



Assessing technology integration in mathematics teaching: Development and preliminary content validation of an integrated didactic tetrahedron-TPACK instrument

Kartono

Universitas Terbuka, INDONESIA

Sudirman*

Universitas Terbuka, INDONESIA

Thesa Kandaga

Universitas Terbuka, INDONESIA

Mery Noviyanti

Universitas Terbuka, INDONESIA

Article Info

Article history:

Received: April 27, 2026

Revised: June 30, 2026

Accepted: Sept 02, 2026

Keywords:

Content Validation;
Didactic Tetrahedron;
Mathematics Teaching;
Technology Integration;
TPACK.

Abstract

Background: Meaningful technology integration in mathematics teaching requires teachers to coordinate technological knowledge with pedagogy, mathematical content, and classroom interactions. Although the Didactic Tetrahedron and Technological Pedagogical Content Knowledge (TPACK) provide complementary perspectives on this process, instruments integrating both frameworks remain limited.

Aim: This study aimed to develop an integrated Didactic Tetrahedron-TPACK instrument and establish preliminary evidence of its content validity for assessing technology integration in secondary mathematics teaching.

Method: As the initial stage of a larger sequential explanatory mixed-methods project, preliminary interviews with in-service mathematics teachers informed instrument development. A 30-item questionnaire covering seven TPACK domains and technology-mediated didactical relations, accompanied by a five-question semi-structured interview protocol, was subsequently evaluated through expert judgment.

Results: Preliminary findings revealed limited meaningful technology integration despite increasing emphasis on deep-learning pedagogy. Expert evaluation yielded an overall mean relevance score of 3.89 out of 4. Twenty-six questionnaire items and all interview questions were rated highly relevant, whereas four items required minor revision, with Technological Content Knowledge receiving the lowest domain score (3.50).

Conclusion: The instrument demonstrates promising preliminary content relevance for assessing technology integration in mathematics teaching, while further multi-expert validation and psychometric testing are required before large-scale implementation.

To cite this article: Kartono., Sudiraman., Kandaga, T. & Noviyanti, M. (2026). Assessing technology integration in mathematics teaching: Development and preliminary content validation of an integrated didactic tetrahedron-TPACK instrument . *Journal of Advanced Sciences and Mathematics Education*, 6(3), 621-642.

INTRODUCTION

Digital transformation has fundamentally reshaped mathematics education, requiring teachers to move beyond content mastery and conventional pedagogy toward the purposeful integration of technology into everyday instructional practices (Sinclair et al., 2021). Digital technologies provide new opportunities for representing mathematical ideas, facilitating exploration, supporting interaction, and creating learning experiences that may be difficult to achieve through conventional classroom resources alone (Cirneanu & Moldoveanu, 2024; Sofroniou, 2025). This transformation is particularly relevant to secondary mathematics because students frequently encounter abstract concepts that require appropriate representations and meaningful learning

*Corresponding author:

Sudarman, Universitas Terbuka, Indonesia
sudirman.officaial@ecampus.ut.ac.id ✉

experiences to develop conceptual understanding. Carefully orchestrated digital tools can bridge abstract mathematical ideas and students' learning experiences by supporting visualization, exploration, and engagement with mathematical concepts. Nevertheless, the presence of technology in classrooms does not necessarily ensure meaningful integration because its educational contribution depends on how teachers connect technological affordances with pedagogical purposes and mathematical content. Previous studies have documented persistent differences between the availability of digital resources and teachers' capacity to employ them for genuine pedagogical transformation rather than superficial augmentation (Mailizar et al., 2020; Stein et al., 2020). These challenges indicate that technology integration should be understood as a pedagogical and didactical process rather than simply as the adoption or frequency of use of digital tools. In Indonesia, this issue has become increasingly relevant following the curricular emphasis on "deep learning" (*pembelajaran mendalam*) as a pedagogical orientation across basic and secondary education. This orientation further increases expectations that teachers create meaningful learning experiences in which technology supports rather than merely accompanies mathematical instruction. Consequently, understanding technology integration in mathematics teaching requires attention to both the professional knowledge teachers mobilize and the ways technology influences interactions among teachers, students, and mathematical content.

Graduate students enrolled in mathematics education programs delivered through open and distance learning occupy a strategic dual position within this transformation. As practising secondary mathematics teachers, they are expected to translate pedagogical innovation and technological developments into meaningful instructional practices in their own classrooms. Simultaneously, as graduate students, they experience technology-mediated instruction themselves, which may influence their beliefs, knowledge, and approaches to classroom teaching (Thurm & Barzel, 2020). Their simultaneous roles as learners and practitioners therefore provide a distinctive context for examining how experiences with technology-mediated education may correspond with technology integration in school mathematics. However, exposure to digital learning environments does not automatically translate into teachers' capacity to select, adapt, and use technology according to specific mathematical and pedagogical purposes. Meaningful integration requires teachers to coordinate knowledge of technology with an understanding of mathematical content, pedagogical strategies, student learning, and classroom interactions. Research on mathematics teacher professional development further suggests that reflective, collaborative, and sustained learning experiences are more effective in strengthening instructional practice than isolated forms of technical training (Lynch et al., 2025; Thirumalai & Sarangapani, 2025). These findings reinforce the importance of examining technology integration as a multidimensional professional capacity rather than as technical proficiency alone. For mathematics teachers undertaking graduate education through open and distance learning, such an examination is particularly relevant because technology simultaneously constitutes a medium for their own learning and a potential resource for their professional teaching practice. An appropriate assessment framework is therefore required to capture both the knowledge teachers need to integrate technology and the didactical relationships through which technology becomes meaningful within mathematics instruction.

The Didactic Tetrahedron and Technological Pedagogical Content Knowledge (TPACK) provide complementary theoretical perspectives for examining these dimensions of technology integration in mathematics teaching. The Didactic Tetrahedron extends the classical didactic triangle of teacher, student, and content by positioning technology as a fourth vertex that mediates and reconfigures relationships among the other three components (Rezat & Geiger, 2024). From this perspective, technology is not merely an additional instructional tool but an element capable of reshaping how mathematical knowledge is represented, accessed, communicated, and constructed within classroom interactions. TPACK, meanwhile, conceptualizes the professional knowledge

required for meaningful technology integration through seven interacting domains: technological knowledge (TK), content knowledge (CK), pedagogical knowledge (PK), pedagogical content knowledge (PCK), technological content knowledge (TCK), technological pedagogical knowledge (TPK), and TPACK itself. A substantial body of research has continued to refine the measurement and validation of TPACK through systematic reviews, meta-analyses, subject-specific assessments, and context-adapted instruments (Backfisch et al., 2025; Fabian et al., 2024; M. Li & Nugraha, 2025). Parallel research has extended the Didactic Tetrahedron to account for artificial intelligence and other emerging technological artifacts within contemporary teaching and learning environments (Chiu, 2025; Dasari et al., 2024; Mani & Zargeh, 2025). Despite these developments, TPACK and the Didactic Tetrahedron have predominantly been applied through distinct theoretical and methodological traditions. TPACK primarily explains the configuration of knowledge teachers require to integrate technology, whereas the Didactic Tetrahedron emphasizes how technology mediates relationships among teachers, students, and mathematical content. Very few studies have operationalized these complementary perspectives within a single measurement instrument, particularly for graduate mathematics teachers in open and distance education, whose dual roles as learners and practitioners remain empirically under-examined (Viberg et al., 2023). Moreover, rigorous content evaluation through structured expert judgment remains essential when developing theoretically integrated instruments to ensure that their items adequately represent the intended constructs before broader empirical administration (Kania et al., 2024; Merino-Soto, 2022).

Previous studies have provided substantial evidence regarding technology integration in mathematics education, although the literature continues to develop along relatively distinct theoretical and methodological trajectories. Research grounded in TPACK has extensively examined teachers' technological, pedagogical, and content knowledge through systematic reviews, empirical investigations, and instrument-development or validation studies, including instruments specifically designed for mathematics teachers across different educational contexts (Kadluba et al., 2025; M. Li et al., 2023; Smith & Zolkowski, 2023). More recent studies have further extended TPACK assessment toward context-specific competencies, enacted TPACK, artificial intelligence integration, and technology-supported instruction, reinforcing the need for measurement approaches that reflect the complexity of teachers' technology-related knowledge (Aumann et al., 2023; Setiawan et al., 2025; Sofyan et al., 2023). In parallel, studies employing the Didactic Tetrahedron have conceptualized digital technology as an artifact that mediates and potentially transforms relationships among teachers, students, and mathematical content in settings involving online instruction, augmented reality, collaborative digital materials, and artificial intelligence (Dasari et al., 2024; Lindenbauer et al., 2024; Liu & Buchbinder, 2024; Mani & Zargeh, 2025). Other research has emphasized the broader challenges of digital technology integration in mathematics teaching and teacher preparation, with some studies drawing upon both TPACK and tetrahedral perspectives to interpret technology-mediated educational practices (Dockendorff & Zaccarelli, 2025; Weinhandl et al., 2021). Nevertheless, the literature represented by these studies predominantly employs TPACK to characterize or measure teachers' knowledge and the Didactic Tetrahedron to conceptualize technology-mediated didactical relationships, rather than operationalizing both perspectives within a unified assessment instrument. Consequently, there remains a methodological and conceptual need for an instrument that simultaneously captures teachers' technological-pedagogical-content knowledge and the relational role of technology among teachers, students, and mathematical content, particularly in secondary mathematics teaching. Addressing this gap, the present study develops and provides preliminary content-validation evidence for an integrated Didactic Tetrahedron-TPACK instrument designed to assess technology integration in mathematics teaching.

Accordingly, this study aimed to develop an integrated Didactic Tetrahedron-TPACK instrument and establish preliminary evidence of its content validity for assessing technology

integration in secondary mathematics teaching. The study constitutes the instrument-development phase of a broader sequential explanatory mixed-methods project examining technology integration among graduate mathematics education students who are also practising secondary mathematics teachers. First, the study sought to identify technology-integration needs and challenges experienced by practising mathematics teachers through a preliminary needs-analysis interview. These findings were intended to provide an empirical foundation for ensuring that instrument development reflected authentic conditions surrounding technology use in mathematics classrooms. Second, the study sought to operationalize the seven TPACK domains alongside technology-mediated relationships represented by the Didactic Tetrahedron within an integrated questionnaire and semi-structured interview protocol. Third, the developed instrument was subjected to structured expert judgment to examine the relevance of its items and identify components requiring refinement before broader administration. Through this integration, TPACK provides a framework for assessing the technological, pedagogical, and content-related knowledge teachers bring to instruction, while the Didactic Tetrahedron provides a relational perspective on how technology interacts with teachers, students, and mathematical content. The study therefore addresses a conceptual and measurement gap by connecting teacher knowledge with technology-mediated didactical relationships within a unified assessment framework. Rather than claiming comprehensive instrument validation at this initial stage, the study focuses specifically on generating preliminary content-validity evidence that can guide subsequent instrument refinement and more extensive psychometric evaluation. Ultimately, the developed instrument is expected to provide a theoretically grounded foundation for examining how technology is meaningfully integrated into secondary mathematics teaching within the subsequent quantitative and qualitative phases of the broader research project.

LITERATURE REVIEW

Technology integration in mathematics teaching extends beyond the availability or routine use of digital devices and involves the purposeful alignment of technological resources with mathematical content, pedagogical strategies, and learning objectives. Digital technologies can broaden opportunities for students to explore mathematical relationships, manipulate representations, test conjectures, and interact with concepts that may otherwise remain abstract (Abrahamson et al., 2020; Pedersen et al., 2021; Sofroniou, 2025). Their contribution is particularly relevant when technology enables mathematical representations and learning experiences that cannot be reproduced as effectively through conventional instructional resources (Abrahamson et al., 2020; Attard & Holmes, 2022; Pedersen et al., 2021). However, the instructional presence of technology does not necessarily indicate that meaningful integration has occurred. Research has demonstrated that mathematics teachers may use digital resources primarily to support existing instructional routines rather than to transform how mathematical concepts are taught and learned (Mailizar et al., 2020). Such patterns suggest that the quality of technology integration depends substantially on teachers' ability to recognize the pedagogical affordances and limitations of particular digital tools. Teachers must also determine whether technology contributes to conceptual understanding, mathematical reasoning, student interaction, or merely functions as an alternative medium for presenting information. Studies of novice mathematics teachers have shown that technology integration is influenced by multiple facilitating and inhibiting conditions, including teacher competence, instructional context, and the perceived usefulness of technological resources (Stein et al., 2020). Consequently, evaluating technology integration requires attention not only to whether teachers use technology but also to the knowledge and instructional reasoning underlying its use. A theoretically grounded assessment should therefore capture how teachers connect

technology with mathematics and pedagogy while accounting for the didactical relationships through which digital resources influence classroom teaching and learning.

Technological Pedagogical Content Knowledge (TPACK) provides an established conceptual framework for explaining the forms of professional knowledge teachers require to integrate technology meaningfully into instruction. Rather than conceptualizing technological competence as an isolated skill, TPACK emphasizes the dynamic interaction among technology, pedagogy, and disciplinary content. The framework comprises technological knowledge (TK), content knowledge (CK), pedagogical knowledge (PK), pedagogical content knowledge (PCK), technological content knowledge (TCK), technological pedagogical knowledge (TPK), and the integrated TPACK domain (Ait Ali et al., 2023; Demeshkant et al., 2022; S. Li et al., 2022; Ning et al., 2024). Within mathematics teaching, TK concerns teachers' understanding of technological resources, whereas CK represents their knowledge of mathematical concepts, structures, and procedures. PCK captures the ability to transform mathematical content into pedagogically accessible forms, while TCK concerns how technology can represent, explore, or transform particular mathematical ideas. TPK emphasizes the relationship between technological affordances and pedagogical approaches, whereas TPACK represents teachers' capacity to coordinate these knowledge domains within specific instructional situations. The framework has consequently become influential in studies examining teacher preparation, professional development, technology integration, and technology-related instructional competence. Nevertheless, systematic and meta-analytic evidence indicates continuing concerns regarding the validity of TPACK measurement, particularly when complex and overlapping knowledge domains are operationalized through self-report instruments (Backfisch et al., 2025). Subject-specific and test-based approaches have therefore been developed to provide more context-sensitive representations of teachers' TPACK, including measures focused specifically on mathematics education (Backfisch et al., 2025; M. Li & Nugraha, 2025). These developments demonstrate that TPACK remains valuable for assessing teacher knowledge while also emphasizing the importance of constructing instruments whose items are theoretically precise and sufficiently sensitive to the distinctive demands of mathematics teaching.

The Didactic Tetrahedron provides a complementary perspective by conceptualizing technology as a structural component of the didactical situation rather than merely as a competency possessed by teachers. It extends the conventional didactic triangle consisting of teacher, student, and content through the inclusion of artifacts or technology as a fourth vertex. This extension recognizes that digital resources can mediate and reconfigure the relationships through which mathematical teaching and learning occur. Technology may alter how teachers present mathematical content, how students interact with mathematical objects, and how communication develops between teachers and learners (Rezat & Geiger, 2024). Accordingly, the framework directs analytical attention toward relational changes rather than treating technology simply as an external instructional resource. In technology-rich mathematics classrooms, digital artifacts may participate actively in mathematical exploration, representation, feedback, and communication, thereby reshaping the learning landscape (Rezat & Geiger, 2024). The growing presence of artificial intelligence has further expanded this theoretical discussion by raising questions about how intelligent digital artifacts occupy and transform the technology vertex within didactical relationships (Dasari et al., 2024; Mani & Zargheh, 2025). Such developments demonstrate the continuing relevance of the Didactic Tetrahedron for interpreting increasingly complex configurations of technology-mediated mathematics instruction. Nevertheless, the framework is primarily relational and does not by itself provide a detailed taxonomy of the technological, pedagogical, and content knowledge teachers require to manage these configurations effectively. The Didactic Tetrahedron therefore offers an important complement to knowledge-oriented frameworks

such as TPACK when the objective is to assess both teachers' competencies and the relational consequences of technology integration.

Integrating TPACK with the Didactic Tetrahedron provides a theoretical basis for examining technology integration simultaneously from knowledge-oriented and relational perspectives. TPACK identifies the configurations of technological, pedagogical, and mathematical knowledge teachers need, whereas the Didactic Tetrahedron explains how technology participates in relationships among teachers, students, and content. These perspectives are complementary because teachers' knowledge becomes educationally meaningful when it is enacted through specific instructional relationships and classroom situations. For example, a teacher may possess technological knowledge but still be unable to select a digital representation that appropriately communicates a difficult mathematical concept to students. Conversely, changes in teacher-student, student-content, or teacher-content relationships resulting from technology use cannot be fully interpreted without considering the professional knowledge guiding teachers' instructional decisions. Recent extensions of technology-oriented frameworks further demonstrate that emerging digital artifacts increasingly challenge conventional boundaries among technological competence, pedagogical reasoning, and classroom relationships (Chiu, 2025; Dasari et al., 2024; Mani & Zargeh, 2025). Within an integrated perspective, the technology vertex of the Didactic Tetrahedron can therefore be interpreted in relation to the knowledge structures represented by the seven TPACK domains. Such integration allows assessment to move beyond determining whether teachers possess technology-related knowledge toward examining whether this knowledge supports meaningful interactions with students and mathematical content. TPACK can consequently be positioned not as a framework independent of the Didactic Tetrahedron but as a knowledge structure that helps explain the substantive characteristics of technology-mediated didactical relationships. Operationalizing this conceptual integration through a unified instrument provides the theoretical foundation for a more comprehensive assessment of technology integration in mathematics teaching.

The theoretical integration of TPACK and the Didactic Tetrahedron requires an assessment instrument whose items adequately represent both teacher-knowledge domains and technology-mediated didactical relationships. Instrument development should therefore begin with explicit construct definitions and systematic alignment between theoretical dimensions, item content, and the educational context in which the instrument will eventually be administered. Previous research on TPACK measurement has demonstrated the importance of developing instruments that are sensitive to subject-specific and contextual characteristics rather than assuming that generic technology-integration measures operate uniformly across teaching populations. Evidence from reviews of TPACK assessments further indicates that instrument validity should be established progressively through multiple sources of evidence rather than inferred solely from the use of an established theoretical framework (Backfisch et al., 2025). Content validation represents an important early stage of this process because it examines whether proposed items adequately correspond with the constructs they are intended to represent. Structured expert judgment can identify ambiguous wording, conceptual overlap, inadequate representation, and items that require revision before empirical administration (Kania et al., 2024). When multiple independent experts are involved, formal indices such as Aiken's V can provide quantitative evidence concerning the degree of agreement regarding item relevance. However, content-validity evidence alone does not establish an instrument's construct validity, reliability, or capacity to discriminate among respondents, making subsequent empirical testing necessary. For an integrated Didactic Tetrahedron-TPACK instrument, preliminary content evaluation is particularly important because the instrument must preserve the conceptual distinctions among the seven TPACK domains while simultaneously representing the relational function of technology within mathematics teaching. Accordingly, preliminary content validation constitutes a foundational rather than final stage toward establishing a theoretically

coherent and psychometrically defensible instrument for assessing technology integration in mathematics teaching.

METHOD

Research Design

This study constitutes the first, instrument-development stage of a larger project that employs a sequential explanatory mixed-methods design to investigate how graduate mathematics education students at a large open university in Indonesia integrate digital technology into their secondary school teaching practice, viewed through the combined lens of the Didactic Tetrahedron and TPACK. Within that broader design, the present stage is itself organized sequentially: a brief qualitative needs-analysis interview was conducted first to ground instrument development in authentic classroom conditions, followed by instrument construction and formal expert content validation, consistent with recommended practice for developing psychometrically sound instruments in mathematics education research. The needs-analysis component was descriptive and qualitative in nature, intended to surface the practical technology-integration challenges experienced by secondary mathematics teachers rather than to test hypotheses. The subsequent instrument-validation component was likewise descriptive, relying on expert judgment rather than field administration, since the purpose at this stage was to establish content validity, not construct or criterion validity, which will be examined once the revised instrument is administered to the full participant sample in the next phase of the project.

Participant

Two distinct sets of participants informed this stage of the study. First, the preliminary needs-analysis interview involved practising secondary mathematics teachers who were consulted informally as key informants to surface real classroom conditions surrounding deep-learning implementation and technology use; this step functioned as a needs analysis rather than a full qualitative study, and its findings were used only to inform instrument content, not as generalizable empirical claims. Second, the content-validation stage involved a single expert validator, a professor of mathematics education with a background in both TPACK-related instrument review and teacher education, who evaluated every item of the questionnaire and interview protocol against criteria of content relevance, construct clarity, and language appropriateness. These two participant groups are deliberately distinct from the 84 graduate mathematics education students, drawn from 39 regional distance-learning units across Indonesia, who constitute the target sample for the subsequent quantitative and qualitative data-collection phases of the broader project. That larger sample, including the five teachers purposively selected for in-depth classroom observation, had not yet been recruited at the time of this report and therefore falls outside the scope of the results presented here.

Data Collection

Data for the preliminary needs-analysis stage were collected through brief, informal semi-structured conversations with in-service teachers, focusing on how deep-learning pedagogy was being implemented in their schools, the extent of technology integration into mathematics instruction, and teachers' familiarity with instructional media designed to make learning enjoyable and meaningful. No audio recordings or verbatim transcripts were produced at this exploratory stage; instead, field notes were used to identify recurring, salient themes that could inform item development. Data for the instrument-validation stage were collected using a structured expert-judgment sheet in which the validator rated each of the 30 questionnaire items, covering the seven TPACK domains and four Tetrahedron relational items, together with the five interview-protocol questions, on a four-point scale (1 = not relevant, 2 = somewhat relevant requiring major revision, 3 = relevant requiring minor revision, 4 = highly relevant). The validator additionally provided written

comments identifying items requiring rewording, the addition of illustrative non-examples, or terminological refinement, and rendered an overall feasibility judgment for the instrument as a whole.

Data Analysis

Findings from the preliminary interviews were analyzed descriptively using a qualitative thematic approach: field notes were read repeatedly to identify recurring patterns across informants, which were then organized into a small number of overarching themes that directly informed the emphasis given to particular TPACK domains and Tetrahedron relations during item writing, particularly the technology-related domains (TK, TCK, TPK). Expert-validation data were analyzed descriptively by computing, for each TPACK domain and each Tetrahedron dimension, the number of items, the sum and mean of the assigned relevance scores, and the corresponding qualitative category (highly relevant, mean ≥ 3.75 ; relevant with minor revision, mean 3.0–3.74; requiring major revision, mean < 3.0). Because only a single expert rater was involved at this stage, inter-rater agreement indices such as Aiken's V, which require multiple independent judges, could not be computed; this limitation, and its implications, are addressed in the Discussion. Written comments were analyzed qualitatively and grouped by domain to generate concrete revision recommendations.

Procedure

The research procedure was conducted sequentially through five interconnected stages: preliminary needs analysis, theoretical construct mapping, instrument development, expert content evaluation, and instrument revision. First, a preliminary needs analysis was conducted through brief informal semi-structured conversations with practising secondary mathematics teachers to identify classroom conditions related to deep-learning implementation, digital technology integration, and the use of instructional media. Field notes from these conversations were reviewed to identify recurring issues that could provide contextual direction for instrument development. Second, the identified needs were mapped onto the theoretical constructs of the Didactic Tetrahedron and TPACK to establish the dimensions to be represented in the instrument. The seven TPACK domains—TK, CK, PK, PCK, TCK, TPK, and TPACK—were combined with technology-mediated didactical relationships derived from the Didactic Tetrahedron to construct an integrated conceptual framework for item development. Third, these theoretical and contextual dimensions were operationalized into an initial instrument consisting of 30 questionnaire items and five semi-structured interview questions intended to assess technology integration in secondary mathematics teaching. Fourth, the initial instrument was submitted to a mathematics education expert for preliminary content evaluation using a four-point relevance scale and written qualitative feedback concerning item relevance, conceptual clarity, language appropriateness, terminology, and necessary refinements. The expert ratings were analyzed descriptively by calculating total and mean relevance scores, while written comments were examined to identify specific items requiring modification. Fifth, items receiving lower relevance ratings or specific recommendations from the expert were revised by refining their wording, clarifying terminology, and incorporating illustrative non-examples where necessary. The revised instrument resulting from these procedures constitutes the version prepared for subsequent administration to the broader participant sample, where further evidence concerning its construct validity, reliability, and psychometric properties can be established.

RESULTS AND DISCUSSION

Results

Preliminary Needs Analysis: Technology Integration in the Context of Deep Learning (RQ1)

The preliminary interviews with in-service secondary mathematics teachers surfaced three interrelated themes, summarized in Figure 1. First, teachers reported that deep-learning pedagogy was already being introduced across school levels, indicating that the policy and curricular groundwork for more meaningful mathematics instruction is present. Second, and in tension with the first theme, only a small proportion of teachers were found to actively integrate digital technology when delivering mathematics content in ways that support this deeper learning; for most teachers, technology use remained peripheral rather than central to instructional design. Third, many teachers reported limited understanding of how to use mathematics instructional media in ways that make lessons enjoyable, which they linked to reduced meaningfulness of learning for their students. These findings reveal a notable gap between teachers' awareness of deep-learning-oriented pedagogy and their capacity to translate its principles into technology-supported mathematics instruction. The challenge appears to extend beyond access to digital tools, as teachers also require pedagogical knowledge for selecting and integrating technology in accordance with mathematical content and learning objectives. Consequently, technology may be present in the instructional environment without necessarily functioning as a meaningful component of students' mathematical learning experiences. The interview findings further suggest that effective technology integration requires teachers to coordinate technological, pedagogical, and content-related considerations rather than treating digital media as an additional instructional feature. Taken together, the preliminary needs analysis provides an empirical basis for examining teachers' technology integration through a framework capable of capturing the relationships among mathematics content, pedagogy, technology, and classroom instructional contexts.

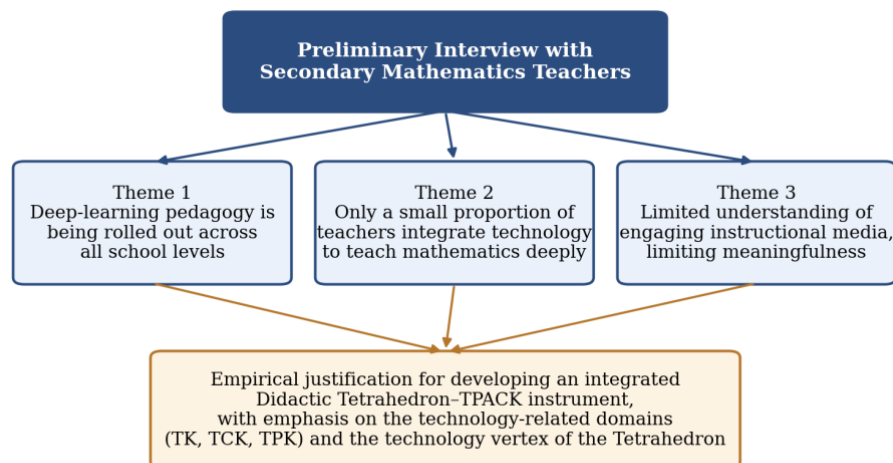


Figure 1. Thematic map of the preliminary needs-analysis interview with secondary mathematics teachers.

Taken together, these themes directly answer the first research question: they justify the development of an integrated instrument by showing that the central problem is not the absence of a supportive pedagogical policy (deep learning is present) but a knowledge gap in how to operationalize that policy through technology, precisely the space jointly described by the

Tetrahedron's technology vertex and the technology-related TPACK domains (TK, TCK, TPK). This finding directly shaped the emphasis given to these domains during item construction, described next.

The Integrated Instrument

Building on the needs-analysis findings and the theoretical frameworks reviewed above, an integrated instrument was developed consisting of two complementary parts. Part A is a 30-item questionnaire using a four-point Likert scale, covering the seven TPACK domains (TK, CK, PK, PCK, TCK, TPK, and TPACK) and three items explicitly targeting the Tetrahedron relation among teacher, student, content, and technology. Part B is a five-question semi-structured interview protocol, administered reflectively after classroom observation, exploring how teachers integrate technology, how the didactic relations among teacher, student, and content shift as a result, what barriers they encounter, and how their own experience as distance-learning graduate students shapes their classroom practice. Figure 2 illustrates how the seven TPACK domains were conceptually mapped onto the technology vertex of the Tetrahedron in the instrument's underlying design. The questionnaire was designed to provide structured evidence of teachers' technological, pedagogical, and content-related knowledge, while the interview protocol was intended to capture contextual explanations that may not be adequately represented through numerical responses alone. The combination of these two components enables the instrument to examine not only what teachers report knowing about technology integration but also how such knowledge is interpreted and enacted within mathematics classroom interactions. Integrating the TPACK domains with the Didactic Tetrahedron further allows technology to be examined in relation to the teacher–student–content system rather than as an isolated instructional component. This conceptual integration is particularly important because the preliminary needs analysis indicated that the presence or use of digital tools does not necessarily translate into meaningful technology-supported learning. Accordingly, the integrated instrument was structured to capture both teachers' knowledge dimensions and the relational dynamics through which technology may influence mathematics teaching and learning.

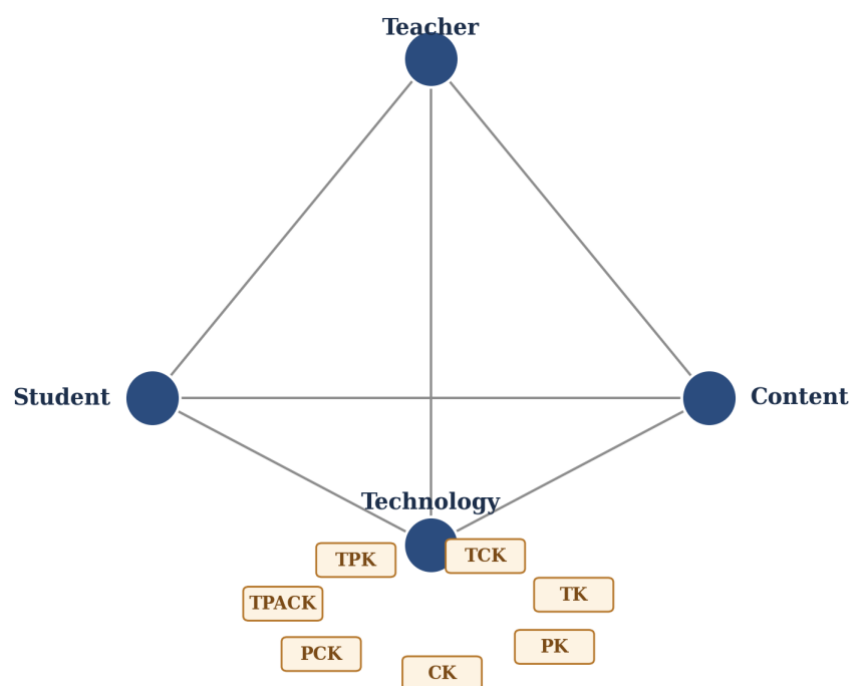


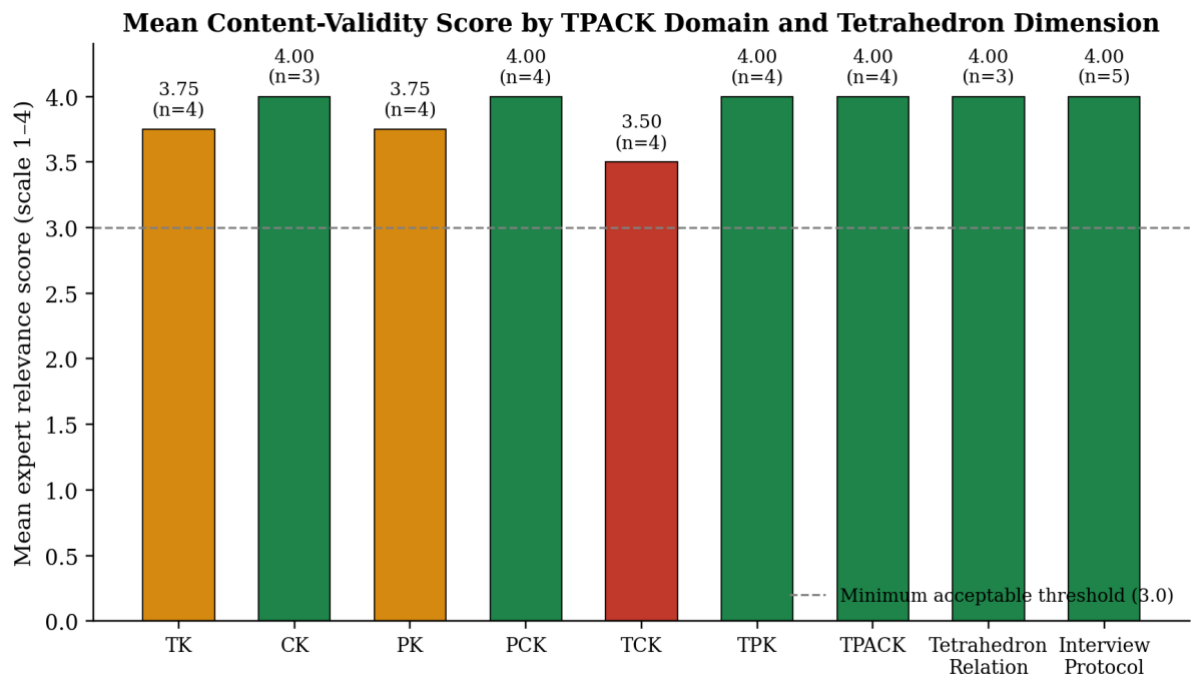
Figure 2. Conceptual mapping of the seven TPACK domains onto the technology vertex of the Didactic Tetrahedron, as operationalized in the integrated instrument.

Expert Content Validation (RQ2)

The instrument was submitted to expert judgment by a professor of mathematics education. Every item across both parts of the instrument received a rating of either 3 (relevant, minor revision) or 4 (highly relevant); no item was rated 1 or 2, indicating that none required major revision or removal. Table 1 summarizes the mean relevance score for each TPACK domain and Tetrahedron dimension, and Figure 3 displays the same results graphically.

Table 1. Expert Content-Validity Results by TPACK Domain and Tetrahedron Dimension

Domain / Dimension	Items (n)	Mean Score (1–4)	Category
Technological Knowledge (TK)	4	3.75	Highly relevant
Content Knowledge (CK)	3	4.00	Highly relevant
Pedagogical Knowledge (PK)	4	3.75	Highly relevant
Pedagogical Content Knowledge (PCK)	4	4.00	Highly relevant
Technological Content Knowledge (TCK)	4	3.50	Relevant, minor revision
Technological Pedagogical Knowledge (TPK)	4	4.00	Highly relevant
TPACK (integrated)	4	4.00	Highly relevant
Tetrahedron Relation (Teacher–Student–Content–Technology)	3	4.00	Highly relevant
Semi-structured Interview Protocol	5	4.00	Highly relevant
Overall (35 items)	35	3.89	Highly relevant

**Figure 3.** Mean content-validity score by TPACK domain and Tetrahedron dimension, with the minimum acceptable threshold of 3.0.

As Table 1 and Figure 3 show, the overall mean relevance score across all 35 items was 3.89 out of 4, well above the conventional minimum acceptable threshold of 3.0 for content validity (Kania et al., 2024). Twenty-six of the thirty questionnaire items, together with all five interview-protocol questions, were rated highly relevant (score = 4), indicating strong alignment between item wording and the underlying TPACK and Tetrahedron constructs. Four items were rated 3 (relevant, minor revision): one item in TK concerning teachers' ability to resolve simple technical problems in the classroom, one item in PK concerning ongoing formative assessment strategies, and two items in TCK concerning the selection of appropriate digital tools for visualizing mathematical concepts and evaluating whether a given technology genuinely supports conceptual understanding rather than serving a merely decorative function.

Domain-level analysis further shows that TCK obtained the lowest mean score (3.50) among all domains, while CK, PCK, TPK, TPACK, the Tetrahedron relation items, and the interview protocol all obtained the maximum mean score of 4.00. This pattern directly answers the second research question: the instrument as a whole is judged content-valid, but validity is not uniformly distributed across domains, it is comparatively weaker precisely at the technology-content interface, the domain that operationalizes how technology mediates mathematical content rather than pedagogy in general.

In response to the third research question, the validator's written comments point to four concrete revisions prior to full-scale administration. First, the TK item on resolving technical problems should be reworded to also capture teachers' awareness of emerging educational technology trends, not only troubleshooting competence. Second, the PK item on continuous assessment should be more explicitly linked to specific formative-assessment strategies rather than left at a general level. Third, the PCK-adjacent item on selecting representations for difficult concepts should be extended to also ask teachers to identify non-examples, strengthening its diagnostic value. Fourth, the two TCK items should be revised to foreground the exploratory and explanatory functions of digital tools, and the item referring to technology as mere "decoration" should be reworded so that the intended construct, technology as a genuine aid to conceptual understanding, is unambiguous. Documentation of the completed expert-validation sheet is presented in Figure 4(a-d).

Taken together, these validation results demonstrate that the proposed instrument has a strong conceptual foundation while also identifying specific areas requiring refinement before empirical implementation. The concentration of lower relevance ratings within TK, PK, and particularly TCK suggests that the principal challenge lies not in the overall structure of the instrument but in the precision with which several technology-related competencies are operationalized. In particular, the comparatively lower TCK score highlights the complexity of distinguishing meaningful technology-content integration from the general use of digital tools during mathematics instruction. This distinction is important because effective technology integration requires teachers to select digital resources according to their capacity to represent, explore, explain, or deepen specific mathematical concepts. The validators' recommendations therefore provide substantive guidance for improving item clarity and construct representation rather than requiring fundamental changes to the integrated TPACK-Tetrahedron framework. Revising the identified items is expected to strengthen the instrument's ability to differentiate between routine technology use and pedagogically meaningful technology integration within mathematics classrooms. Consequently, the expert-validation stage serves not only as evidence of content validity but also as a formative mechanism for ensuring that the final instrument more precisely captures the multidimensional and relational nature of technology-supported mathematics teaching.

LEMBAR VALIDASI INSTRUMEN PENELITIAN

LEMBAR VALIDASI INSTRUMEN OLEH AHLI
Transformasi Praktik Mengajar Guru Sekolah Menengah dalam Era Digital dalam Kerangka
Tetrahedron Didaktis dan TPACK

Mahasiswa Pascasarjana Pendidikan Matematika, Universitas Terbuka

A. Identitas Validator

Nama Validator	Prof. Dr. Samul Hamy, M.Pd.
NIP / NIDN	0307088501
Instansi	UHAMKA
Bidang Keahlian	Pendidikan Matematika
Jabatan Akademik	Profesor
Tanggal Validasi	17 Juli 2026

B. Petunjuk Pengisian

Bapak/Ibu dimohon memberikan penilaian terhadap setiap butir instrumen (Bagian A: Angket TPACK dan Tetrahedron Didaktis; Bagian B: Pedoman Wawancara Semi-Terstruktur) ditinjau dari kesesuaian isi, konstruk, dan kejelasan bahasa. Penilaian dilakukan dengan memberi tanda centang (✓) pada kolom angka yang sesuai, dengan skala sebagai berikut:

1 = Tidak Relevan/Tidak Sesuai
2 = Kurang Relevan/Kurang Sesuai, perlu revisi besar
3 = Relevan/Sesuai, perlu revisi kecil
4 = Sangat Relevan/Sangat Sesuai

Kolom "Saran Perbaikan" dimohon diisi apabila butir yang bersangkutan memerlukan revisi redaksi, substansi, atau dihapus.

C. Bagian A — Validasi Angket TPACK dan Tetrahedron Didaktis

No	Pernyataan	Skor Penilaian				Saran Perbaikan
		1	2	3	4	
A. Technological Knowledge (TK)						
1	Saya menguasai pengoperasian berbagai perangkat digital (komputer, laptop, tablet, smartphone) untuk keperluan pembelajaran.				✓	
2	Saya mampu mengoperasikan aplikasi/platform digital baru dengan cepat tanpa banyak kesulitan.				✓	
3	Saya rutin mengikuti perkembangan teknologi digital yang relevan dengan pembelajaran matematika.				✓	
4	Saya dapat mengatasi masalah teknis sederhana (jaringan, aplikasi error, dll.) saat menggunakan teknologi di kelas.			✓		Terdapat klem- an yang menyebabkan kesulitan dalam penggunaan
B. Content Knowledge (CK)						
5	Saya menguasai konsep-konsep matematika sekolah menengah yang saya ajarkan secara mendalam.				✓	Kata "mendalam" diikuti dengan "memahami"

Halaman 1 dari 4

Figure 4a. Expert validation sheet, page 1 (identity of the validator and Part A, TK–CK domains).

LEMBAR VALIDASI INSTRUMEN PENELITIAN

No	Pernyataan	Skor Penilaian				Saran Perbaikan
		1	2	3	4	
6	Saya memahami keterkaitan antar-topik matematika dalam kurikulum yang saya ajarkan.				✓	
7	Saya mampu menjelaskan asal-usul dan penalaran di balik suatu prosedur/rumus matematika, bukan sekadar hasil akhirnya.				✓	
C. Pedagogical Knowledge (PK)						
8	Saya mampu merancang strategi pembelajaran yang sesuai dengan karakteristik siswa saya.				✓	
9	Saya mampu mengelola kelas agar siswa terlibat aktif dalam proses pembelajaran.				✓	
10	Saya mampu melakukan asesmen untuk mengetahui pemahaman siswa secara berkelanjutan.			✓		Konsep asesmen diapresiasi oleh karakteristiknya
11	Saya mampu menyesuaikan pendekatan mengajar ketika strategi yang digunakan kurang efektif.				✓	
D. Pedagogical Content Knowledge (PCK)						
12	Saya mampu memilih representasi (contoh, ilustrasi, model) yang tepat untuk menjelaskan konsep matematika yang sulit.				✓	- tambahkan non contoh
13	Saya mampu mengantisipasi kesalahpahaman (miskonsepsi) siswa terhadap topik matematika tertentu.				✓	
14	Saya mampu merancang urutan pembelajaran matematika yang membangun pemahaman konsep secara bertahap.				✓	
15	Saya mampu mengaitkan materi matematika dengan konteks kehidupan sehari-hari siswa.				✓	
E. Technological Content Knowledge (TCK)						
16	Saya mengetahui aplikasi/software yang sesuai untuk memvisualisasikan konsep matematika tertentu (misalnya geometri, fungsi, statistik).			✓		- tambahkan Explorasi dan Eksplorasi
17	Saya memahami bagaimana teknologi dapat mengubah cara suatu konsep matematika direpresentasikan.			✓		
18	Saya mampu memilih alat digital yang paling sesuai dengan karakteristik topik matematika yang diajarkan.				✓	
19	Saya mampu mengevaluasi apakah suatu alat teknologi benar-benar mendukung pemahaman konsep matematika, bukan sekadar hiasan.			✓		- Kata "hiasan" bisa diganti "alat hias"
F. Technological Pedagogical Knowledge (TPK)						
20	Saya mampu merancang aktivitas pembelajaran yang memanfaatkan teknologi untuk mendorong keterlibatan siswa.				✓	
21	Saya mampu menggunakan teknologi untuk memberikan umpan balik kepada siswa secara lebih cepat dan personal.				✓	

Halaman 2 dari 4

Figure 4b. Expert validation sheet, page 2 (Part A, PK–TCK domains).

LEMBAR VALIDASI INSTRUMEN PENELITIAN

No	Pernyataan	Skor Penilaian				Saran Perbaikan
		1	2	3	4	
22	Saya mampu memfasilitasi kolaborasi antar-siswa melalui platform digital.				✓	
23	Saya mampu mengubah peran saya dari sumber informasi utama menjadi fasilitator ketika menggunakan teknologi di kelas.				✓	
G. Technological Pedagogical Content Knowledge (TPACK)						
24	Saya mampu mengintegrasikan teknologi, pedagogi, dan konten matematika secara terpadu dalam satu rancangan pembelajaran.				✓	
25	Saya mampu memilih strategi mengajar berbasis teknologi yang paling sesuai untuk topik matematika dan karakteristik siswa tertentu.				✓	
26	Saya secara rutin merefleksikan efektivitas integrasi teknologi dalam pembelajaran matematika yang saya lakukan.				✓	
27	Saya merasa kompetensi TPACK saya berkembang signifikan sejak menempuh studi pascasarjana di UT.				✓	
H. Relasi Tetrahedron Didaktis (Guru-Siswa-Materi-Teknologi)						
28	Kehadiran teknologi mengubah cara saya berinteraksi dengan siswa selama pembelajaran matematika berlangsung.				✓	
29	Teknologi berperan sebagai mediator yang memperkuat hubungan antara siswa dan materi matematika yang dipelajari.				✓	
30	Saya mampu mengorkestrasi (mengatur) penggunaan teknologi agar tidak mengganggu, melainkan memperkuat relasi didaktis guru-siswa-materi.				✓	

D. Bagian B — Validasi Pedoman Wawancara Semi-Terstruktur

No	Butir Pertanyaan Wawancara	Skor Penilaian				Saran Perbaikan
		1	2	3	4	
1	Bagaimana Bapak/Ibu mengintegrasikan teknologi digital dalam merancang dan melaksanakan pembelajaran matematika selama ini?				✓	
2	Perubahan apa yang Bapak/Ibu rasakan dalam relasi antara guru, siswa, dan materi ajar setelah menggunakan teknologi tersebut?				✓	
3	Apa hambatan utama yang Bapak/Ibu hadapi dalam mengintegrasikan teknologi ke dalam pembelajaran matematika, dan bagaimana strategi mengatasinya?				✓	
4	Bagaimana pengalaman Bapak/Ibu sebagai mahasiswa pascasarjana yang mengikuti pembelajaran jarak jauh memengaruhi cara Bapak/Ibu mengajar di kelas?				✓	
5	Menurut Bapak/Ibu, kompetensi TPACK apa yang paling berkembang selama menempuh studi ini, dan bagaimana proses perkembangannya terjadi?				✓	

Halaman 3 dari 4

Figure 4c. Expert validation sheet, page 3 (Part A, TPK–Tetrahedron Relation domains, and Part B interview protocol).

LEMBAR VALIDASI INSTRUMEN PENELITIAN

E. Penilaian Kelayakan Instrumen Secara Umum

☐ Layak digunakan tanpa revisi
☒ Layak digunakan dengan revisi sesuai saran
☐ Tidak layak digunakan

F. Saran dan Komentar Umum

- Secara umum butir-butir Instrumen sudah baik. Akan tetapi, ada beberapa hal yg perlu ditambahkan pada instrumen tersebut.
 Jakarta, 17 Juli 2026
 Validator,
 Prof. Dr. Samud Nadiy, M.P.
 NIP/NIDN. 0307088501

Halaman 4 dari 4

Figure 4d. Expert validation sheet, page 4 (overall feasibility judgment and validator's general comments).

Discussion

The findings provide preliminary evidence that the integrated Didactic Tetrahedron-TPACK instrument has strong content relevance for assessing technology integration in secondary mathematics teaching, as reflected by an overall expert-validation mean of 3.89 on a four-point scale and the absence of items rated below 3.00. This result indicates that the initial item pool generally corresponds with the theoretical constructs intended to be represented, although it should not be interpreted as evidence of comprehensive psychometric validity. Such caution is consistent with recent TPACK measurement literature showing that apparently well-established instruments may still demonstrate uneven validity evidence when examined across different populations, contexts, and forms of measurement (Backfisch et al., 2025), in particular, emphasize the importance of accumulating validity evidence progressively rather than treating a single evaluation procedure as sufficient confirmation of instrument quality. Similar concerns are reflected in subject-specific TPACK research, where the quality of measurement depends not only on theoretical alignment but also on whether items adequately represent the particular disciplinary context in which technological knowledge is enacted (Fabian et al., 2024; M. Li & Nugraha, 2025). The present findings therefore extend this literature by applying preliminary content evaluation to an instrument that combines teacher-knowledge dimensions with technology-mediated didactical relationships rather than measuring TPACK as an independent construct. This distinction is important because technology integration in mathematics teaching involves not only what teachers know about technology, pedagogy, and mathematical content but also how these forms of knowledge are mobilized within interactions involving teachers, students, content, and digital artifacts. Nevertheless, the high overall score obtained at this stage should be interpreted primarily as evidence that the proposed items are substantively appropriate from the perspective of the participating expert. Further validation involving multiple experts and subsequent empirical administration is required to determine whether the theoretical structure proposed during instrument development is reproduced in participants' response patterns. Thus, the current findings establish a promising foundation for instrument refinement while reinforcing contemporary recommendations that content relevance should constitute one component of a broader and cumulative validation process.

The domain-level results provide a more differentiated interpretation, particularly because Technological Content Knowledge (TCK) obtained the lowest mean relevance score of 3.50 and accounted for two of the four questionnaire items requiring minor revision. The identified items concerned the selection of digital tools for visualizing mathematical concepts and the distinction between technology that genuinely supports conceptual understanding and technology that merely adds a digital component to an otherwise unchanged lesson. This pattern is theoretically important because TCK represents the intersection at which teachers must understand how specific technological affordances interact with the characteristics and representations of particular mathematical content. Comparable complexity, whose subject-specific and test-based investigation demonstrated that technology-related knowledge in mathematics cannot necessarily be inferred reliably from teachers' broader perceptions of their technological and pedagogical competence (Fabian et al., 2024). The present findings complement this evidence from an instrument-development perspective, suggesting that the complexity of TCK emerges not only when teachers respond to assessments but also when researchers attempt to formulate items capable of representing the construct precisely. Context-specific measurement research similarly indicates that TPACK dimensions need to be operationalized in ways that remain sensitive to disciplinary and instructional contexts rather than being transferred uncritically across educational settings (M. Li & Nugraha, 2025). In mathematics, this requirement is especially consequential because a technological tool may be technically functional and pedagogically attractive while contributing little

to the representation, exploration, or understanding of the mathematical concept being taught. The lower TCK score therefore should not simply be interpreted as an instrument weakness but as evidence of a construct boundary requiring greater conceptual precision during item formulation. Refining these items to distinguish technological availability from mathematically meaningful affordances may improve the instrument's capacity to identify whether teachers can connect specific technologies with particular forms of mathematical reasoning and representation. Accordingly, the concentration of revision recommendations within TCK supports previous arguments that technology integration should be assessed through discipline-sensitive constructs and reinforces the need for further empirical examination of how these items function among practising mathematics teachers.

The preliminary needs-analysis findings further contextualize the validation results by indicating a discrepancy between institutional expectations for innovative pedagogy and teachers' capacity to enact these expectations through meaningful technology integration. Teachers consulted during the exploratory stage indicated that deep-learning pedagogy was already being promoted, while meaningful incorporation of technology into mathematics teaching remained comparatively limited and familiarity with instructional media varied considerably. Although these preliminary observations were derived from an informal needs analysis and should not be generalized beyond the instrument-development context, they correspond with patterns reported in previous studies of mathematics teachers' technology integration. That novice mathematics teachers' technology integration is shaped by interacting facilitating and inhibiting conditions, indicating that positive attitudes toward digital resources do not automatically result in sustained classroom implementation (Stein et al., 2020). Similarly, uneven readiness among secondary mathematics teachers to incorporate ICT into their instructional practices despite recognition of its potential educational value (Arhin et al., 2024). These findings are consistent with broader research showing that access to digital resources may result primarily in the augmentation of existing instructional practices when teachers have insufficient pedagogical preparation to use technology transformatively (Mailizar et al., 2020). Within the Indonesian context, the challenge becomes particularly relevant as current pedagogical directions increasingly emphasize meaningful and deeper learning experiences, while teachers' readiness to translate these orientations into classroom practice remains an important implementation concern. The alignment between the preliminary needs analysis and previous empirical research therefore supports the decision to emphasize technology-related dimensions such as TK, TCK, and TPK during instrument construction. More importantly, it suggests that an assessment of technology integration should distinguish the availability or frequency of technology use from teachers' capacity to employ digital resources for conceptual, pedagogical, and relational purposes. The integrated instrument developed in this study responds to this distinction by attempting to capture not merely whether technology is present in mathematics instruction but how teachers' knowledge supports its meaningful positioning within teaching and learning interactions.

The integration of the Didactic Tetrahedron and TPACK provides an additional theoretical contribution because the findings suggest that teacher knowledge and technology-mediated classroom relationships can be represented within a common assessment framework. Contemporary applications of the Didactic Tetrahedron emphasize that digital technologies increasingly function as active components of the instructional environment by reshaping access to mathematical objects, communication, feedback, representation, and relationships among participants (Rezat & Geiger, 2024). This relational interpretation has become even more relevant with the emergence of artificial intelligence, as digital artifacts can assume increasingly interactive roles within mathematics teaching rather than functioning merely as passive instructional resources (Dasari et al., 2024; Mani & Zargheh, 2025). In comparison, contemporary TPACK research continues to focus primarily on the

configurations of knowledge teachers require to coordinate technological resources with pedagogy and disciplinary content (Fabian et al., 2024; M. Li & Nugraha, 2025). The present study connects these perspectives by conceptualizing the TPACK domains in relation to the technology vertex of the Didactic Tetrahedron rather than treating the two frameworks as independent explanations of technology integration. This theoretical positioning allows technological knowledge to be examined in relation to the interactions through which digital resources influence teachers, students, and mathematical content. The strong preliminary relevance judgments for the Tetrahedron-related component suggest that this conceptual integration can be translated into assessable items, although expert relevance alone cannot establish that respondents will interpret the items according to the intended relational structure. Consequently, the next empirical phase should examine whether the theoretically distinct TPACK and Tetrahedron dimensions remain distinguishable in response data or whether substantial overlap emerges between knowledge-oriented and relational items. Such investigation is especially important because contemporary digital environments increasingly blur conventional distinctions between possessing technological knowledge and interacting pedagogically with intelligent or adaptive technological artifacts (Dasari et al., 2024; Mani & Zargeh, 2025). The present findings therefore advance the discussion from parallel application of the two frameworks toward their operational integration, while leaving the empirical structure of that integration as a question requiring subsequent psychometric examination.

Methodologically, the results must be interpreted within the limitation that preliminary content evaluation was conducted by a single expert, which restricts the strength of the validity claims that can be made at this stage. Contemporary instrument-development research emphasizes that content validity is strengthened when judgments are obtained independently from multiple experts who possess relevant theoretical, disciplinary, and measurement expertise (Kania et al., 2024). The present ratings can therefore indicate item relevance according to one qualified expert but cannot demonstrate the degree of agreement that would emerge across a broader panel of judges. This limitation is particularly relevant to the TCK items because their comparatively lower ratings suggest that different experts might interpret their conceptual boundaries or wording differently. The findings nevertheless serve a useful developmental function by identifying where revision should be concentrated before the instrument is exposed to broader expert and respondent evaluation. Beyond instrument validation, the combined questionnaire and interview protocol also corresponds with recent professional-development literature emphasizing that mathematics teachers' technology-related competence cannot be understood adequately through technical knowledge alone. Recent evidence indicates that sustained professional learning involving reflection, pedagogical content knowledge, and engagement with instructional practice is more consequential than narrowly technical approaches to teacher development (Lynch et al., 2025; Thirumalai & Sarangapani, 2025). The inclusion of a semi-structured interview protocol alongside the questionnaire may consequently provide opportunities in the subsequent phase to examine the reasoning and classroom experiences underlying teachers' quantitative responses rather than interpreting scale scores in isolation. Future validation should therefore expand the expert panel, refine the technology-content items, examine reliability and construct structure using the planned participant sample, and investigate whether qualitative evidence converges with the patterns identified quantitatively. Taken together, the findings support the integrated Didactic Tetrahedron–TPACK instrument as a theoretically grounded and preliminarily relevant approach to assessing technology integration in mathematics teaching, while positioning the current study as an initial validation stage that requires further expert, psychometric, and classroom-based evidence before stronger claims regarding instrument validity can be established.

Implications

This study provides theoretical, methodological, and practical implications for assessing technology integration in mathematics teaching. The integration of the Didactic Tetrahedron and TPACK offers a broader perspective by connecting teachers' professional knowledge with technology-mediated relationships among teachers, students, and mathematical content. Methodologically, the developed instrument provides an initial framework for assessing the seven TPACK domains alongside didactical relationships through questionnaire and interview components. The lower relevance score for TCK indicates that particular attention should be given to measuring teachers' ability to connect digital technologies with specific mathematical concepts. Further validation involving multiple experts and empirical testing is necessary before the instrument can be applied more broadly. Practically, the instrument may help teacher-education programs identify strengths and weaknesses in teachers' technology-integration competencies. The findings may also inform professional development that emphasizes meaningful technology use rather than technical proficiency alone. In open and distance graduate education, the instrument can support investigation of how teachers' experiences as technology-mediated learners relate to their classroom practices. Combining questionnaire and interview evidence may provide a more comprehensive understanding of teachers' instructional decisions. Such information can support the refinement of mathematics teacher-education curricula and technology-focused professional learning. The framework may also contribute to evaluating whether technology supports mathematical reasoning, conceptual understanding, and meaningful classroom interactions. Overall, this study provides a preliminary foundation for developing a theoretically integrated and empirically robust approach to assessing technology integration in mathematics teaching.

Limitations and Suggestions for Future Research

This study has several limitations that should be considered when interpreting its findings. Content validation involved only one expert, so the results represent preliminary rather than comprehensive evidence of instrument validity. The needs analysis was also based on brief informal interviews without audio recordings or verbatim transcripts, limiting the depth of qualitative interpretation. In addition, the instrument has not yet been administered to the planned sample of graduate mathematics teachers. Consequently, its construct validity, reliability, and psychometric performance remain to be established. The lower relevance score for TCK also indicates that several technology-content items require further refinement. Future research should involve multiple independent experts to strengthen content-validation evidence. The revised instrument should subsequently be tested with a larger and more diverse sample of mathematics teachers. Further psychometric analysis is needed to examine its factor structure, internal consistency, and relationships among the proposed dimensions. Recorded interviews and classroom observations could also provide deeper evidence regarding teachers' technology-integration practices. Future studies should further examine whether the TPACK and Didactic Tetrahedron dimensions remain empirically distinguishable when integrated within a single instrument. Overall, iterative expert review, broader empirical testing, and classroom-based investigation are required before the instrument can be recommended for wider application.

CONCLUSION

This study reported the instrument-development phase of a larger sequential explanatory mixed-methods project investigating how graduate mathematics education students in an open and distance learning program integrate digital technology into their secondary school teaching practice. Grounded in a preliminary needs-analysis interview that revealed a gap between the presence of

deep-learning pedagogy and teachers' limited capacity to enact it through technology, an integrated 30-item questionnaire and five-question interview protocol spanning the seven TPACK domains and four Tetrahedron relations was developed and subjected to expert content validation. The instrument achieved a high overall mean relevance score (3.89 out of 4), with the majority of items rated highly relevant and only four items, concentrated in the technology-content knowledge domain, recommended for minor revision. These findings directly answer the study's three research questions, establishing both the empirical justification for the instrument and the specific refinements needed before its full-scale administration.

Beyond its immediate methodological contribution, this study illustrates how needs-analysis interviews, theoretical integration of the Didactic Tetrahedron and TPACK, and formal expert judgment can be combined into a coherent, transparent instrument-development pathway for mathematics education research, one that other graduate researchers working within similar frameworks may adapt. The study's principal limitation, reliance on a single expert validator rather than a multi-rater panel with formal agreement indices such as Aiken's *V*, means that the content-validity evidence reported here should be regarded as preliminary rather than conclusive. Future work will expand the validator panel, revise the technology-content items as recommended, and proceed to the questionnaire, focus-group, and classroom-observation phases of the broader project involving 84 graduate mathematics teachers across Indonesia, which will allow the instrument's construct validity and its capacity to capture genuine transformation in teaching practice to be examined empirically.

AUTHOR CONTRIBUTIONS STATEMENT

Kartono conceived the study, developed the conceptual framework, designed the research methodology, and contributed to drafting the manuscript. Sudirman coordinated the research process, contributed to instrument development, data analysis, interpretation of the findings, and critical revision of the manuscript. Thesa Kandaga contributed to the development of the Didactic Tetrahedron-TPACK instrument, preliminary needs analysis, data organization, and manuscript preparation. Mery Noviyanti contributed to instrument refinement, interpretation of the expert-validation results, literature synthesis, and final review of the manuscript. All authors read and approved the final version of the manuscript.

REFERENCES

- Abrahamson, D., Nathan, M. J., Williams-Pierce, C., Walkington, C., Ottmar, E. R., Soto, H., & Alibali, M. W. (2020). The future of embodied design for mathematics teaching and learning. *Frontiers in Education*, 5. <https://doi.org/10.3389/educ.2020.00147>
- Ait Ali, D., El Meniari, A., El Filali, S., Morabite, O., Senhaji, F., & Khabbache, H. (2023). Empirical research on technological pedagogical content knowledge (TPACK) framework in health professions education: A literature review. *Medical Science Educator*, 33(3), 791–803. <https://doi.org/10.1007/s40670-023-01786-z>
- Arhin, J., Boateng, F. O., Akosah, E. F., & Gyimah, K. (2024). Perceptions and readiness of high school mathematics teachers for integration of ICT tools in the teaching and learning of mathematics. *Pedagogical Research*, 9(1), em0179. <https://doi.org/10.29333/pr/14032>

- Attard, C., & Holmes, K. (2022). An exploration of teacher and student perceptions of blended learning in four secondary mathematics classrooms. *Mathematics Education Research Journal*, 34(4), 719–740. <https://doi.org/10.1007/s13394-020-00359-2>
- Aumann, A., Schnebel, S., & Weitzel, H. (2023). The EnTPACK rubric: Development, validation, and reliability of an instrument for measuring pre-service science teachers' enacted TPACK. *Frontiers in Education*, 8. <https://doi.org/10.3389/feduc.2023.1190152>
- Backfisch, I., Scherer, R., Schneider, J., & Lachner, A. (2025). How valid, really? A meta-analysis of the validity evidence of technological pedagogical and content knowledge (TPACK) self-report assessments. *Educational Research Review*, 49, 100718. <https://doi.org/10.1016/j.edurev.2025.100718>
- Chiu, T. K. F. (2025). Developing intelligent-TPACK (I-TPACK) framework from unpacking AI literacy and competency: Implementation strategies and future research direction. *Interactive Learning Environments*, 33(7), 4189–4192. <https://doi.org/10.1080/10494820.2025.2545053>
- Cirneanu, A.-L., & Moldoveanu, C.-E. (2024). Use of digital technology in integrated mathematics education. *Applied System Innovation*, 7(4). <https://doi.org/10.3390/asi7040066>
- Dasari, D., Hendriyanto, A., Sahara, S., Suryadi, D., Muhaimin, L. H., Chao, T., & Fitriana, L. (2024). ChatGPT in didactical tetrahedron, does it make an exception? A case study in mathematics teaching and learning. *Frontiers in Education*, 8. <https://doi.org/10.3389/feduc.2023.1295413>
- Demeshkant, N., Trusz, S., & Potyrała, K. (2022). Interrelationship between levels of digital competences and technological, pedagogical and content knowledge (TPACK): A preliminary study with Polish academic teachers. *Technology, Pedagogy and Education*, 31(5), 579–595. <https://doi.org/10.1080/1475939X.2022.2092547>
- Dockendorff, M., & Zaccarelli, F. G. (2025). Successfully preparing future mathematics teachers for digital technology integration: A literature review. *International Journal of Mathematical Education in Science and Technology*, 56(5), 948–979. <https://doi.org/10.1080/0020739X.2024.2309273>
- Fabian, A., Fütterer, T., Backfisch, I., Lunowa, E., Paravicini, W., Hübner, N., & Lachner, A. (2024). Unraveling TPACK: Investigating the inherent structure of TPACK from a subject-specific angle using test-based instruments. *Computers & Education*, 217, 105040. <https://doi.org/10.1016/j.compedu.2024.105040>
- Kadluba, A., Strohmaier, A., Schons, C., & Obersteiner, A. (2025). How much C is in TPACK? A systematic review on the assessment of TPACK in mathematics. *Educational Studies in Mathematics*, 118(2), 169–199. <https://doi.org/10.1007/s10649-024-10357-x>
- Kania, N., Kusumah, Y. S., Dahlan, J. A., Nurlaelah, E., Gürbüz, F., & Bonyah, E. (2024). Constructing and providing content validity evidence through the aiken's V index based on the experts' judgments of the instrument to measure mathematical problem-solving skills. *REID (Research and Evaluation in Education)*, 10(1), 64–79. <https://doi.org/10.21831/reid.v10i1.71032>
- Li, M., Noori, A. Q., & Li, Y. (2023). Development and validation of the secondary mathematics teachers' TPACK scale: A study in the Chinese context. *Eurasia Journal of Mathematics, Science and Technology Education*, 19(11), em2350. <https://doi.org/10.29333/ejmste/13671>
- Li, M., & Nugraha, M. G. (2025). Validating a context-specific TPACK scale for primary mathematics education in China. *International Electronic Journal of Mathematics Education*, 20(2), em0819. <https://doi.org/10.29333/iejme/15934>
- Li, S., Liu, Y., & Su, Y.-S. (2022). Differential analysis of teachers' technological pedagogical content knowledge (TPACK) abilities according to teaching stages and educational levels. *Sustainability*, 14(12). <https://doi.org/10.3390/su14127176>

- Lindenbauer, E., Infanger, E.-M., & Lavicza, Z. (2024). Enhancing mathematics education through collaborative digital material design: Lessons from a national project. *European Journal of Science and Mathematics Education*, 12(2), 276–296. <https://doi.org/10.30935/scimath/14323>
- Liu, J., & Buchbinder, O. (2024). Instructional technology tetrahedron and network visualization: Conceptualizing online teaching through a lens of reflective noticing. *Digital Experiences in Mathematics Education*, 10(3), 431–461. <https://doi.org/10.1007/s40751-024-00151-y>
- Lynch, K., Gonzalez, K., Hill, H., & Merritt, R. (2025). A Meta-Analysis of the experimental evidence linking mathematics and science professional development interventions to teacher knowledge, classroom instruction, and student achievement. *AERA Open*, 11, 23328584251335302. <https://doi.org/10.1177/23328584251335302>
- Mailizar, Almanthari, A., Maulina, S., & Bruce, S. (2020). Secondary school mathematics teachers' views on e-learning implementation barriers during the COVID-19 pandemic: The case of indonesia. *Eurasia Journal of Mathematics, Science and Technology Education*, 16(7), em1860. <https://doi.org/10.29333/ejmste/8240>
- Mani, J., & Zargeh, C. (2025). AI artifacts in the mathematics didactical tetrahedron: A developed model. *Eurasia Journal of Mathematics, Science and Technology Education*, 21(5), em2630. <https://doi.org/10.29333/ejmste/16307>
- Merino-Soto, C. (2022). Coeficientes V de Aiken: Diferencias en los juicios de validez de contenido. *MHSalud: Revista en Ciencias del Movimiento Humano y Salud*, 20(1), 1–10. <https://doi.org/10.15359/mhs.20-1.3>
- Ning, Y., Zhang, C., Xu, B., Zhou, Y., & Wijaya, T. T. (2024). Teachers' AI-TPACK: Exploring the relationship between knowledge elements. *Sustainability*, 16(3). <https://doi.org/10.3390/su16030978>
- Pedersen, M. K., Bach, C. C., Gregersen, R. M., Højsted, I. H., & Jankvist, U. T. (2021). Mathematical representation competency in relation to use of digital technology and task Design-A literature review. *Mathematics*, 9(4). <https://doi.org/10.3390/math9040444>
- Rezat, S., & Geiger, V. (2024). The role of digital technologies in transforming student learning landscapes. In *Handbook of Digital Resources in Mathematics Education* (pp. 587–608). Springer, Cham. https://doi.org/10.1007/978-3-031-45667-1_21
- Setiawan, A., Soeharto, S., Wijaya, T. T., Korenova, L., & Lavicza, Z. (2025). Measuring teachers' competencies for AI integration: Development and validation of the AI-TPACK in vocational education. *Computers and Education Open*, 9, 100319. <https://doi.org/10.1016/j.caeo.2025.100319>
- Sinclair, S., Hack, T. F., MacInnis, C. C., Jaggi, P., Boss, H., McClement, S., Sinnarajah, A., & Thompson, G. (2021). Development and validation of a patient-reported measure of compassion in healthcare: The Sinclair compassion questionnaire (SCQ). <https://doi.org/10.1136/bmjopen-2020-045988>
- Smith, P. G., & Zelkowski, J. (2023). Validating a TPACK instrument for 7-12 mathematics in-service middle and high school teachers in the United States. *Journal of Research on Technology in Education*, 55(5), 858–876. <https://doi.org/10.1080/15391523.2022.2048145>
- Sofroniou, A. (2025). Advancing conceptual understanding: A meta-analysis on the impact of digital technologies in higher education mathematics. *Education Sciences*, 15(11). <https://doi.org/10.3390/educsci15111544>
- Sofyan, S., Habibi, A., Sofwan, M., Yaakob, M. F. M., Alqahtani, T. M., Jamila, A., & Wijaya, T. T. (2023). TPACK-UotI: The validation of an assessment instrument for elementary school teachers. *Humanities and Social Sciences Communications*, 10(1), 55. <https://doi.org/10.1057/s41599-023-01533-0>

- Stein, H., Gurevich, I., & Gorev, D. (2020). Integration of technology by novice mathematics teachers - what facilitates such integration and what makes it difficult? *education and information technologies*, 25(1), 141-161. <https://doi.org/10.1007/s10639-019-09950-y>.
- Thirumalai, B. R., & Sarangapani, P. M. (2025). Teacher professional development at scale: Mathematics teachers' community of practice action research study. *Contemporary Education Dialogue*, 22(2), 267–305. <https://doi.org/10.1177/09731849251330685>
- Thurm, D., & Barzel, B. (2020). Effects of a professional development program for teaching mathematics with technology on teachers' beliefs, self-efficacy and practices. *ZDM*, 52(7), 1411–1422. <https://doi.org/10.1007/s11858-020-01158-6>
- Viberg, O., Grönlund, Å., & Andersson, A. (2023). Integrating digital technology in mathematics education: A Swedish case study. *Interactive Learning Environments*, 31(1), 232–243. <https://doi.org/10.1080/10494820.2020.1770801>
- Weinhandl, R., Houghton, T., Lindenbauer, E., Mayerhofer, M., Lavicza, Z., & Hohenwarter, M. (2021). Integrating technologies into teaching and learning mathematics at the beginning of secondary education in Austria. *EURASIA Journal of Mathematics, Science and Technology Education*, 17(12). <https://doi.org/10.29333/ejmste/11428>