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Addressing misconceptions in ratio and proportion: A constructivist approach for enhancing grade 9 learners' understanding

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Abstract

Backgorund: Proportionality, deeply rooted in the fundamental principles of ratio and proportion, is a core component of mathematics education that supports cognitive processes such as problem-solving and reasoning. Despite its inclusion in global curricula, empirical research suggests that students often face significant challenges in applying ratio and proportion in problem-solving scenarios. These challenges are largely attributed to the use of flawed cognitive strategies, resulting in persistent misconceptions.
Aim: This study examines the misconceptions regarding ratio and proportion among Grade 9 learners in the Lebowakgomo District, Limpopo Province, and aims to explore how these learners conceptualize and solve related mathematical problems through the lens of constructivist learning theory.
Method: Employing a qualitative case study approach, this research utilized tests and semi-structured interviews to gather data from 30 Grade 9 learners. A representative subset was selected for qualitative analysis, with five learners chosen for follow-up interviews to ensure diverse perspectives.
Results: The study revealed that learners struggled primarily with problems requiring proportional reasoning, resorting to incorrect strategies such as intuitive thinking, additive methods, incomplete problem-solving approaches, and erroneous use of cross-multiplication.
Conclusion: These misconceptions are primarily due to a lack of foundational mathematical skills and conceptual understanding, which impede learners' ability to engage in proportional reasoning. Based on these findings, the study advocates for enhancing teachers' pedagogical content knowledge in ratio and proportion to improve the instructional support provided to learners.

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INTRODUCTION

Ratio and proportion are foundational mathematical concepts that are integral to both real-world applications and advanced mathematical understanding. The study of ratios and proportions is a key component of the mathematical curriculum (Smith, 2022; Boye & Agyei, 2023). It is a fundamental subject that is more significant in most sectors of advanced study, including engineering, business, and the arts (Jitendra et al., 2019). Even at lower levels, such as in the high school curriculum, courses like geography, the arts, and sciences require an understanding of ratio and proportion. Despite their importance, many grade 9 learners face significant challenges in understanding these concepts, often due to misconceptions that are deeply ingrained and impede their learning progress. According to Moyo and Machaba (2021), these misconceptions often arise from previous learning experiences, everyday interpretations, or a lack of conceptual understanding, which can result in errors in problem-solving and reasoning. Malatjie and Machaba (2019) describe conceptual understanding as the knowledge that is constructed through restructuring and reorganising the prior related concept with the new incoming knowledge, resulting in better understanding and grasping of the learnt material.

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Global performance data from the Trends in International Mathematics and Science Study (TIMSS) revealed that South Africa's grade 9 Mathematics results fell significantly below international benchmarks, particularly when compared to countries like Japan and the Netherlands. (Mullis & Martin, 2017). Again, TIMMS project ended their report by pointing out what high-ranked nations in mathematics education do, in terms of their high mathematics better performance than South Africa. It stated that South Africa as a nation is still ranked lowly in its mathematics performance globally. According to TIMSS (2018) it is suggested that, in general, mathematics seems to be in high regard regionally, nationally and internationally; however, learners' performance remains very poor.

The Department of Basic Education (DBE, 2011a) in the Annual National Assessment (ANA) reports emphasizes that senior-phase learners should be able to formulate ratios from given quantities and compute quantities based on given ratios. Bango (2020), agrees with the view that learners show a complete lack of understanding of the basic concept of ratio; hence learners just use meaningless algorithms in trying to solve ratios encountered in everyday contextual problems. However, the reports (ANA) according to Bango (2020), propose the following interventions: at the senior phase the conceptual understanding and symbolic representation of ratio should form part of the basic knowledge learners are expected to have. It also proposed that learners should be given varied applications involving the calculations of ratios and equivalent forms of ratios should be included in these applications.

This study seeks to identify common misconceptions in ratio and proportion among Grade 9 learners using a constructivist approach, which fosters active learning by encouraging students to construct their understanding through exploration, discussion, and reflection. By leveraging this approach, educators can create meaningful learning experiences that help students correct their misunderstandings and develop a deeper, more accurate comprehension of ratio and proportion. Addressing these misconceptions and implementing effective instructional strategies are crucial for enhancing students' mathematical proficiency and boosting their confidence. This study will explore prevalent errors, analyse their underlying causes, and propose constructivist-based interventions to enhance students' conceptual clarity and problem-solving skills in ratio and proportion. This research will help all alternative stakeholders like subject advisors, syllabus planners to come up with ways to impart the best teaching of mathematics, especially of ratios and proportions.

THEORETICAL FRAMEWORK

This study is grounded in the constructivist learning theory, which posits that learners actively construct knowledge based on their experiences and prior understanding. Constructivism, as articulated by theorists such as Piaget (1991) and Vygotsky (1978), emphasizes the importance of meaningful learning experiences where students engage in exploration, reflection, and social interaction to develop a deeper understanding of mathematical concepts. According to Malatjie and Machaba (2019), constructivism's view emphasises that the construction of knowledge depends on what the child already has, what he to conceptualise and, hence, in the process, the learner understands what he will be doing as an active participant of the learning process. Abakah (2019), states that knowledge is constructed through re-structuring and re- organising the prior knowledge with the new incoming knowledge. He suggests that misconceptions do not mean that learners are dull or stupid and empty, but that is how knowledge is constructed within the mental structure of a learner during the teaching and learning processes. Abakah (2019) points out that in the process of constructing knowledge by learners through social interaction, insights, meanings, and misconceptions can occur. That is when the teacher comes in so that he becomes the facilitator or guide of the learning process.

In the context of ratio and proportion, misconceptions often arise due to fragmented or superficial knowledge acquired through rote learning rather than conceptual understanding. Piaget (1991) theory of cognitive development suggests that learners must progress through stages of development to construct more sophisticated mathematical reasoning. At the formal operational stage, typical of grade 9 learners, students should be able to think abstractly and logically about proportional relationships. However, persistent misconceptions may indicate gaps in their conceptual development, requiring targeted interventions that align with constructivist principles. Vygotsky (1978) concept of the Zone of Proximal Development (ZPD) further supports the need for instructional scaffolding to help students bridge the gap between their current understanding and the desired level of competence. Through guided learning, collaborative problem-solving, and strategic questioning, educators can facilitate conceptual change and encourage learners to reconstruct their understanding of ratio and proportion (Mufit & Fauzan, 2023; Valsecchi et al., 2024). The study also draws on Bruner's theory of discovery learning, which underscores the significance of active engagement and exploration in the learning process (Arsyad et al., 2024). By encouraging students to investigate and analyse proportional relationships in real-world contexts, educators can help them develop a more intuitive and flexible understanding of these concepts.

Furthermore, misconceptions in ratio and proportion can be analysed through the lens of conceptual change theory, which suggests that learners must first recognize inconsistencies in their existing knowledge before they can adopt new, more accurate understandings. Strategies such as cognitive conflict, hands-on activities, and reflective discussions align with the constructivist approach to challenge and replace erroneous beliefs with correct conceptual frameworks. In summary, this study is framed by constructivist learning theories that advocate for an active, student-centred approach to addressing misconceptions in ratio and proportion. By leveraging the insights from Piaget, Vygotsky, and Bruner, the research aims to identify common errors, understand their cognitive origins, and implement pedagogical strategies that foster meaningful and lasting learning outcomes among grade 9 learners.

LITERATURE REVIEW

Definition of Ratio and Proportion

Lestari and Murtafiah (2020) define a ratio as a measure comparing two quantities, which becomes a rate when the quantities have different units. Dougherty et al. (2017) further explain ratios can be expressed as fractions, decimals, and percentages. Phuong & Loc (2020) emphasize the connection between ratios and fractions, which aids learners in understanding equivalent fractions. Olivier (1992) distinguishes between ratios as comparisons within a set and as properties defining proportionality between sets, with relationships such as $y = kx$. Hino & Kato (2019) categorize ratios, rates, percentages, and proportions under the Multiplicative Conceptual Field (MCF), emphasizing their interconnectivity. Jitendra et al. (2021) further supports the view that ratio, rate, proportion, and percentages form an inseparable web of mathematical concepts.

Understanding Misconceptions

Aliustaoğlu et al. (2018) define misconceptions as systematic misunderstandings that hinder learners' comprehension of mathematical concepts. Sibanda (2021) distinguishes errors from misconceptions, noting that errors are systematic mistakes, whereas misconceptions stem from flawed cognitive frameworks. Ginsburg (1977) identifies faulty rules and erroneous assumptions as the basis of misconceptions and underscores the importance of addressing these issues early. Vanluydt et al. (2022) highlight the importance of proportional reasoning in everyday life, while emphasizes its application across various disciplines. Sawatzki et al. (2019) argue that proportional reasoning is critical for practical applications such as financial literacy and fairness.

Common Misconceptions in Ratio and Proportion

Pedersen and Bjerre (2021) argue that misconceptions arise from incorrect strategies learners employ to solve ratio problems. Olivier (1989) stresses that ineffective teaching strategies contribute to persistent misconceptions. Burgos and Godino (2020; 2021) identify several incorrect strategies used by learners, including:

- Additive Strategy: Misinterpreting ratios by adding or subtracting terms rather than scaling proportionally.
- Incomplete Strategy: Using only one measure space, leading to incorrect solutions.
- Guesswork/Intuitive Strategy: Providing baseless answers without processing information.
- Magical Doubling Strategy: Incorrectly applying doubling with an additive approach.
- Incorrect Multiplication Strategy: Applying multiplication with the wrong factors
- Incorrect Cross-Multiplication Strategy: Setting up proportions incorrectly, leading to flawed solutions.
- Build-Up Strategy: Incorrectly applying proportional relationships within a ratio.
- Routine Errors: Applying direct proportion instead of proportional division.

Causes of Misconceptions

Ginsburg (1977) categorizes misconceptions into those arising from insufficient basic skills, procedural learning, and overgeneralization. Burgos and Godino (2020; 2021) suggest that learners' reliance on disjointed rules leads to misunderstanding. Copur-Gencturk (2021) distinguishes between procedural and conceptual knowledge, noting that an overemphasis on procedures can hinder deep understanding. Frimpong (2021) argues that both procedural and conceptual knowledge are essential for mathematical comprehension. However, many educational systems focus heavily on procedural instruction, resulting in learners who rely on rote memorization. Olivier (1989) warns against oversimplification of prior knowledge, which can lead to errors when applied to more complex concepts.

In the South African context, Thompson and Hare (2021) highlight a key misconception arising from the separation of ratios and rates in the curriculum, unlike international curricula that consider rates a subset of ratios. Misunderstandings also stem from language and notation inconsistencies, such as interpreting "three-fifths" incorrectly. Petit (2020) recommends encouraging exploration and independent thinking to combat misconceptions. Constructivist approaches emphasize the teacher's role as a facilitator, guiding learners to build their conceptual understanding rather than relying on memorized rules. In conclusion, addressing misconceptions in ratio and proportion requires a multifaceted approach, incorporating conceptual teaching, early detection, and encouraging learners to develop a deep understanding of mathematical relationships.

METHOD

A research paradigm provides a framework within which reality is constructed and interpreted. Voce (2014) describes a paradigm as a foundational framework that influences how a researcher perceives the world, establishes their viewpoint, and defines knowledge connections. In this study, misconceptions and misunderstandings were considered abstract and dynamic processes that humans create and experience within mathematical contexts. The study aimed to uncover learner misconceptions and misunderstandings before seeking patterns and deeper meanings within mathematics education.

Among the commonly used research paradigms interpretivism, social constructionism, critical paradigms, and positivism and this study was guided by Interpretivism. As Cohen et al., (2000) suggest, interpretivism focuses on human research where individuals interpret their social roles within relationships to derive meaning from both their roles and those of others. This paradigm allowed for an in-depth exploration of grade 9 learners' misconceptions about ratio and proportion.

Research Approach

Guided by the interpretivist paradigm, the research approach for this study was qualitative. According to Tracey (2020), qualitative research includes methods such as interviews, textual analysis, and observations, offering a rich and holistic understanding of phenomena. Qualitative research is characterized by its ability to:

- Provide a deep understanding of the phenomenon in its natural context.
- Capture the lived experiences of participants.
- Offer insights into processes over time.
- Honor participants' local meanings and interpretations.

The study employed qualitative methods to analyse learners' responses to tests and semi-structured interviews conducted within the classroom environment. This approach was deemed appropriate as it provided detailed insights into learners' mathematical thinking processes. The natural setting of the classroom was utilized to ensure authenticity in the collected data.

Research Design

The research design adopted for this study was a case study. According to Yin (2013), case studies are effective in answering "how" and "why" questions and enable researchers to obtain holistic perspectives on real-world phenomena. This design was chosen to explore misconceptions about ratio and proportion among grade 9 learners in a rural school in the Mphahlele circuit. Piekkari & Welch (2018) support the case study approach for its ability to provide an in-depth analysis of qualitative data, allowing the researcher to interpret patterns and draw meaningful conclusions. The case study design enabled an exploration of learners' mathematical understanding without influencing their behaviour.

Research Setting

The study was conducted in the Mphahlele circuit, a rural community in the Lebowakgomo District of Limpopo Province, South Africa. The circuit comprises nine public secondary schools with approximately 400 grade 9 learners and 12 mathematics teachers. This location was selected due to the lack of prior studies focusing on misconceptions related to ratio and proportion within grade 9 classrooms. Krishna (2002) defines a population as a group targeted for study, while a sample represents a subset of that population. The total population for this study included 400 grade 9 learners, comprising 190 males and 210 females. 30 learners from Grade 9 class were with the intent of capturing a representative subset for qualitative analysis.

The sampling process involved selecting a school randomly from the circuit, followed by choosing a class of 30 learners. Each learner was assigned a number (01-30) to maintain anonymity. Five learners were then selected from the test group for interviews, ensuring a diverse representation of perspectives. Purposeful sampling allowed for an in-depth exploration of misconceptions within the selected population.

According to Cohen et al. (2000), common data collection methods in case studies include observations, interviews, tests, and document analysis. This study employed two primary methods: tests and semi-structured interviews. Tests were used to elicit learners' misconceptions regarding ratio and proportion. Reynolds et al., (2021) define tests as instruments designed to measure knowledge and skills. In this study, a direct exploration approach was adopted, allowing learners to provide open-ended responses that offered valuable insights into their understanding. A seven-item test, based on Bloom's taxonomy (Agarwal, 2019), was administered to assess learners' conceptual grasp of ratio and proportion. Learners were assured that the test was solely for research purposes to encourage honest participation. The test was conducted within a one-hour timeframe and focused on conceptual understanding and application.

Following the test, semi-structured interviews were conducted with five selected learners. These interviews aimed to explore learners' thought processes and the factors contributing to their misconceptions. The interviews were recorded with participants' consent and provided rich qualitative data for analysis. The combination of test and interview methods allowed for triangulation, ensuring a comprehensive understanding of the learners' challenges in mastering ratio and proportion concepts.

RESULTS AND DISCUSSION

The study uncovered several common misconceptions among learners, including:

- Additive Strategy Misconception: Learners often applied additive reasoning instead of the appropriate multiplicative reasoning when comparing ratios.
- Incorrect Cross-Multiplication: Many learners misapplied cross-multiplication, leading to incorrect solutions.
- Incomplete Strategy: Some learners relied on partial solutions without fully comprehending the relationship between quantities.
- Guesswork and Intuitive Reasoning: A significant number of learners resorted to guessing rather than applying logical reasoning.
- Misinterpretation of Ratio Language: Learners faced challenges with the mathematical language, leading to incorrect problem interpretations.

These findings indicate that misconceptions stem from a lack of foundational knowledge, ineffective teaching strategies, and limited exposure to real-world applications of ratio and proportion. The analysis focuses on learners' performance and highlights misconceptions identified for each question. Learners were instructed to show all necessary workings, as explicitly stated on the question paper and emphasized before the test.

Misconceptions Related to Question 1

Question 1: Onion soup recipe for 8 people:

- 2 pints of water
- 4 cubes of chicken soup
- 2 spoons of butter
- 1/2 pint of cream

Learners were asked to adjust the recipe for different serving sizes:

(a) How much water is needed for 4 people?
(b) How many cubes of chicken soup are needed for 4 people?
(c) How much water is needed for 6 people?
(d) How many cubes of chicken soup are needed for 6 people?
(e) How much cream is needed for 6 people?

Learners' responses to Questions 1(a) and 1(b) are summarized in Table 3, while Table 4 presents responses for Questions 1(c), 1(d), and 1(e). The tables categorize responses based on strategies identified in the research literature.

Table 1. Learners' Responses to Questions 1(a) and 1(b)

Question	Correct (%)	Incomplete Strategy (%)	Incorrect Operation (%)	Incorrect Multiplicative Strategy (%)	Total (%)
1(a)	93.33	0.0	3.33	3.33	100
1(b)	90	3.3	6.7	0.0	100

For Questions 1(a) and 1(b), which required halving the original ingredient quantities, over 90% of learners provided correct responses. However, approximately 7% of errors were attributed to incorrect operations and cross-multiplication. Notably, some learners multiplied instead of dividing by 2, arriving at incorrect answers such as 4 and 8. To be precise, learners increased rather than halved. As shown in the table, the learner's improper cross-multiplication of 1(b) falls under this category. The following is Learner 5's response to Question 1(a), which includes further analysis to clarify the mistakes and misconceptions:

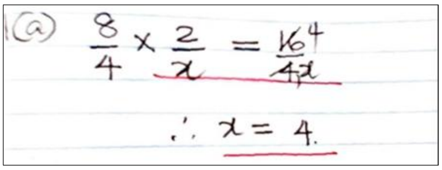

Figure 1. Learner 5’s extracted solution

A deeper analysis of Learner 5's response highlights a misunderstanding of cross-multiplication principles. The learner used a multiplication sign instead of an equal sign when forming equivalent ratios, leading to an erroneous solution. The interview with Learner 5 revealed confusion between cross-multiplication and fraction multiplication. Below is the interview:

Researcher : *Your answer to this question 1(b) is you got 4 pints, isn’t it? Tell me how you obtained it, please.*

Learner 5 : *I wrote 2 over x, which is the unknown fraction for the pints of water, and 8 over 4, which is the number of persons for whom I wish to make the soup. I then multiplied the two fractions I formulated together so that I can get the value of the unknown x. I then cross-multiplied, multiplied numerator by numerator, then over denominator by denominator. I got* $\frac{16}{4X}$*. I divided by 4, and I simplified it to* $\frac{4}{X}$*, meaning my final answer is 4.*

Researcher : *Okay, you said you got 4/x and finally your answer is 4.*

Learner 5 : *Yes sir, x=4.*

Researcher : *Thank you for the interview.*

Learner 5 : *My pleasure.*

Learner 5 committed a slip-type of misunderstanding. According to the learner script, the misconception occurs when a learner confuses cross multiplication with procedural multiplication involving fractions. This form of misunderstanding, according to Brodie (2010), is a conceptual framework that the learner creates that fits with her or his past knowledge but does not correspond to the learner's conventional mathematical understanding, according to Frimpong (2021), Learner 5 appears to mix up cross multiplication with fraction multiplication. Thus, the participant misunderstands the cross-multiplication approach or another mathematical procedure. Below are more examples of the same beliefs in script form from other learners:

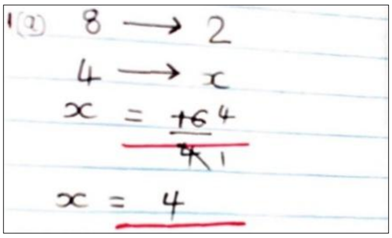

Figure 2. Learner 12’s extracted solution

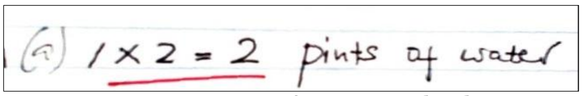

Figure 3. Learner 23’s extracted solution

Learner 21 provided the wrong answer of 16, which is displayed below:

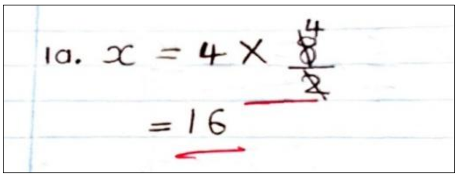

Figure 4. Learner 21's extracted solution

The maths solution seems not quite obvious, and it appears that the learners multiplied the soup's serving size of 4 by the number of persons in the recipe, 8, before dividing the result by 2 (amount of water needed to make soup for eight people). The ratio employed appeared to have been a factor drawn from many measure spaces, hence this misunderstanding is known as an inaccurate multiplicative method. Nearly 3% of participants in Question 1(a) and nearly 7% in Question 1(b) were found to have misperception. The following texts from learners who shared the same misunderstanding are displayed:

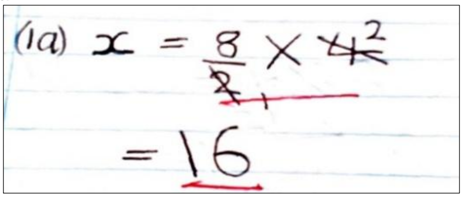

Figure 5. Learner 14's extracted solution

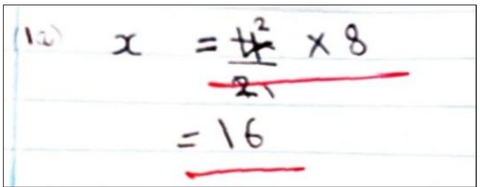

Figure 6. Learner 18's extracted solution

To get the ingredients needed for six people, learners were typically instructed to halve the answer (to get ingredients for four people), halve it again (to get ingredients for two people), and then add the two responses. Finding solutions to these issues requires a build-up approach. For instance, to determine how many cubes are required to make soup for six people, students had to split 4 cubes (the number of cubes required to make soup for eight people) in half to get 2 cubes (cubes needed to make soup for four people). They had to divide the two cubes in half once more to get one (cubes needed to make soup for two people) The total amount of chicken soup required for six persons would then be 2 cubes plus 1 cube, or 3 cubes. The following table shows the learners' responses based on the categorisation of strategies previously elaborated in the literature of my research.

Table 2. Learners' Responses to Questions 1(c), 1(d), and 1(e)

Question	Correct (%)	Incomplete Strategy (%)	Incorrect Cross-Multiplication (%)	Incorrect Multiplicative Strategy (%)	Other Incorrect Strategies (%)	Total (%)
1(c)	53.3	6.7	6.7	6.7	13.3	100
1(d)	53.33	30	3.33	3.33	10	100
1(e)	6.7	30	16.7	3.3	40	100

Responses to Questions 1(c), 1(d), and 1(e) revealed significant challenges, with only 53% of learners correctly answering Questions 1(c) and 1(d), and just 7% correctly answering Question 1(e). Errors were largely attributed to incomplete strategies and incorrect cross-multiplication. For instance, Learner 16 computed the answer wrong for Question 1(e), as shown below:

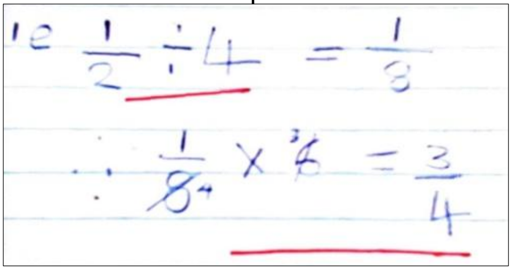

Figure 7. Learner 16's extracted solution

Interviews with learners, such as Learner 16, demonstrated confusion in applying proportional reasoning correctly. Misconceptions stemmed from misapplying numerical values, reliance on given ingredient values, and misunderstanding fraction division. Below is a transcript of an interview with Learner 16:

Researcher	: *I see you got ¾ for your answer to question 1(e), kindly elaborates on your solution.*
Learner 16	: *divided* $\frac{1}{2}$ *by 4 and I got* $\frac{1}{8}$ *sir.*
Researcher	: *Okay. Why did you divide by 4? [Interviewer probes]*
Learner 16	: *Hmmmmm… I don't understand how I took and used that 4, but sir it's given on the ingredients list.*
Researcher	: *No problem, you can continue.*
Learner 16	: *I then divided by the number of 6 people for the soup preparation; finally, I got* $\frac{3}{4}$
Researcher	: *Thank you*
Learner 16	: *Pleasure sir.*

Overall, the analysis indicates a recurring trend of learners struggling with ratio and proportion concepts due to conceptual misunderstandings, procedural errors, and reliance on incorrect strategies. Addressing these challenges requires targeted interventions to strengthen foundational mathematical skills and enhance instructional methodologies.

Learner Conceptualisation Responses on Ratio and Proportion Items

This section addresses the research question: *How do Grade 9 learners comprehend the concepts of ratio and proportion?* Understanding learners' thought processes and conceptualisation of these mathematical ideas is crucial. To explore this, learners' responses were analysed from both written tests and interviews, providing insight into their approaches to solving ratio and proportion problems. A test was administered to all 30 learners, followed by unstructured interviews with five selected participants. This approach allowed for a comprehensive understanding of their perspectives, conceptualisation, and problem-solving strategies.

The findings indicate that a significant number of learners exhibited a lack of conceptual understanding of ratio and proportion. This was consistently evident across the test items and interview responses. Many learners equated ratios with fractions or vice versa, a common misconception. According to Beckmann & Izsák (2015), a ratio is a measure comparing two quantities with the same unit, whereas a rate compares quantities with different units, and a proportion represents an equality of two ratios. Dougherty et al. (2017) further explain that ratios can be expressed as decimals and percentages, emphasizing the distinction between ratio and proportion. Conversely, Van De Walle (2016) defines fractions as parts of a whole divided into equal portions.

It was observed that learners struggled with differentiating between ratio and fraction, ratio and rate, and ratio and proportion, often using these concepts interchangeably. This confusion reflects a fundamental gap in their conceptual understanding, which hinders their ability to apply these concepts effectively in problem-solving contexts. Beckmann & Izsák (2015) emphasize the importance of clear differentiation between these terms to facilitate deeper understanding.

How Grade 9 Learners Solve Ratio and Proportion Problems

The test questions were designed to assess learners' ability to manipulate and solve ratio and proportion problems, ranging from basic to more abstract concepts. Learners displayed varying levels of difficulty, with simpler questions often prompting them to write answers without showing calculations. This suggests an intuitive approach rather than a structured problem-solving process. For more complex questions that required deductive reasoning, many learners struggled to apply the appropriate strategies. Instead, they resorted to intuitive, additive, and incorrect multiplication techniques to arrive at answers. Nasir (2018) cautions against the overemphasis on procedural teaching without fostering proportional reasoning, which was evident in the learners' responses.

The South African Mathematics Curriculum Assessment Policy (CAPS) document (DBE, 2011) outlines essential skills such as logical reasoning, communication, and applying mathematical knowledge in various contexts. However, the findings suggest that learners have yet to fully develop these skills, leading to misconceptions and incorrect problem-solving approaches.

Misconceptions in Ratio and Proportion Demonstrated by Learners

The study identified several misconceptions among learners when solving ratio and proportion problems. These misconceptions include:

- Incorrect Use of Cross Multiplication: Learners often applied cross multiplication incorrectly, treating it as simple fraction multiplication. For instance, one learner incorrectly calculated the ratio 8:4 and 2:x as 16/4x, misunderstanding the concept and leading to an incorrect solution.
- Additive Strategy Instead of Multiplicative Reasoning: Some learners applied an additive approach instead of recognizing the multiplicative nature of ratio and proportion. For example, when given a problem involving proportional enlargement, they simply added values instead of scaling proportionally.
- Incomplete Strategy Implementation: Learners often extracted numbers from given ratios without considering their relationships, leading to incorrect answers. This strategy resulted in inconsistent and illogical solutions.
- Reliance on Intuitive (Guesswork) Strategies: A significant number of learners relied on guesswork without showing any working or logical reasoning, leading to inconsistent and incorrect answers.

The misconceptions identified highlight the need for targeted interventions to address these conceptual gaps and provide learners with a solid foundation in ratio and proportion concepts.

CONCLUSION

The findings of this study underscore the critical importance of understanding and applying the concepts of ratio and proportion, both in theoretical contexts and in solving practical, real-world problems. Mastery of ratio and proportion is essential not only for effective problem-solving in mathematics but also for applications in related disciplines, forming the foundational skills required for advanced mathematical reasoning and complex problem-solving. This study suggest that a thorough understanding of ratio and proportion can significantly enhance students' analytical skills, empowering them to approach both academic and real-world problems with greater confidence and precision. Educators and curriculum developers are encouraged to prioritize the teaching of ratio and proportion, ensuring that students develop the necessary mathematical competencies for both academic and real-life applications. Based on the findings, it is recommended that future teaching methodologies incorporate more interactive, hands-on activities and real-world applications of ratio and proportion. Such approaches could enhance student engagement, motivation, and conceptual understanding, fostering a deeper connection to the material. Additionally, further research should investigate innovative instructional strategies, examining their effectiveness in improving student learning outcomes in ratio and proportion, and their long-term impact on students' mathematical proficiency.

AUTHOR CONTRIBUTIONS STATEMENT

Masilo France Machaba spearheaded the conceptual framework and research design, taking the lead in analyzing the results and drafting the introduction and conclusion sections. Daniel Madzingira played a pivotal role in managing data collection and conducting analysis. He also contributed to interpreting the findings and refining the methodology section. As the corresponding author, he facilitated all communications with the journal and ensured smooth submission and revision processes.

REFERENCES

Abakah, F. (2019). *Exploring mathematics learners' problem-solving skills in circle geometry in South African schools*: *(a case study of a high school in the Northern Cape Province)* (Doctoral dissertation, University of South Africa). https://doi.org/10.1037/edu0000282

Agarwal, P.K. (2019). Retrieval practice & bloom's taxonomy: Do students need fact knowledge before higher order learning? *Journal of Educational Psychology*, *111*(2), 189.

Arsyad, S. N., Tangkin, W. P., Sumartono, S., & Astuti, B. (2024). Implications of bruner's cognitive theory on elementary school education in the 21st century. *Klasikal: Journal of education, language teaching and science*, *6*(3), 697-704. https://doi.org/10.52208/klasikal.v6i3.1225

Aliustaoğlu, F., Tuna, A. & Biber, A.Ç. (2018). The misconceptions of sixth-grade secondary school students on fractions. *International Electronic Journal of Elementary Education*, *10*(5), 591-599. https://doi.org/10.26822/iejee.2018541308

Bango, S. (2020). *An investigation into Grade 7 learners' knowledge of ratios* (Master dissertation, University of Pretoria).

Beckmann, S., & Izsák, A. (2015). Two perspectives on proportional relationships: Extending complementary origins of multiplication in terms of quantities. *Journal for Research in Mathematics Education, 46*(1), 17-38. https://doi.org/10.5951/jresematheduc.46.1.0017

Boye, E. S., & Agyei, D. D. (2023). Effectiveness of problem-based learning strategy in improving teaching and learning of mathematics for pre-service teachers in Ghana. *Social Sciences & Humanities Open*, *7*(1), 100453. https://doi.org/10.1016/j.ssaho.2023.100453

Brodie, K. (2010). *Teaching mathematical reasoning in secondary school classrooms*. New York: Springer. https://doi.org/10.1007/978-0-387-09742-8

Burgos, M. & Godino, J.D. (2020). Prospective primary school teachers' competence in analysing the difficulties in solving proportionality problems. *Mathematics Education Research Journal*, 1-23. https://doi.org/10.1007/s13394-020-00344-9

Burgos, M. & Godino, J.D. (2021). Prospective primary school teachers' competence for the cognitive analysis of students' solutions to proportionality tasks. *Journal für Mathematik-Didaktik*, 1-30. https://doi.org/10.1007/s13138-021-00193-4

Cohen, L, Manion, L & Morrison, K. (2000). *Research Methods in Education*. 5th edition. Routledge Falmer, London, UK: Falmer.

Department of Basic Education (DBE). (2011). Curriculum and assessment Policy statement Grades 7-9. *Mathematics. Pretoria, Department of Basic Education*. *15*(1).

Department of Basic Education (DBE). (2011a). *Report on the annual national assessment of 2010*. Pretoria: DBE.

Dougherty, B., Bryant, D.P., Bryant, B.R., & Shin, M. (2017). Promoting understanding of ratios and proportional reasoning for middle school students with persistent mathematics difficulties. *Teaching Exceptional Children, 49*(2), 96-105. https://doi.org/10.1177/0040059916674897

Frimpong, T.I. (2021). *An exploration of the extent Grade 9 mathematics teachers engages with learners' errors in the teaching and assessment of mathematics*. (Dissertation, University of KwaZulu-Natal).

Ginsburg, H. (1977). *Children's arithmetic: The learning process*. New York: D. Van Nostrand.

Hino, K. & Kato, H. (2019). Teaching whole-number multiplication to promote children's proportional reasoning: a practice-based perspective from Japan. *ZDM, 51*(1), 125-137. https://doi.org/10.1007/s11858-018-0993-6

Jitendra, A. K., Harwell, M. R., Im, S. H., Karl, S. R., & Slater, S. C. (2019). Improving student learning of ratio, proportion, and percent: A replication study of schema-based instruction. *Journal of Educational Psychology*, *111*(6), 1045. https://doi.org/10.1037/edu0000335

Jitendra, A.K., Harwell, M.R., Karl, S.R., Im, S.H. & Slater, S.C. (2021). Investigating the generalizability of schema-based instruction focused on proportional reasoning: A multi-state study. *The Journal of Experimental Education, 89*(4), 587-604. https://doi.org/10.1080/00220973.2020.1751580

Krishna S. O.R. (2002). *Methodology of Research in Social Sciences*. Delhi: Himalaya Publishing House.

Lestari, N.D.S. & Murtafiah, W. (2020, October). Exploring ethnomathematics activities in tobacco farmers' community at Jember, East Java, Indonesia. *Journal of Physics: Conference Series, 1657*(1), 012027. https://doi.org/10.1088/1742-6596/1657/1/012027

Machaba, F. M. (2016). The concepts of area and perimeter: Insights and misconceptions of Grade 10 learners. *Pythagoras, 37*(1), 1-11. https://doi.org/10.4102/pythagoras.v37i1.304

Mahlabela, P.T. (2012). *Learner errors and misconceptions in ratio and proportion: a case study of grade 9 learners from a rural KwaZulu-Natal school*. (Doctoral dissertation, University of KwaZulu Natal).

Mahlabela, P. T., & Bansilal, S. (2015). An exploration of learners' theorems-in-action used in problems on ratio and proportion. *Pythagoras, 36*(2), 1-10. https://doi.org/10.4102/pythagoras.v36i2.252

Malatjie, F. & Machaba, F. (2019). Exploring mathematics learners' conceptual understanding of coordinates and transformation geometry through concept mapping. *EURASIA Journal of Mathematics, Science and Technology Education, 15*(12), em1818. https://doi.org/10.29333/ejmste/110784

Misailidou, C., & Williams, J. (2003). Diagnostic assessment of children's proportional reasoning. *Journal of Mathematics Behaviour, 22*, 335-368. https://doi.org/10.1016/S0732-3123(03)00025-7

Moyo, M. & Machaba, F.M. (2021). Grade 9 learners' understanding of fraction concepts: Equality of fractions, numerator and denominator. *Pythagoras, 42*(1), 13. https://doi.org/10.4102/pythagoras.v42i1.602

Mufit, F., & Fauzan, A. (2023). The effect of cognitive conflict-based learning (CCBL) model on remediation of misconceptions. *Journal of Turkish Science Education*, *20*(1), 26-49. https://doi.org/10.36681/tused.2023.003

Mullis, I. V., & Martin, M. O. (2017). *TIMSS 2019 Assessment Frameworks.* ERIC.

Nasir, R. (2018). Identifying the Students' Proportional Reasoning. *International Journal of Educational Science and Research (IJESR), 8*(2), 71-78. https://doi.org/10.24247/ijesrapr201811

Olivier, A. (1989). Handling pupils' misconceptions. *Pythagoras, 21*, 10-19.

Pedersen, P.L. & Bjerre, M. (2021). Two conceptions of fraction equivalence. *Educational Studies in Mathematics, 107*(1), 135-157. https://doi.org/10.1007/s10649-021-10030-7

Petit, M.M., Laird, R.E., Wyneken, M.F., Huntoon, F.R., Abele-Austin, M.D. & Sequeira, J.D. (2020). A focus on ratios and proportions: Bringing mathematics education research to the classroom. *Routledge*. https://doi.org/10.4324/9780429353611

Phuong, N.T. & Loc, N.P. (2020). Solving word problems involving "ratio" concept of the fifth-Grade students: A study in Vietnam. *Universal Journal of Educational Research, 8*(7), 2937-2945. https://doi.org/10.13189/ujer.2020.080722

Piaget, J. (1964a). Cognitive development in children: Piaget development and learning. *Journal of Research in Science Teaching, 2*(3), 176–186. https://doi.org/10.1002/tea.3660020306

Piekkari, R. & Welch, C. (2018). The case study in management research: Beyond the positivist legacy of Eisenhardt and Yin. The SAGE Handbook of qualitative business and management research methods: History and Traditions (pp. 345-358). SAGE. https://doi.org/10.4135/9781526430212.n21

Sibanda, E. (2021). *Exploration of Grade 8 learners' misconceptions in learning surface area and volume of prisms at a high school in Johannesburg East District* (Doctoral dissertation, University of South Africa).

Smith, J. L. (2022). *Student reasoning, argumentation, and positioning relative to identity in inquiry-oriented undergraduate mathematics*. The Florida State University.

Thompson, P.W. & Harel, G. (2021). Ideas foundational to calculus learning and their links to students' difficulties. *ZDM–Mathematics Education, 53*(3), 507-519. https://doi.org/10.1007/s11858-021-01270-1

TIMSS, P. (2018). *TIMSS 2011 International Results in Mathematics.* IEA Secretariat.

Tracey, K. (2020). 'Several choice collections' in geometry, astronomy, and chronology: Using and collecting mathematics in early modern England. *In Reading Mathematics in early modern Europe* (pp. 266-293). Routledge. https://doi.org/10.4324/9781003102557-12

Valsecchi, W. M., Delfino, J. M., Santos, J., & Faraj, S. E. (2024). A problem-based learning activity for enhancing inquiry skills and facilitating conceptual change in a biological chemistry course. *Chemistry Education Research and Practice*, *25*(2), 438-457.

Van de Walle, J.A., Karp, K.S., & Bay-Williams, J.M. (2016). *Elementary and middle school mathematics*. London: Pearson Education UK.

Vanluydt, E., Verschaffel, L. & Van Dooren, W. (2022). The role of relational preference in early proportional reasoning. *Learning and Individual Differences*, 93, 102108. https://doi.org/10.1016/j.lindif.2021.102108

Vygotsky, L. S. (1987). *The collected works of LS Vygotsky: The fundamentals of defectology*. Springer Science & Business Media.

Yin, R.K. (2013). Validity and generalization in future case study evaluations. *Evaluation*, *19*(3), 321-332. https://doi.org/10.1177/1356389013497081