Figure 4 has shown the ability to visually represent using a tree diagram to map the road from City A to City B, then City B to City C according to what the subject already knows.

First Indicator: in solving a problem, a visual representation will be used in the high disposition group (T1)

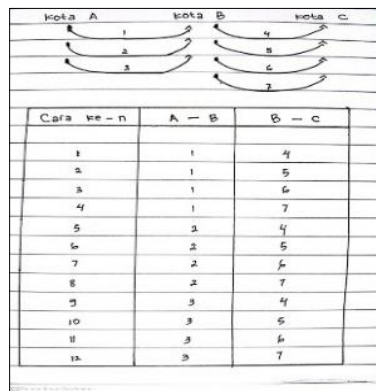


Figure 5. Subject T1's Answers in Solving the First Problem

Based on the answers written by subject T1 in Figure 5, it has also brought up the ability of visual representation by using the form of a table to show how many ways to map roads from City A to City B, then City B to City C specifically.

Figures 3, 4, and 5 above illustrate students' ability to solve problems involving the alignment of several types of images. All students with high and low mathematical and moderate abilities could answer the first question correctly. Therefore, it was reported that almost all students could show and count the number of ways of travel starting from the route from city A to city C by passing through city B.

Students' Mathematical Representation Ability in the Second Indicator

Second Indicator: presenting data/information from a representation in the form of charts, graphs, or tables and solving a problem orally or in writing in the low disposition group (L1)

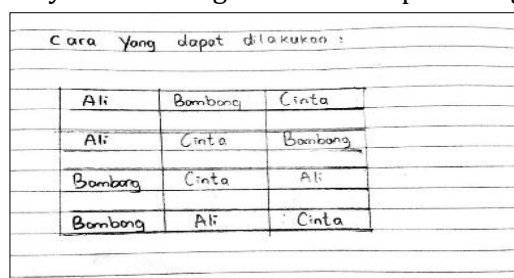


Figure 6. Subject L1's Answer to Solving the Second Problem

Second Indicator: presenting data/information from a representation in the form of charts, graphs, or tables and solving a problem orally or in writing in the moderate disposition group (S1)

$$\begin{aligned} \text{Cara} &= \frac{3!}{2!} \\ &= \frac{3 \times 2 \times 1}{2 \times 1} \\ &= \frac{6}{2} \\ &= 3 \text{ Cara} \end{aligned}$$

Figure 7. Subject L1's Answer to Solving the Second Problem

Based on students' answers shown in Figures 6 and 7, this Indicator shows students' ability to solve problems by presenting data in numerical form and identifying problems using words. Most students with high math ability could answer the second question correctly. On the other hand, almost all students with low manipulative ability could not solve the second problem correctly. This implies that students with weak mathematical abilities can present data in numerical form and solve problems using words or text. Meanwhile, for students with average mathematical ability, the number of students who can answer is almost the same as the number of wrong students.

Furthermore, the second problem is combined with the second Indicator of mathematical representation, namely presenting data or information from the representation into diagrams, graphs, or tables and answering questions using words or written text. This Indicator shows students' ability to solve problems by presenting data in numbers and identifying problems using words. Most students with high mathematical disposition ability could answer the second question correctly. On the other hand, almost all students with low mathematical disposition abilities could not solve the second problem correctly. This implies that students with low mathematical dispositions struggle to present data in the form of numbers and solve problems using words or written text. Meanwhile, the number of students with moderate mathematical ability who can answer correctly is almost the same as that of students who answer incorrectly.

Students' Mathematical Representation Ability in the Third Indicator

Third Indicator: developing mathematical equations or models from the given representations and solving problems involving mathematical expressions in the low disposition group (L1)

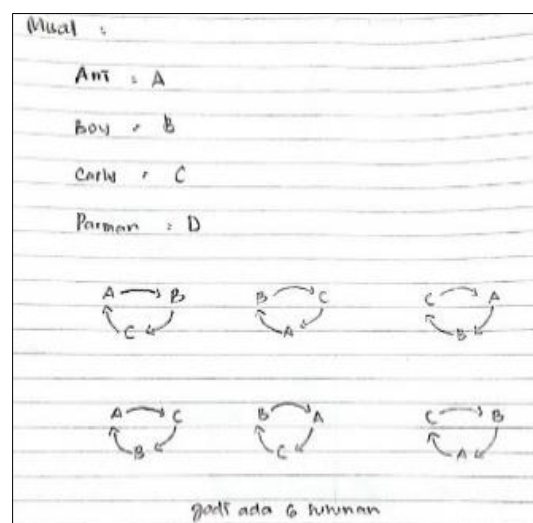


Figure 8. Subject S1's Answers in Solving the Third Problem

Third Indicator: developing mathematical equations or models from the given representations and solving problems involving mathematical expressions in the moderate disposition group (S1)

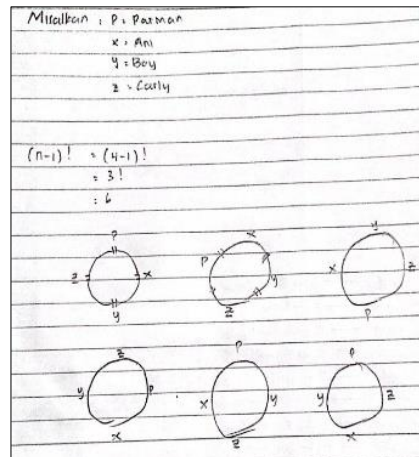


Figure 9. Subject S1's Answers in Solving the Third Problem

Third Indicator: developing mathematical equations or models from the given representations and solving problems involving mathematical expressions in the high disposition group (T1)

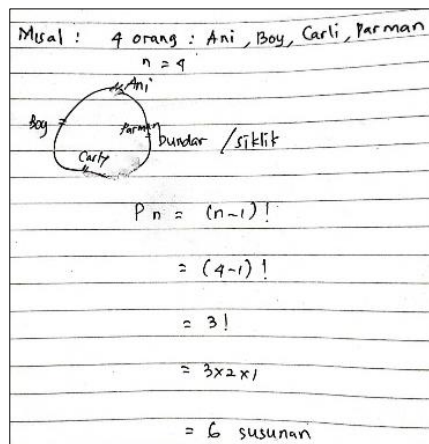


Figure 10. Subject T1's Answers in Solving the Third Problem

The third problem is the third mathematical representation index, which is developing mathematical equations or models and solving problems involving mathematical expressions shown in figures 8, 9, and 10. It shows the ability of students to create given mathematical equations or models and solve problems using mathematical expressions. Based on Table 3, most of the students have high and moderate mathematical abilities and good problem-solving abilities. This shows that students have made mathematical models and solved problems using appropriate mathematical expressions.

In addition, the fourth problem concerns the fourth index of mathematical descriptions, namely, drawing patterns to clarify problems and facilitate problem-solving. This metric describes a student's ability to draw in a given situation. Overall, only eight students were able to draw patterns based on the given situation. This shows that the students' ability to draw patterns is still weak. The final problem then assesses five competency indicators in mathematical representation, namely working on word problems in certain situations. This Indicator describes students' ability to form questions orally or in writing, depending on the given situation.

According to the test results, students' ability to form verbal questions in certain situations is as bad as drawing patterns. Only eight students can ask questions with the given scenario words. Based on the research results, most students are competent in the first metric, namely the ability to use visual representations to solve mathematical problems. They already know and can distinguish the form of determining the traverse road route. This observation confirms Apriani (2016) that students prefer to use patterns to solve problems well. Students also use pictures to clarify what is in their minds so that, from the pictures, students come up with ideas to solve problems. The research of Kholiqowati et al. (2016) concluded that participants could have a good visual representation category for each type of student characteristic.

Students' Mathematical Representation Ability in the Fourth and Fifth Indicators

The data in Table 12 shows that the students' ability to solve the 4th question is lower than the 5th question. This shows the difficulty of students in solving the two questions. In other words, a misunderstanding prevented them from properly handling both matters.

Figure 12 shows the responses of students with high processing speeds to solve the fourth problem.

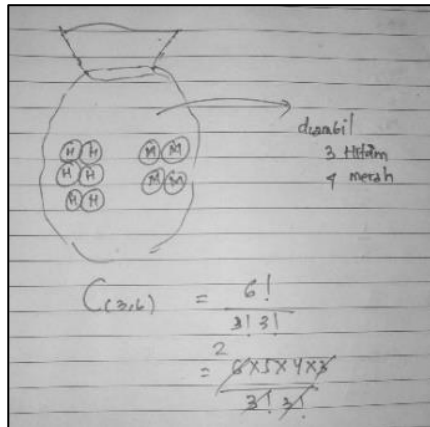


Figure 11. Subject T1's Answer to Solving the Fourth Problem

Figure 11 shows that students cannot correctly draw (sketch) the given situation. The sketches drawn by students do not match the given situation. This illustrates that students have not been able to draw a pattern of counting rules to clarify and facilitate completion. In the next question, students are asked to write down the steps they need to count as many ways as possible if at least three black balls are drawn. The answer is just a solution without any explanation to solve the problem. However, students cannot determine many ways that can be done correctly to perform calculations with the combination concept. Moreover, students have not been able to solve math problems from the examples that have been given. Likewise, students with moderate and weak mathematical abilities also.

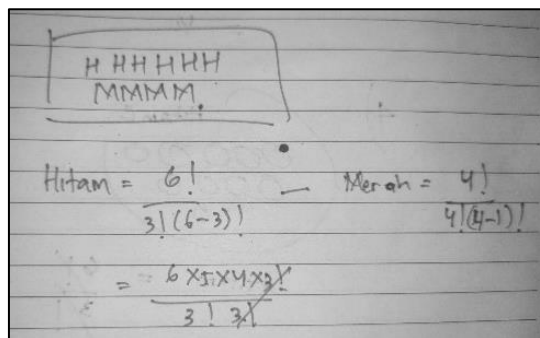


Figure 12. Subject S1's Answers in Solving the Fourth Problem

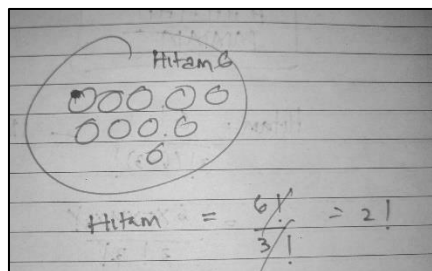


Figure 13. Subject L1's Answer to Solving the Fourth Problem

In Figure 12, it can be seen that students with average sorting skills cannot draw sketches of certain situations. The sketch is not suitable for the given problem situation. In addition, students cannot explain the procedure for determining the number of ways on the ball. This directly calculates the fourth problem to determine the height of the tower. As a result, an error occurs when specifying the dimensions and comparing the values with the given faces. In addition, students with low mathematical proficiency could not describe the given situation correctly, as shown in Figure 13. Students writing

down the steps to count how many ways on the ball were also incorrect. Therefore, calculating the number of balls is not very suitable.

Based on the student's answer, it can be concluded that the student cannot draw the given situation. Most of the students failed to represent the situation in their sketches and did not understand the appropriate pairs. They didn't write the proper procedure to solve the problem. They also have difficulty determining the appropriate partner in the calculation.

The results of this study strengthen the conclusion of Husna (2015), which concludes that the difficulties faced when solving words related to mathematical representations are finding keywords and understanding relevant information. Dewi and Sopiany (2017) research also concludes that students' misconceptions in solving problems in the ability index to ask questions or collect data are given. Below, examples of student responses to the five high, medium, and low problem-solving problems are shown in Figures 14, 15, and 16.

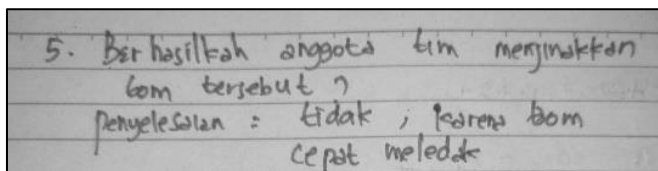


Figure 14. Subject T1's Answer to Solving the Fifth Problem

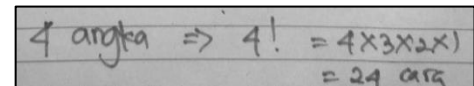


Figure 15. Subject S1's Answer to Solving the Fifth Problem

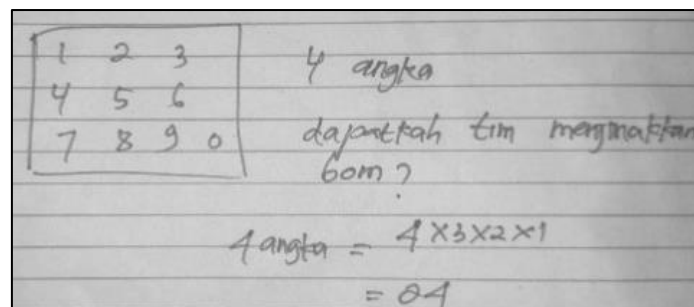


Figure 16. Subject L1's Answer to Solving the Fifth Problem

Students with a high ability to express mathematics cannot create problems involving homogeneity using words or written texts. The answer does not fit the given situation, so the answer is wrong. In addition, students with average math skills demonstrated the ability to correctly defuse a bomb despite the erroneous assumption that there was still a next step in determining the time required. Students who are weak in the mathematical layout can identify the required number digits. However, he assumed that the time needed was 24 seconds. It should not be 24 seconds but 24 ways. Since one method takes 2 seconds, $24 \times 2 = 48$ seconds is faster than 2 minutes in the time it takes to defuse the bomb.

Based on the answers in Figure 16, they generally assume that the team cannot defuse the bomb. The team can defuse the bomb. They were only asked to calculate the time required for each method used in the numbers. This research is in line with the findings of Dewi and Sopiany (2017), which state that students have relatively weak mathematical representations, especially the ability to create problem-based situations given data or representations. Students have difficulty asking questions according to the information or data provided. Because it only focuses on the image without considering other information, so the problem formulation is wrong. Furthermore, Herlina et al. (2017) concluded that, generally, the representation ability of high school students has a low average ability.

The focus of the discussion of this research is on students' mathematical representation ability, mathematical disposition, and MLAM assessment. From the observations made during the assessment, most students asked many questions before starting the assessment from several previous assessments. This shows that students learn on their own instead of tutoring by giving very positive and negative feedback. From the research that has been done, it is hoped that other researchers can develop it using different material topics.

CONCLUSION

Based on the research, the results show that 75% of students who have taken MLAM have completed their course. In addition, 70% of students who took the MLAM achieved an overall score or higher on their final question. It should be emphasized here that the 3-year rate is recorded for students who have taken MLAM in the course of study. A greater worth than their failure It was found that students who went through the MLAM process and failed to achieve the passing grades set by YALC engaged in self-discipline, namely negative attitudes towards subjects and a lack of seriousness in lessons and classroom activities, and school activities.

The mathematical representation ability of high school students using the MLAM method can be built as follows: 1) The majority of students with high, medium, and low mathematical representation abilities achieve the first and third mathematical indexes, namely by using visual representations to solve problems and develop mathematical equations or models, or by solving problems involving mathematical expressions. 2) Students with strong mathematical skills can present data or information in diagrams, graphs, or tables and solve problems using written versions of words or text. On the other hand, weak students in mathematics can't achieve this target. As for students with moderate learning power, it is relatively balanced. 3) Most students with high, medium, and low mathematical abilities failed to meet the fourth and fifth mathematical representations, namely writing mathematical steps in words and solving problems with mathematical expressions; and creating problematic situations based on the data or representations provided. Further research is recommended to improve students' mathematical representation and mathematical liking ability. Using a learning model as an independent variable in research can support the learning process.

AUTHOR CONTRIBUTIONS STATEMENT

MS and RD shape the ideas presented. MS develops theory and performs calculations. MS and RD experimented. MS wrote scripts with NR and RS support. GH and HI compile into MLAM ide. MS develops theoretical formalism, performs analytical calculations, and performs numerical simulations. Both MS and RD authors contributed to the final version of the manuscript. All authors provided critical comments and helped shape the study, analysis, and manuscript. MS and RD designed models and frameworks for computing and data analysis. MS designs and performs experiments, models, and data analysis. RD is supported by XYZ measurement, and NR helps to perform XYZ simulation. MS and RD drafted the manuscript with NR, RS, and NH. MS design project, main concept idea, and proof outline. RD did almost all the suggested experiments' technical details and numerical calculations. With the help of NH, NR goes beyond the limits of quantum mechanics. RS verifies XYZ numerical results with independent implementation. RS and NH proposed the XYZ trial in discussion with MS, RD, NR, RS, and NH writing the manuscript.

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