



Exploring students' metacognitive processes in scaffolded integration problem solving

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Abstract

Background: Integration techniques are fundamental to advanced mathematics, yet students frequently encounter difficulties in recognizing problem structures, selecting appropriate integration methods, and regulating their problem-solving processes. Scaffolding may facilitate these processes by supporting students' metacognitive awareness, monitoring, and evaluation.

Aim: This study aimed to explore students' metacognitive processes during scaffolded integration problem solving.

Method: A qualitative case study was conducted with three Mathematics Education students enrolled in an Integral Calculus course at UIN Walisongo. Participants represented different performance levels and were examined through open-ended integration problems and follow-up interviews. Their problem-solving processes were analyzed according to three metacognitive dimensions: awareness, monitoring, and evaluation.

Results: The findings revealed varying levels of metacognitive engagement across participants. Awareness and monitoring were more evident among higher-performing students, particularly in recognizing problem characteristics, selecting appropriate integration strategies, and controlling procedural steps. In contrast, evaluation remained consistently underdeveloped, as students rarely verified their answers or reflected on the effectiveness of their strategies.

Conclusion: Scaffolded integration problem solving can facilitate the emergence of metacognitive awareness and monitoring; however, explicit evaluative prompts are needed to strengthen verification and reflection during mathematical problem solving.

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INTRODUCTION

Integration techniques constitute a fundamental component of calculus because they provide students with conceptual and procedural foundations for solving mathematical problems and progressing toward more advanced areas of mathematics. Mastery of integration requires more than computational proficiency, as students must recognize the structure of an integrand, connect relevant mathematical concepts, and determine an appropriate technique before executing a solution (Cervantes-Barraza et al., 2025; Maries & Singh, 2023; Özgeldi & Aydın, 2021). Common approaches, including substitution, integration by parts, partial fractions, trigonometric integrals, and trigonometric substitution, impose different reasoning demands depending on the characteristics of a given problem. Consequently, integration problem solving involves strategic decision making in which students must coordinate conceptual understanding, procedural knowledge, and the selection of suitable mathematical operations. Nevertheless, students frequently experience difficulties in

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interpreting integral problems, determining appropriate procedures, and maintaining accuracy throughout the solution process. Previous evidence indicates that these difficulties can prevent students from determining appropriate steps when confronting integral problems with different mathematical characteristics (Ke & M. Clark, 2020; McDonald & Smith, 2020). Difficulties may become particularly apparent when students are required to move beyond direct application of formulas and recognize relationships among functions, derivatives, and alternative solution procedures. Such conditions suggest that successful integration problem solving depends not only on knowing available techniques but also on determining when, why, and how a particular technique should be applied. Addressing these challenges therefore requires learning environments that provide students with sufficient support while gradually encouraging them to assume greater responsibility for their mathematical reasoning. In this context, scaffolding represents a relevant instructional approach because it can provide temporary and responsive assistance that helps students bridge the gap between their current capabilities and the demands of complex mathematical problems.

Scaffolding can be implemented through guiding questions, hints, feedback, worked examples, or prompts that direct students toward relevant mathematical relationships without immediately providing complete solutions. In integration problem solving, such assistance can encourage students to inspect the characteristics of an integrand, identify relationships between functions and their derivatives, compare possible integration techniques, and reconsider strategies that do not produce mathematically consistent results. Through structured support, students can gradually develop more systematic problem-solving strategies while receiving assistance that corresponds to their individual learning needs (Liu et al., 2025; Mas' et al., 2025; Riva, 2026; W.-S. Wang et al., 2025; Zhong & Zhan, 2024). Recent evidence further indicates that scaffolding can facilitate mathematical understanding through interaction, communication, and instructional guidance that enables learners to construct meaning rather than merely reproduce procedures. However, obtaining a correct solution after receiving instructional support does not necessarily demonstrate that students understand or regulate the reasoning underlying the solution process. Mathematical problem solving also requires students to recognize what they know, determine whether a selected strategy remains appropriate, identify errors as they emerge, and critically examine the validity of the final result. These regulatory activities are closely associated with metacognition, which enables learners to become aware of and exercise control over their cognitive processes during mathematical tasks. Metacognition is particularly relevant to integration problems because students often need to choose among multiple possible techniques and continuously determine whether their chosen procedures correspond to the mathematical structure of the problem. Research has shown that metacognitive processes contribute to mathematical problem solving by enabling learners to regulate their thinking and coordinate strategies while completing mathematical tasks. Accordingly, examining integration problem solving through a metacognitive perspective can provide insight not only into whether students arrive at correct solutions but also into how they recognize, regulate, and reconsider their mathematical decisions.

Within the framework proposed by Wilson and Clarke, metacognitive processes can be examined through three interconnected dimensions: awareness, monitoring, and evaluation. Awareness concerns students' recognition of problem characteristics, available information, prior knowledge, and potentially appropriate strategies, whereas monitoring involves controlling the progression of a solution, checking procedural consistency, and identifying errors that arise during problem solving. Evaluation extends this regulation by requiring students to verify the correctness of a completed solution, assess the effectiveness of the strategy employed, and reflect on whether alternative approaches could provide a more appropriate solution. Contemporary research has increasingly highlighted metacognition and metacognitive scaffolding in various learning

environments, indicating continued interest in supporting learners' regulation of complex cognitive activities. Studies have also shown that students with stronger mathematical problem-solving abilities tend to demonstrate more developed metacognitive characteristics, reinforcing the close relationship between mathematical performance and regulation of thinking (Tanreso et al., 2025). Despite these developments, existing studies provide comparatively limited explanations of how awareness, monitoring, and evaluation unfold specifically during scaffolded integration problem solving, particularly among university students who demonstrate different levels of problem-solving performance (Gao et al., n.d.; Im et al., 2026; Vo et al., 2024; C.-Y. Wang & Huang, 2026). This limitation is important because students may successfully recognize an appropriate integration technique yet fail to monitor its implementation, or they may complete a mathematically correct procedure without subsequently evaluating the validity of their answer. Examining overall performance alone may therefore obscure important differences among the metacognitive processes underlying students' mathematical solutions. A process-oriented investigation is needed to trace how students identify appropriate strategies, regulate procedural decisions, recognize inconsistencies, and evaluate their results while receiving scaffolded support. Such an investigation can extend existing discussions of metacognition beyond general mathematical achievement by positioning awareness, monitoring, and evaluation as observable dimensions of students' reasoning during a specific and cognitively demanding calculus task. Therefore, exploring these processes within scaffolded integration problem solving can provide a more nuanced understanding of where students demonstrate metacognitive strengths and where more explicit instructional support remains necessary.

Recent studies have increasingly emphasized the contribution of metacognition and metacognitive scaffolding to learning and problem solving, including investigations of metacognitive scaffolding in generative AI-supported learning, project-based and engineering learning, inquiry-based environments, programming, physics, chemistry, and higher-order thinking development (Michalsky, 2024; Vo et al., 2024; W.-S. Wang et al., 2025; Wu et al., 2025). Other studies have examined metacognitive knowledge, monitoring, control, and regulation as important mechanisms underlying students' learning and problem-solving processes (Rivas et al., 2022; Rivers et al., 2020; Zepeda & Nokes-Malach, 2023), while research in mathematics education has highlighted problem-solving strategies, creativity, and metacognitive considerations in mathematical tasks (Khalid et al., 2020; Szabo et al., 2020; Verschaffel et al., 2020). However, these strands of research provide limited insight into how distinct metacognitive processes unfold specifically during scaffolded integration problem solving in undergraduate mathematics education, particularly when students must recognize problem structures, select appropriate integration techniques, regulate procedural decisions, detect inconsistencies, and verify their solutions. Furthermore, existing studies tend to examine overall metacognitive performance, general problem-solving competence, or the effects of scaffolding in domains other than integral calculus, leaving the interplay among awareness, monitoring, and evaluation during integration problem solving comparatively underexplored. This distinction is important because integration problems may require students to coordinate conceptual understanding with strategic choices among substitution, integration by parts, partial fractions, and trigonometric techniques rather than merely execute a predetermined procedure. Therefore, this study addresses this gap by qualitatively exploring students' awareness, monitoring, and evaluation during scaffolded integration problem solving across different performance levels, providing a process-oriented understanding of metacognitive regulation within a specific mathematics learning context.

Responding to this research gap, the present study aims to explore students' metacognitive processes during scaffolded integration problem solving, with particular attention to awareness, monitoring, and evaluation. The study examines how students recognize the mathematical

characteristics of integration problems and connect these characteristics with relevant prior knowledge and available integration techniques. It further investigates how students determine whether substitution, integration by parts, partial fractions, trigonometric integration, or other strategies are appropriate for the structure of a given problem. Attention is also directed toward how students monitor the consistency of their mathematical reasoning, control procedural steps, and identify conceptual or computational errors during the solution process. In addition, the study explores students' evaluation processes by examining whether they verify final answers, reconsider the effectiveness of selected strategies, and reflect on the mathematical validity of their solutions. These metacognitive processes are investigated across students representing different levels of problem-solving performance to identify similarities and variations in their regulatory patterns. Rather than considering correct answers as the sole indicator of successful problem solving, the study emphasizes the reasoning processes through which students plan, regulate, and evaluate their mathematical actions. This perspective is consistent with contemporary mathematics education research emphasizing that metacognitive regulation constitutes an important component of effective mathematical problem solving. By examining students' written solutions alongside their explanations of the reasoning underlying those solutions, the study seeks to provide a process-oriented account of metacognitive activity within a scaffolded calculus learning context. Ultimately, the findings are expected to inform the development of more targeted scaffolding strategies that support not only the selection and execution of integration techniques but also students' capacity for verification, reflection, and increasingly independent mathematical problem solving.

LITERATURE REVIEW

Integration problem solving is a central component of calculus learning because it requires students to coordinate conceptual understanding, procedural knowledge, and strategic decision making. Unlike routine exercises in which a procedure is explicitly indicated, integration problems may require learners to first recognize the mathematical structure of an integrand before determining an appropriate technique. Students may need to distinguish whether a problem is more appropriately addressed through substitution, integration by parts, rational-function decomposition, trigonometric integration, or trigonometric substitution (Msomi & Bansilal, 2022; Weldeana et al., 2023). This decision-making process becomes challenging when different techniques appear plausible or when the structure of the integrand must first be transformed. Previous research has shown that students continue to experience difficulties in solving integral problems because of limitations in conceptual understanding and in the execution of appropriate solution procedures (Braithwaite & Sprague, 2021; Ncube & Luneta, n.d.). These difficulties may influence students' capacity to determine the sequence of steps required for problems with different mathematical characteristics. Effective integration problem solving therefore extends beyond memorizing formulas or reproducing previously demonstrated procedures. It requires students to interpret mathematical information, connect the problem with relevant prior knowledge, select a defensible strategy, and maintain consistency throughout the solution process. Such characteristics position integration problems as suitable contexts for investigating how students regulate their mathematical thinking while navigating multiple strategic possibilities. Accordingly, understanding students' integration problem solving requires attention not only to the accuracy of their final answers but also to the cognitive and metacognitive processes underlying their strategic decisions.

Metacognition provides a theoretical lens for examining how learners become aware of and regulate their thinking during mathematical problem solving. Within mathematics education, metacognitive activity is particularly important when a task cannot be completed successfully

through direct procedural recall and instead requires strategic choices and continuous regulation. Wilson and Clarke conceptualize metacognition in mathematical problem solving through three interrelated dimensions: awareness, monitoring, and evaluation (Mannan et al., 2025; Prastiti et al., 2026). Awareness concerns learners' recognition of the requirements of a problem, relevant prior knowledge, available strategies, and their own understanding before or during the solution process. In integration problems, awareness may be manifested when students identify characteristics of an integrand and associate those characteristics with a potentially appropriate integration technique. Monitoring refers to the ongoing regulation of reasoning, including checking procedural steps, assessing whether the selected method remains appropriate, and detecting errors or inconsistencies as a solution develops. Evaluation occurs when students reconsider the completed solution, verify its mathematical correctness, assess the effectiveness of the strategy employed, and reflect on alternative approaches. Research in mathematics education has demonstrated the relevance of metacognitive processes to mathematical problem solving because learners must coordinate cognitive activity with regulation of their strategies (Ferreira et al., 2022; Hegedus & Otálora, 2023; Kholid et al., 2024; Mannan et al., 2026; Prastiti et al., 2026). Students with stronger mathematical problem-solving abilities have also been reported to exhibit more developed metacognitive characteristics than students with lower problem-solving abilities (Tanreso et al., 2025). Thus, awareness, monitoring, and evaluation provide a useful analytical structure for explaining differences in how students approach, regulate, and reflect upon integration problems.

Although the three metacognitive dimensions are conceptually related, they may not necessarily emerge with the same strength during a mathematical problem-solving episode. A student may demonstrate awareness by correctly recognizing the structure of an integral and selecting an appropriate technique while showing limited monitoring of subsequent algebraic or computational procedures. Similarly, a learner may successfully monitor a sequence of solution steps but terminate the process immediately after obtaining an answer without independently verifying its validity. These distinctions are important because the correctness of a final response alone may conceal differences in the regulatory processes through which that response was produced. Metacognitive awareness establishes an initial orientation toward the task by enabling students to recognize what is known, what must be determined, and which knowledge may be relevant. Monitoring then operates throughout problem solving by allowing learners to assess whether their mathematical actions remain consistent with the intended strategy. Evaluation extends this regulation beyond procedural completion by encouraging students to verify results and reconsider the effectiveness of the approach they employed. In integration problem solving, the interaction among these processes is especially relevant because an initially reasonable technique may require modification when subsequent transformations become inefficient or mathematically inconsistent (Garte et al., 2025; Greengrass, 2026; Malenkov et al., 2021; J. Zhang et al., 2020). Consequently, analyzing metacognition as differentiated but interconnected processes can reveal aspects of mathematical reasoning that may remain invisible when students are classified solely according to correct or incorrect answers. A process-oriented perspective therefore provides an appropriate basis for examining how different students regulate their reasoning while working through scaffolded integration problems.

Scaffolding offers an instructional mechanism through which these metacognitive processes can be elicited and supported during mathematical problem solving. Rather than providing students with complete solutions, scaffolding involves temporary assistance that can be adjusted according to learners' current difficulties and gradually reduced as greater independence develops (Azizah et al., 2025). In mathematics learning, such support can take the form of guiding questions, prompts, hints, feedback, representations, or structured steps that direct attention toward relevant features of a problem. Structured assistance has been associated with the development of more systematic

problem-solving strategies because learners receive support corresponding to the difficulties they encounter during mathematical tasks. Scaffolding can also facilitate mathematical understanding through instructional interaction and communication that encourages students to participate actively in constructing solutions (Su, 2026). Within integration problem solving, prompts may direct students to examine the form of an integrand, identify relationships among functions, compare alternative techniques, or reconsider a procedure when an inconsistency appears. From a metacognitive perspective, these prompts can potentially stimulate awareness by directing attention to problem characteristics and monitoring by encouraging students to examine the appropriateness of their ongoing procedures. Scaffolding may also support evaluation when prompts explicitly require students to verify their final results, justify the effectiveness of a selected technique, or consider an alternative solution. However, the presence of instructional support does not automatically imply that students independently engage in all dimensions of metacognitive regulation, particularly when verification and reflection are not explicitly demonstrated. Therefore, examining students' responses to scaffolded integration problems is necessary to distinguish between successful procedural performance and the actual manifestation of awareness, monitoring, and evaluation.

The intersection of integration problem solving, metacognition, and scaffolding provides the conceptual foundation for the present study. Existing literature demonstrates that integration problems present conceptual and procedural challenges for students, while metacognitive research emphasizes the importance of regulating thought processes during mathematical problem solving (Susanna et al., 2026; Toikka et al., 2024; Zhao & Saleh, 2025). Research on scaffolding further indicates that structured instructional assistance can support students in navigating demanding mathematical tasks and developing more systematic problem-solving processes (Su, 2026). Nevertheless, these research strands have often emphasized mathematical difficulties, metacognitive ability, or scaffolding as related but relatively separate concerns. Comparatively less attention has been devoted to documenting how the specific dimensions of awareness, monitoring, and evaluation are manifested while university students solve integration problems within a scaffolded environment. This issue becomes particularly important when students at different performance levels may arrive at different solutions through substantially different patterns of metacognitive regulation. Examining only test scores or final answers may consequently provide an incomplete representation of how students recognize integration structures, regulate their procedures, identify errors, and verify their results. A qualitative examination of written solutions complemented by students' explanations can provide richer evidence regarding the processes underlying these differences. Accordingly, the present study integrates scaffolded integration problem solving with the awareness-monitoring-evaluation framework to examine metacognition as a dynamic component of students' mathematical reasoning. This conceptual integration provides the basis for exploring how students with different problem-solving performance levels manifest metacognitive processes and for identifying dimensions of regulation that may require more explicit instructional support.

METHOD

Research Design

This study employed a qualitative approach with a case study design to explore students' metacognitive processes during scaffolded integration problem solving. The qualitative approach was selected because the study sought to obtain an in-depth understanding of students' thinking processes rather than merely quantify their problem-solving performance. The investigation focused on how students recognized mathematical problems, selected appropriate integration techniques,

monitored the progression of their solutions, and evaluated the results obtained. Scaffolding was incorporated into integral calculus instruction through step-by-step guidance intended to support students when they encountered difficulties in solving integration problems. The case study design enabled the researchers to examine students' written responses and explanations in detail and to identify variations in metacognitive processes across different levels of problem-solving performance. Metacognition was examined through the three dimensions proposed by Wilson and Clarke: awareness, monitoring, and evaluation. These dimensions provided the analytical framework for interpreting students' reasoning throughout the problem-solving process. Accordingly, the study emphasized not only whether students obtained mathematically correct answers but also how they selected, controlled, and reflected on their solution strategies. The combination of written problem-solving evidence and follow-up interviews provided complementary information about students' metacognitive activity. Thus, the research design was intended to generate a process-oriented account of metacognition within scaffolded integration problem solving.

Participant

The study was conducted in the context of an Integral Calculus course involving 96 Mathematics Education students at UIN Walisongo in 2025. During the course, students received integral calculus instruction incorporating scaffolding in the form of step-by-step guidance for solving integration problems. The problem-solving instrument was subsequently administered to 32 students, who completed six open-ended integration problems within the allocated assessment period. Based on their performance on the instrument, three students were selected for more intensive qualitative investigation. The selection represented contrasting levels of problem-solving performance, consisting of the student with the highest score, a student representing the middle score, and the student with the lowest score. These participants were coded as R1, R2, and R3, respectively, to distinguish the cases throughout the analysis. Selecting participants from different performance levels enabled the study to examine possible variations in awareness, monitoring, and evaluation during integration problem solving. The three selected students became the primary participants for the analysis of written solutions and subsequent interviews. Their written responses provided evidence of the strategies and procedures employed, while interviews were used to obtain further explanations of their reasoning. The manuscript does not specify how the 32 students were selected from the initial group of 96 students; therefore, this aspect should be clarified in the final version to strengthen the transparency of participant selection.

Instrument and Data Collection

The study employed open-ended integration problems and follow-up interviews as the principal sources of data. The written instrument consisted of six open-ended questions representing various integration techniques and had undergone pilot testing to establish its validity and reliability before administration. To guide the examination of students' metacognitive processes, nine indicators were developed based on the three dimensions of metacognition adopted in the study. For awareness, the indicators comprised understanding the problem, recognizing the type of problem, and acknowledging relevant existing knowledge. For monitoring, the indicators included controlling the problem-solving process, checking the suitability of the selected method, and detecting errors during solution development. For evaluation, the indicators comprised reviewing the completed solution, verifying the accuracy of the procedures employed, and drawing conclusions from the solution. The six open-ended questions were administered to 32 students, who were given 90 minutes to complete the assessment. Students' written responses were collected as the primary evidence for identifying the manifestation of awareness, monitoring, and evaluation during integration problem solving. After R1, R2, and R3 had been selected, follow-up interviews were conducted to clarify reasoning that could not be fully captured through written work alone. The

interviews also served to verify the interpretations derived from the open-ended responses and provide deeper information regarding the participants' metacognitive processes.

Data Analysis

Data analysis was organized around the three metacognitive dimensions of awareness, monitoring, and evaluation and their corresponding nine indicators. The written solutions of R1, R2, and R3 were examined to identify the mathematical strategies selected, the sequence of procedures performed, and any conceptual or procedural difficulties encountered during problem solving. Awareness was identified from evidence that participants understood the problem, recognized its mathematical characteristics, activated relevant prior knowledge, and selected an appropriate integration strategy. Monitoring was examined by tracing how participants controlled successive solution steps, maintained consistency between mathematical transformations, assessed the suitability of the selected technique, and detected errors during the problem-solving process. Evaluation was analyzed by determining whether participants reviewed their completed solutions, verified the accuracy of the procedures and results, and formulated appropriate conclusions. Particular attention was given to discrepancies between correct final answers and the presence or absence of explicit metacognitive activity because successful procedural completion did not automatically indicate that all three dimensions had been demonstrated. Interview responses were used to clarify students' reasoning and to verify interpretations derived from their written solutions. Evidence from the written responses and interviews was then considered together to characterize the metacognitive processes demonstrated by each participant. The patterns identified for R1, R2, and R3 were subsequently compared to examine similarities and differences across the three performance levels. The analysis ultimately produced a descriptive profile of each participant's attainment in awareness, monitoring, and evaluation during scaffolded integration problem solving.

Research Procedure

The research was implemented through a sequential series of stages beginning with the preparation of the metacognitive framework and research instruments. First, the researchers established nine indicators representing awareness, monitoring, and evaluation to provide a systematic basis for examining students' metacognitive processes. Second, six open-ended questions covering different integration techniques were developed and subjected to pilot testing for validity and reliability. Third, integral calculus instruction was conducted with students receiving scaffolding in the form of step-by-step guidance for solving integration problems. Fourth, the open-ended problem-solving instrument was administered to 32 students within a 90-minute period to obtain evidence of their performance and metacognitive processes. Fifth, the students' responses were examined and three participants representing the highest, middle, and lowest performance levels were selected and designated as R1, R2, and R3. Sixth, the written solutions of these three participants were examined according to the awareness, monitoring, and evaluation indicators. Seventh, follow-up interviews were conducted with R1, R2, and R3 to verify interpretations derived from their written responses and obtain deeper explanations of their reasoning. Eighth, evidence from the written solutions and interviews was integrated to develop individual descriptions of each participant's metacognitive processes. Finally, the patterns across the three cases were compared to identify differences and similarities in awareness, monitoring, and evaluation during scaffolded integration problem solving.

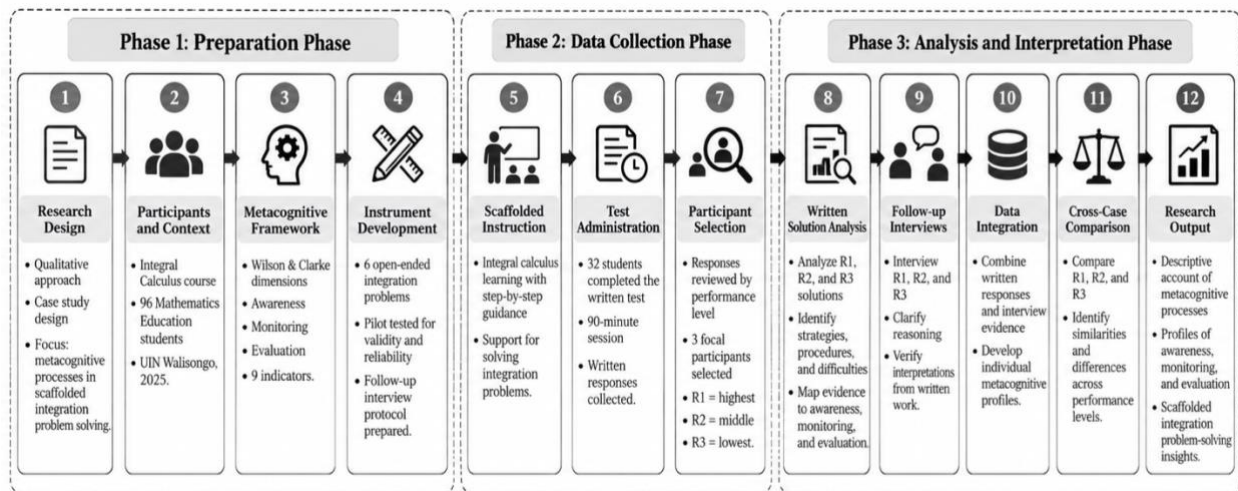


Figure 1. Research Method Flow

RESULTS AND DISCUSSION

Results

This section discusses the metacognitive abilities of mathematics education students—specifically regarding awareness, monitoring, and evaluation—as they solve problems involving integration techniques, based on Wilson's framework.

$$\begin{aligned}
 &\int x 2^{x^2} dx \\
 &\text{Misal: } u = x^2 \\
 &\frac{du}{dx} = 2x \\
 &du = 2x dx \\
 &x dx = \frac{1}{2} du \quad \checkmark \\
 &\int x 2^{x^2} dx = \frac{1}{2} \int 2^u du \\
 &\int a^u du = \frac{a^u}{\ln a} + C \\
 &\frac{1}{2} \int 2^u du = \frac{1}{2} \cdot \frac{2^u}{\ln 2} + C = \frac{2x^2}{2 \ln 2} + C \\
 &\text{Jadi, integral dari } \int x 2^{x^2} dx \text{ adalah } \frac{2x^2}{2 \ln 2} + C
 \end{aligned}$$

Figure 1. Solution by Student R1 for Question 1

At the awareness stage, student R1 demonstrated an awareness of the problem's characteristics and was able to identify the most appropriate strategy. This was evident when student R1 immediately set $u = x^2$. The choice of this substitution indicates that student R1 recognized the relationship between x^2 (as the function within the exponent) and the factor $x dx$ (which is the derivative of x^2). This awareness shows that student R1 recognized an integral pattern suitable for solution via the substitution method. However, the answer sheet did not show an explanation for why the substitution strategy was chosen; student R1 simply wrote "Let $u = x^2$ " without providing a conceptual rationale. Thus, while the awareness indicator was met, it was not explicitly demonstrated because the thought process remained implicit.

The monitoring indicator was more clearly evident. Student R1 was able to systematically control the solution process through the following stages: 1) determining the substitution; 2) differentiating the function to obtain $du = 2x dx$; 3) transforming the differential form to $x dx = \frac{1}{2} du$; 4) converting the entire integral expression into the new variable; 5) correctly applying the exponential integral formula; and 6) reverting the variable u back to x^2 . This sequence of steps

demonstrates that student R1 continuously monitored the relationship between successive steps, thereby avoiding errors in transforming the integral's form. Wilson and Clarke explain that monitoring involves controlling the thought process during problem-solving by checking strategies, the relationships between procedures, and the consistency of the steps taken. Consequently, the monitoring indicator can be considered successfully met.

The evaluation indicator was not optimally demonstrated. Although Student R1 obtained the correct answer, their worksheet showed no evidence of evaluation activities—such as re-checking the result via differentiation, verifying whether the answer satisfied the original integral, comparing it with another method, or justifying the correctness of the result. Wilson and Clarke define evaluation as the ability to reflect on the effectiveness of a strategy and re-examine the solution. The student's response merely stated, "So, the integral of ... is ...," without any verification process. Consequently, the evaluation indicator was not fully met; the student tended to stop after obtaining the final result without reflecting on the correctness of the answer.

$$\begin{aligned}
 & 2) a. \int_1^4 e^{\sqrt{x}} dx \\
 & \hookrightarrow u = \sqrt{x} \\
 & x = u^2 \\
 & dx = 2u du \\
 & \int_1^4 e^{\sqrt{x}} dx = 2 \int_1^2 u e^u du \\
 & \text{Integral Partial} \\
 & \int u e^u du = u e^u - e^u \\
 & 2 \int_1^2 u e^u du = 2 [(u-1) e^u]_1^2 \\
 & = 2 [(2-1)e^2 - (1-1)e^1] \\
 & = 2 (e^2 - 0) \\
 & = 2e^2 \\
 & \text{Jadi, integral dari } \int_1^4 e^{\sqrt{x}} dx \text{ adalah } 2e^2
 \end{aligned}$$

Figure 2. Solution by Student R1 for Question 2

Regarding the awareness aspect, student R1 demonstrated the ability to recognize the characteristics of the problem and select an appropriate strategy. This was evident when the student realized that the exponential form involving $\sqrt{x}$ would be easier to integrate using the substitution $\sqrt{x}$. Student R1 also understood that, since the given integral was a definite integral, the limits of integration had to be converted to the new variable. The awareness shown in changing the limits from $x=1$ to $x=4$ into $u=1$ to $u=2$ indicates that the student understood the relationship between the substitution concept and definite integrals. Furthermore, the student recognized that the substitution resulted in the form $\int u e^u du$, which could not be solved via standard integration but required integration by parts. This demonstrates an awareness of the function's characteristics and the corresponding solution strategy. Based on these findings, the awareness indicator can be considered successfully met.

Monitoring capabilities were evident throughout student R1's solution process. The student was able to: 1) determine the correct substitution, 2) change the variable and its differential, 3) change the limits of integration, 4) select the integration by parts method, and 5) complete the integration by parts to reach the final form. This sequence of steps shows that the student could systematically control the solution process and ensure alignment between the chosen method and the problem's form. No conceptual errors were found regarding strategy selection or the transformation of the integral. Thus, the monitoring indicator was well-demonstrated.

The evaluation aspect was the weakest indicator. Student R1 did not review the calculation process after obtaining the final result. Additionally, the student did not verify the answer—for instance, by double-checking each step. Yet, according to Wilson and Clarke (2004), evaluation

involves reflecting on the solution and verifying the accuracy of both the strategy and the resulting answer. Therefore, the evaluation indicator was not optimally achieved.

$$\int \frac{3x-13}{x^2+3x-10} dx$$

$$= \int \frac{3x-13}{(x+5)(x-2)} dx$$

$$= \int \frac{A}{x+5} + \frac{B}{x-2} dx$$

$$= A \int \frac{1}{x+5} dx + B \int \frac{1}{x-2} dx$$

$$= A \ln|x+5| + B \ln|x-2| + C$$

Menentukan A dan B

$$\frac{3x-13}{(x+5)(x-2)} = \frac{A}{x+5} + \frac{B}{x-2}$$

$$3x-13 = A(x-2) + B(x+5)$$

$$3x-13 = Ax-2A+Bx+5B$$

$$3x-13 = (A+B)x + (-2A+5B)$$

Koefisien x

$$A+B=3$$

Konstanta

$$-2A+5B=-13$$

Sehingga diperoleh

$$A+B=3$$

$$-2A+5B=-13$$

$$A=3-B$$

Substitusi kepers. 2

$$-2(3-B)+5B=-13$$

$$-6+2B+5B=-13$$

$$-6+7B=-13$$

$$7B=-7$$

$$B=-1$$

Koreksi A dan B

$$A+B=3$$

$$A+(-1)=3$$

$$A=4$$

Integrasi

$$\int \frac{3x-13}{x^2+3x-10} dx = \int \frac{4}{x+5} - \frac{1}{x-2} dx$$

$$= 4 \int \frac{1}{x+5} dx - \int \frac{1}{x-2} dx$$

$$= 4 \ln|x+5| - \ln|x-2| + C$$

Figure 3. Solution by Student R1 for Question 3

The awareness indicator is evident when student R1 is able to recognize the characteristics of the problem and select an appropriate strategy. Prior to integration, student R1 recognizes that the expression $\frac{3x-13}{x^2+3x-10}$ is a rational function with a denominator that can be factored into two linear

factors. This awareness prompts student R1 to choose the partial fraction method rather than substitution or integration by parts. This strategic choice demonstrates that student R1 understands the characteristics of rational functions and knows that partial fraction decomposition is the most efficient method for solving the problem. Furthermore, student R1 understands that once the denominator is factored, the next step is to determine constants A and B by equating coefficients. The ability to select a strategy from the outset indicates that student R1 possesses a high level of metacognitive awareness regarding the task's requirements. Based on the work produced, the awareness indicator has been met exceptionally well.

The monitoring aspect is demonstrated by student R1's ability to systematically control each step of the solution process. After establishing the partial fraction form, student R1 performs algebraic manipulations correctly, equates the numerators, and sets up the system of equations $A+B=3$ and $-2A+5B=-13$. Student R1 then solves this system to obtain the values $A=4$ and $B=-1$. Subsequently, these constants are substituted back into the partial fraction form before integration takes place. The solution process proceeds logically and without conceptual errors. Student R1 also demonstrates the ability to monitor the relationships between steps, ensuring that each procedure is interconnected. These activities align with the monitoring indicators defined by Wilson and Clarke (2004): controlling the flow of the thought process, checking the consistency of solution steps, and ensuring the chosen strategy remains aligned with the objective. Thus, the monitoring indicator can be categorized as excellent. Although student R1 performs well in the awareness and monitoring aspects, the evaluation aspect is not addressed; there is no evidence of a review of the procedures performed, as student R1 simply draws a conclusion.

e) $\int \sin^3 x \, dx$

$\hookrightarrow \sin^3 x = \sin x \cdot \sin^2 x$

$\sin^2 x = 1 - \cos^2 x$

$\int \sin^3 x \, dx$

$= \int \sin x (1 - \cos^2 x) \, dx$

$= \int (\sin x - \sin x \cos^2 x) \, dx$

Substitusikan

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$du = -\sin x \, dx$

$\int \sin x (1 - \cos^2 x) \, dx$

$= \int (1 - u^2) (-du)$

$= -\int (1 - u^2) \, du$

$= -\int 1 \, du + \int u^2 \, du$

$= -u + \frac{u^3}{3} + C$

Kembalikan ke variabel $u = \cos x$

$-u + \frac{u^3}{3} + C$

$= -\cos x + \frac{\cos^3 x}{3} + C$

Jadi, integral dari $\int \sin^3 x \, dx$ adalah $-\cos x + \frac{\cos^3 x}{3} + C$

Figure 4. Solution by Student R1 for Question 4

The awareness aspect is evident when the student recognizes the characteristics of the function to be integrated and selects an appropriate strategy. Student R1 realizes that the integral $\int \sin^3 x \, dx$ cannot be solved directly using basic integration formulas. Consequently, the student first transforms the function using trigonometric identities. This decision demonstrates an understanding of the relationship between trigonometric functions and their derivatives, enabling the selection of the most effective strategy. Thus, the awareness indicator has been met very well.

The monitoring aspect is apparent as Student R1 executes each step of the solution. The student systematically transforms the function, performs a variable substitution, establishes the differential relationship, converts the integral into the new variable, integrates the resulting function, and finally reverts to the original variable. The entire procedure is carried out sequentially and consistently. No conceptual errors are found regarding the use of trigonometric identities or substitution. The student also maintains coherence between steps, ensuring a logical progression of the solution process. According to Wilson and Clarke (2004), monitoring is the ability to control the thought process by checking the consistency of each step. Based on the work, Student R1 demonstrates excellent monitoring skills, as every procedure is performed systematically, leading to the correct answer. Therefore, the monitoring indicator is categorized as excellent.

In contrast to the previous two indicators, the evaluation aspect is not optimally demonstrated. After obtaining the final result, the student immediately concludes the answer without reviewing the outcome. There is no verification process—such as differentiation—or reflection on the effectiveness of the chosen strategy. Yet, according to Wilson and Clarke (2004), evaluation involves reviewing the solution, assessing the appropriateness of the strategy, and reflecting on the thought process employed. The absence of such checks indicates that the student has not yet fully developed metacognitive evaluation skills. Consequently, the evaluation indicator has not been optimally achieved, even though the final answer is correct.

b) $\int x 2^{x^2} \, dx$ (subs)

Misal $u = x^2$

$\frac{du}{dx} = 2x$

$x \, dx = \frac{1}{2} \, du$

$\int x 2^{x^2} \, dx = \frac{1}{2} \int 2^u \, du$

$\frac{1}{2} \int 2^u \, du = \frac{1}{2} \cdot \frac{2^u}{\ln 2} + C$

$\int x 2^{x^2} \, dx = \frac{2^{x^2}}{2 \ln 2} + C$

Figure 5. Solution by Student R2 for Question 1

The awareness indicator is evident in student R2's ability to recognize the characteristics of the problem and select an appropriate solution strategy. At the start of the process, student R2

realized that the exponent x^2 has a derivative proportional to the factor x located outside the exponential function. This awareness prompted student R2 to choose a substitution $u = x^2$ that transformed the integral into a simpler form. This strategic choice demonstrates that student R2 understands the relationship between composite functions and the substitution method. Student R2 did not attempt to use integration by parts or other less suitable methods but immediately selected the most efficient strategy. Based on the work, the awareness indicator was achieved very well.

The monitoring aspect is apparent in student R2's ability to systematically control each step of the solution. After determining the substitution, student R2 calculated the differential $du = 2xdx$ and then rearranged it to $xdx = \frac{1}{2}du$. Student R2 subsequently succeeded in transforming the integral into the new variable and correctly applying the exponential function integration formula to obtain the final result. The entire procedure was carried out sequentially, free from algebraic manipulation errors or conceptual mistakes. Student R2 also maintained consistency in variable usage throughout the solution process. Based on the work, the monitoring indicator was achieved very well.

The evaluation indicator relates to the ability to assess both the result and the solution process. In student R2's work, there was no evidence of activities such as verifying the final result through differentiation or justifying that the result aligned with the original function. Although student R2's answer was correct, the process of reflecting on the result was not explicitly demonstrated. Student R2 proceeded directly to the conclusion without verifying the answer. According to Wilson and Clarke (2004), evaluation encompasses activities such as reassessing results, testing the accuracy of procedures, and drawing conclusions. Therefore, the evaluation indicator was not optimally achieved.

2). A. $\int_1^4 e^{\sqrt{x}} dx$
 Misal kon
 $u = \sqrt{x}$
 $x = u^2, dx = 2u du$
 Sehingga $2 \int_1^2 u e^u du$
 $\int u e^u du = u e^u - \int e^u du = u e^u - e^u = e^u (u - 1)$
 $= 2 [e^u (u - 1)]_1^2$
 $= 2 (e^2 (2 - 1) - e^1 (1 - 1)) = 2 (e^2 - 0) = 2e^2$
 Jadi integral dari $\int_1^4 e^{\sqrt{x}} dx = 2e^2$.

Figure 6. Solution by Student R2 for Question 2

The awareness aspect is evident in student R2's ability to recognize the problem's characteristics from the outset. Student R2 realized that the definite integral form $\int_1^4 e^{\sqrt{x}} dx$ could not be solved directly using basic integration formulas. Consequently, the student chose the substitution strategy $u = \sqrt{x}$ to eliminate the radical term, thereby simplifying the integral. Furthermore, the student recognized that the substitution resulted in the form $\int u e^u du$, which required the integration by parts method. This demonstrates the student's ability to recognize changes in the problem's characteristics following a transformation and to select an appropriate subsequent strategy. According to Wilson and Clarke (2004), awareness entails the ability to recognize available

information, understand problem requirements, and select an appropriate strategy. In this instance, student R2 successfully identified that the solution required a combination of two methods: substitution and integration by parts. Therefore, the awareness indicator was met very effectively.

The monitoring indicator is demonstrated by student R2's ability to systematically control the solution process. The student not only performed the substitution correctly but also adjusted the integral limits from the variable x to the variable u . This step indicates an understanding of definite integral procedures when using substitution. Subsequently, the student applied integration by parts in a logical sequence, arriving at the form $ue'' - e''$. The student then correctly substituted the integral limits, yielding a result of $2e^2$. This logical sequence of steps shows that the student consistently monitored each procedure and maintained the coherence between the steps of the solution. Based on this work, the monitoring indicator was met very effectively.

The evaluation aspect relates to the ability to assess both the results and the solution process. The student's work shows that student R2 recorded the final result and provided a conclusion. However, there was no evidence of checking the result via differentiation or reflecting on the strategy employed. The student did not demonstrate a verification process to confirm that the integral result corresponded to the original function, nor did they explain why the combination of substitution and integration by parts was the most efficient strategy. Based on the work, the evaluation indicator was not optimally demonstrated, even though the final answer was correct.

$$\int \frac{3x-13}{x^2+3x-10} dx$$

$$x^2+3x-10 = (x+5)(x-2)$$

$$\frac{3x-13}{(x+5)(x-2)} = \frac{A}{x+5} + \frac{B}{x-2}$$

$$3x-13 = A(x-2) + B(x+5)$$

mis $x=2$

$$3x-13 = 3 \cdot 2 - 13$$

$$= 6 - 13$$

$$= -7$$

B

mis $x=-5$

$$3x-13 = 3 \cdot (-5) - 13$$

$$= -15 - 13$$

$$= -28$$

$$\int \left(\frac{4}{x+5} - \frac{1}{x-2} \right) dx = 4 \ln|x+5| - \ln|x-2| + C$$

Figure 7. Solution by Student R2 for Question 3

The awareness indicator was demonstrated when student R2 was able to recognize the characteristics of the problem and choose an appropriate strategy. Student R2 realized that the denominator $x^2 + 3x - 10$ could be factored into $(x+5)(x-2)$, making the rational function suitable for solving using partial fractions. In this case, student R2 was able to understand the problem and recognize the problem's form. However, student R2 incorrectly applied the method to determine the values of the constants A and B. Student R2 did not use the concept of solving equations, but when integrating, obtained the correct result, which student R2 admitted to obtaining with the help of a friend. Therefore, the awareness indicator was in the sufficient category.

In the monitoring aspect, student R2 was unable to control the solution process, the appropriateness of the method, and was unable to detect errors, particularly when determining the values of the constants A and B. Student R2 did not use the substitution or elimination method to find the simultaneous values of the system of linear equations in two variables, resulting in an incorrect result. Therefore, student R2's monitoring aspect in this problem was in the poor category.

The evaluation aspect relates to the ability to review results and reflect on the solution process. In student R2's work, this indicator was not optimal. Student R2 immediately wrote the final result

without rechecking it through differentiation. verify that the derivative of the integral result is the same as the original integrand function; reflect whether the strategy used is the most efficient. Therefore, the evaluation indicator has not been achieved optimally.

$$\begin{aligned}
 \int \sin^3 x \, dx &= & \text{misal } u &= \sin x \\
 &= \int u^3 \cdot \frac{1}{\cos x} \, du & \frac{du}{dx} &= \cos x \\
 & & du &= \cos x \, dx \\
 &= \frac{1}{\cos x} \int u^3 \, du & \frac{du}{\cos x} &= dx \\
 &= \frac{1}{\cos x} \cdot \frac{1}{3+1} u^{3+1} + C \\
 &= \frac{1}{\cos x} \cdot \frac{1}{4} u^4 + C \\
 &= \frac{1}{4 \cos x} u^4 + C \\
 &= \frac{1}{4 \cos x} \cos^{-1} x \cdot \sin^4 x + C
 \end{aligned}$$

Figure 8. Solution by Student R2 for Question 4

Based on the work of student R2 shown in the image, the problem being solved is $\int \sin^3 x \, dx$. However, unlike some previous attempts, the student did not arrive at the correct answer in this instance. The student proceeded directly to substitution by setting $u = \sin x$ and $du = \cos x \, dx$, rewriting the integral as $\int \sin^3 x \, dx = \int \frac{u^3}{\cos x} \, dx$. This step reveals a conceptual error; after substituting for variable u , the variable x still remained in the form of $\int u^2 \frac{1}{\cos x} \, du$. Consequently, the integral still contained two different variables, rendering the substitution mathematically invalid. Student R2 then treated $\frac{1}{\cos x}$ as a constant and obtained the result $\frac{\sin^4 x}{4 \cos x} + C$ even though this was not the correct outcome for $\int \sin^3 x \, dx$.

Regarding the "awareness" indicator, student R2 demonstrated an understanding that the problem required a solution strategy and opted for the substitution method. This indicates that student R2 understood that trigonometric functions with exponents can often be solved via a change of variables. However, this awareness was not fully developed. The student failed to recognize that the chosen substitution must result in an integral expressed entirely in terms of the new variable. The student chose a term $u = \sin x$ whose derivative yields $du = \cos x \, dx$ a factor not present $\cos x$ in the integrand. Consequently, the substitution process failed to simplify the problem. According to Wilson and Clarke (2004), awareness entails not only selecting a strategy but also understanding why that strategy suits the problem's characteristics. In this instance, the student demonstrated initial awareness but was unable to identify the most appropriate strategy. Thus, the awareness indicator falls into the "sufficient" category.

The monitoring aspect relates to the ability to control the thought process while working toward a solution. In this instance, Student R2 consistently applied the substitution procedure but failed to monitor whether each step adhered to the fundamental principles of substitution. Student R2 proceeded with the calculation even though the integral simultaneously contained both variables specifically, x and u . Mathematically, this form should have signaled an error in a preceding step; however, Student R2 did not recognize the inconsistency and even treated $\cos x$ as a constant with respect to the variable u . Consequently, the student's monitoring ability was not optimally developed, as they failed to ensure consistency in variable usage.

The evaluation aspect involves the ability to assess both the solution process and the final result. Student R2's work showed no evidence of reviewing the final answer; the student simply wrote down $\int \frac{\sin^4 x}{\cos x} + C$ as the final result without verification—an incorrect answer. According to Wilson and Clarke (2004), evaluation entails reflecting on the solution process and verifying the correctness of the result. Since this process was not undertaken, the evaluation indicator was not met.

The image shows a student's handwritten work for the integral $\int x \cdot 2^{x^2} dx$. The student uses the integration-by-parts formula $\int uv = u \cdot v - \int v du$. They choose $u = x$ and $dv = 2^{x^2}$. They calculate $\frac{du}{dx} = 1$ so $du = dx$. For v , they incorrectly write $v = \frac{1}{x^2+1} 2^{x^2+1}$. Then they substitute into the formula: $\int x \cdot 2^{x^2} = 1 \cdot 2^{x^2} - \int 2^{x^2} dx$, which simplifies to $2^{x^2} - \frac{1}{x^2+1} 2^{x^2+1}$. The final conclusion is that the result is $2^{x^2} - \frac{1}{x^2+1} 2^{x^2+1}$.

Figure 9. Solution by Student R3 for Question 1

At the awareness stage, student R3 is expected to recognize the characteristics of the problem, identify known information, and select an appropriate solution strategy. The answer sheet reveals that student R3 immediately applied the integration-by-parts formula $\int u dv = uv - \int v du$ by selecting specific terms for $u = x$ and $dv = 2^{x^2} dx$. This choice indicates that student R3 was aware of the integration-by-parts method. However, the student failed to realize that the exponential function 2^{x^2} involved a composite function structure that was actually better suited for solution via substitution. This error in strategy selection indicates that the student experienced difficulties during the problem-solving planning stage. In other words, while student R3 possessed declarative knowledge regarding various integration methods, they were unable to link the problem's characteristics to the most appropriate method. According to Wilson and Clarke, this situation indicates that the awareness indicator was only partially achieved; the student recognized that the problem required a specific integration technique but failed to determine the most effective strategy.

Monitoring refers to the ability to track each step of the solution process as it unfolds. Several indicators in the student's response appeared suboptimal. The student wrote down $dv = 2^{x^2} dx$ and then attempted to write down $v = \int 2x^{x^2} dx$. However, the student subsequently obtained $v = \frac{1}{x^2+1} 2^{x^2+1}$, even though the integral of $\int 2x^{x^2} dx$ does not have an elementary antiderivative. This error indicates that student R3 did not verify the integral result obtained. Had student R3 monitored the process effectively, they should have realized that the result could be checked via differentiation—which would not yield 2^{x^2} . Furthermore, the student proceeded with the integration-by-parts process despite having obtained an illogical result. This error demonstrates that student R3 failed to verify the integration result obtained. Had the student engaged in effective monitoring, they should have realized that the result could be tested via differentiation—which would not have yielded the original integrand. Furthermore, the student persisted with the integration-by-parts process despite obtaining an illogical result. According to Wilson and Clarke, the student failed to demonstrate effective monitoring skills because they did not check the suitability of the formula used, identify erroneous steps, or adjust their strategy when encountering obstacles. Consequently, the monitoring indicator was not achieved.

The evaluation stage involves the ability to reassess the final outcome of the solution. Student R3 concluded $\int x2^{x^2} dx = 2^{x^2} - \frac{1}{x^2+1} 2^{x^2+1}$ even though that result was incorrect. This indicates that student R3 had not yet developed the habit of reflecting on their work. According to Wilson and Clarke, the evaluation skill remained undeveloped because the student failed to re-examine the answer, evaluate its correctness, compare the result against established concepts, or refine the strategy used. Thus, the evaluation indicator was not achieved.

a) $\int e^{\sqrt{x}} dx \rightarrow \int e^{x^{1/2}} dx$
 misal $u = \sqrt{x}$ atau $x^{1/2}$
 $\frac{du}{dx} = \frac{1}{2} x^{-1/2}$
 $\frac{du}{dx} = \frac{1}{2} x^{-1/2}$
 $dx = \frac{2}{x^{-1/2}} du$
 $dx = 2 \cdot \frac{1}{x^{-1/2}} du$
 $\Rightarrow \int e^u du = \int e^u \cdot 2 \cdot \frac{1}{x^{-1/2}} du$
 $= 2 \int e^u \cdot x^{1/2} du$
 $= 2 \left[e^u \cdot \frac{1}{\frac{1}{2}} x^{3/2} \right]_1^4$
 $= 2 \left[e^u \cdot \frac{2}{3} x^{3/2} \right]_1^4$
 $= 2 \left[e^{\sqrt{x}} \cdot \frac{2}{3} \sqrt{x^3} \right]_1^4$
 $= 2 \left[\left(e^{\sqrt{4}} \cdot \frac{2}{3} \sqrt{4^3} \right) - \left(e^{\sqrt{1}} \cdot \frac{2}{3} \sqrt{1^3} \right) \right]$
 $= 2 \left[\left(e^2 \cdot \frac{2}{3} \cdot \sqrt{64} \right) - \left(e^1 \cdot \frac{2}{3} \cdot 1 \right) \right]$
 $= 2 \left[\left(e^2 \cdot \frac{2}{3} \cdot 8 \right) - \left(1 \cdot \frac{2}{3} \cdot 1 \right) \right]$
 $= 2 \left[\left(e^2 \cdot \frac{16}{3} - \frac{2}{3} \right) \right]$
 $= 2 \left[e^2 \cdot \frac{16}{3} \right]$
 $= e^2 \cdot \frac{28}{3}$ atau $\frac{28}{3} e^{\sqrt{2}}$
 Jadi, Integral dari $\int e^{\sqrt{x}} dx$ adalah $\frac{28}{3} e^{\sqrt{2}}$

Figure 10. Solution by Student R3 for Question 2

The awareness aspect relates to a student's ability to recognize problem characteristics, identify known information, and select an appropriate solution strategy. The answer sheet reveals that student R3 recognized the presence of a composite function $e^{\sqrt{x}}$, chose to perform a substitution

$u = \sqrt{x}$, then finding the differential of that u and obtaining $\frac{du}{dx} = \frac{1}{2} x^{-1/2}$; this demonstrates an

understanding of the relationship between the original and new variables. However, student R3 failed to realize that, after performing the substitution, the variable x should have been converted into the form u . In the solution, the variable x was still found being used alongside the variable u (specifically $2 \int e^u x^{1/2} du$), even though $u = \sqrt{x}$ is equivalent to $x^{1/2} = u$. This error indicates that student R3 did not fully grasp the concept of substitution as a complete change of variables. Based on Wilson and Clarke's indicators, the awareness aspect is considered "partially achieved"; while the student successfully selected the correct strategy, they lacked a complete understanding of the variable-change concept.

Monitoring refers to the ability to oversee one's thought process while solving a problem. At this stage, student R3 proceeded with calculations without noticing inconsistencies in variable usage

$2 \int e^u x^{1/2} du$ specifically, integrating an expression while failing to adjust the integration limits $x=1 \rightarrow u=1$ and $x=4 \rightarrow u=2$. The student retained the original limits x (1 to 4) rather than updating them. This error indicates a failure to monitor the consistency of the solution procedure. According to Wilson and Clarke, the monitoring indicator was not met, as the student failed to verify

the alignment of steps with the substitution concept, manage the variable change, check for updated integration limits, or detect errors during the process.

Evaluation involves the ability to reassess the obtained solution. The student did not evaluate their errors—namely, the failure to adjust integration limits and the completion of the integration process while the original variable remained present. Consequently, the evaluation indicator was not achieved.

$$\begin{aligned} f) \int \frac{3x-13}{x^2-3x-10} dx \\ \int \frac{3x-13}{(x-5)(x+2)} &= \frac{A}{x-5} - \frac{B}{x+2} \quad (\text{Kali}) \\ &= \frac{A(x+2) - B(x-5)}{(x-5)(x+2)} \quad (\text{Golong}) \\ &= \frac{(x+2)A - (x-5)B}{(x-5)(x+2)} \\ \Rightarrow B &= -13 \\ \Rightarrow (x+2)A &= 3 \\ 13-2 \\ \Rightarrow \int (x+2)A dx - \int (x-5)B dx \\ &= \int -13x + 65 dx \\ &= \int -13x dx + \int 65 dx \\ &= \left(-\frac{13}{2}x^2\right) + 65x \\ &= -\frac{13}{2}x^2 + 65x \end{aligned}$$

Figure 11. Solution by Student R3 for Question 3

The "awareness" aspect reflects the student's ability to understand the characteristics of the problem, identify necessary concepts, and select an appropriate solution strategy. The answer sheet shows that student R3 began by factoring the denominator. Subsequently, R3 wrote down the partial fraction form $\frac{3x-13}{x^2-3x-10} = \frac{A}{x-5} + \frac{B}{x+2}$. This step demonstrates R3's ability to recognize that a rational function with a factored denominator can be solved using the partial fraction method, indicating an understanding of the relevant basic integration techniques. However, the student then wrote $A(x+2) - B(x-5)$ instead of the correct form $A(x+2) + B(x-5)$. This sign error indicates that R3 did not fully grasp the algebraic principles involved in equating the numerators. When questioned, R3 admitted to being confused about whether to use a plus or minus sign, ultimately forcing a minus sign because the expression on the left-hand side contained a minus sign. Thus, based on Wilson and Clarke's indicators, the awareness aspect was only partially achieved; while R3 could select the correct strategy, they did not fully understand the fundamental concepts behind formulating the partial fraction equation.

"Monitoring" refers to the student's ability to control their thought process and check each step while solving the problem. Several weaknesses were evident at this stage. Student R3 did not proceed with equating coefficients to determine the values of the constants A and B. Instead, R3 simply wrote down $B = -13$ the values without showing the logical calculation process. This step suggests that R3 had lost sight of the underlying logic of the partial fraction method. According to Wilson and Clarke, the monitoring indicator was not met because R3 failed to: re-examine the process of determining

constants, ensure consistency in the partial fraction form, detect algebraic manipulation errors, or realize that the integral of a rational function should result in a logarithmic form.

"Evaluation" refers to the student's ability to reassess the solution obtained. The answer sheet showed no attempt to verify the final result or compare it with the fundamental properties of rational function integrals. Student R3 obtained a quadratic function $\frac{13}{2}x^2 + 65$ as the result, whereas the integral of a rational function with a linear denominator almost invariably yields a logarithmic form. Had R3 evaluated the result, they should have realized that it was inconsistent with the theory of partial fraction integration. Student R3 also failed to differentiate the final result to verify that the derivative returned to the original function. Therefore, the evaluation indicator defined by Wilson and Clarke has not been met.

The image shows handwritten work for Question 4. On the left, under 'e)', the student attempts to integrate $\sin^3 x$ by splitting it into $\sin x \cdot \sin^2 x$. They use the identity $\sin^2 x = 1 - \cos^2 x$ and write:

$$\Rightarrow \int \sin x \, dx - \int \sin^2 x \, dx$$

$$= -\cos x - \left(\int 1 - \cos^2 x \, dx \right)$$

$$= -\cos x - (1 - 1 - \cos x)$$

$$= -\cos x - (-\cos x)$$

$$= \cos^2 x$$
 On the right, under 'f)', the student lists trigonometric identities:

$$\sin^2 x + \cos^2 x = 1$$

$$\sin^2 x = 1 - \cos^2 x$$

$$\cos^2 x = 1 - \sin^2 x$$
 The student's final answer is $\cos^2 x$, which is incorrect for the integral of $\sin^3 x$.

Figure 12. Solution by Student R3 for Question 4

The awareness aspect relates to the student's ability to understand the characteristics of the problem, recognize relevant concepts, and select an appropriate solution strategy. The answer sheet reveals that student R3 wrote down the trigonometric identities for $\sin^2 x + \cos^2 x = 1$ and $\sin^2 x = 1 - \cos^2 x$. This indicates that student R3 understood the fundamental trigonometric identity concepts required to solve integrals involving functions raised to an odd power. Such knowledge suggests that student R3 was aware of the relationship between sine and cosine functions. However, student R3 incorrectly applied the concept of integrating the product of two functions, resulting in a flawed solution process where the outcome appeared forced. This error demonstrates that the student was unable to correctly link trigonometric identities with the substitution strategy. According to Wilson and Clarke, this situation indicates that the awareness indicator was only partially achieved; while student R3 recognized the necessary concepts, they failed to devise a solution strategy suited to the problem's structure.

Monitoring refers to student R3's ability to track their thought process and identify errors during the solution phase. The work shows that after obtaining $\int \sin x \, dx - \int \sin^2 x \, dx$, the student proceeded to write $-\cos x - (1 - (1 - \cos x))$, indicating algebraic and integral manipulations performed without regard for applicable mathematical rules. Student R3 appeared to conflate integration results with trigonometric identities, bypassing correct procedural steps. Furthermore, the student failed to realize that the expression $\int \sin^2 x \, dx$ could not be solved simply by substituting $\sin^2 x$ trigonometric identities; it required further transformation—such as using double-angle identities or the substitution method after expressing the integrand in terms of

$\sin^3 x = \sin x(1 - \cos^2 x)$. Student R3 also showed no attempt to verify the consistency of each step in the solution. Based on Wilson and Clarke's indicators, the monitoring ability was not achieved, as the student failed to monitor the accuracy of algebraic manipulations, identify conceptual errors at an early stage, properly manage the use of trigonometric identities, or adjust the strategy upon encountering an incorrect result.

Evaluation refers to the student's ability to assess the final result and reflect on the solution process undertaken. The student did not verify the answer through differentiation or compare the result against the known characteristics of trigonometric function integrals. The absence of reflection on the final result indicates that the student's evaluation ability remains low. According to Wilson and Clarke, the evaluation indicator has not been met because students are not yet able to verify the final result, compare their answers with the correct concepts, or correct the errors they have made.

Table 1. criteria for the attainment of metacognitive skills

	Awareness	Monitoring	Evaluation
R1	Good	Good	Not good
R2	Good	Enough	Not good
R3	Enough	Enough	Not good

The results indicate that students in the high- and middle-performing groups demonstrated good metacognitive abilities at the awareness and monitoring stages; however, all groups faced limitations at the evaluation stage. This suggests that students were capable of recognizing what they knew, identifying the demands of the problem, and selecting appropriate solution strategies before attempting the questions (awareness). Furthermore, during the problem-solving process, they were able to monitor their thinking, check the appropriateness of their steps, and make adjustments upon encountering errors or obstacles (monitoring). Achieving these two indicators demonstrates that the students possessed sufficient declarative and procedural knowledge to control their thinking processes while solving problems. Nevertheless, the evaluation aspect was not fully realized, as students did not conduct a comprehensive review. Upon obtaining an answer, they tended to cease their thinking process immediately without re-evaluating whether the chosen strategy was the most effective, whether a more efficient method existed, or whether the result was mathematically correct. Yet, according to Wilson and Clarke, evaluation is a crucial component of metacognition, as it involves assessing strategy effectiveness and result accuracy, as well as using that experience as a basis for improving future problem-solving efforts.

These findings align with the view that metacognition encompasses not only awareness of the thinking process but also the ability to control and evaluate that process (Flavell, 1979). Students who fail to perform evaluations tend to lack the ability to reflect on the quality of their strategies or to address weaknesses in their problem-solving processes. Therefore, mathematics instruction should be designed to provide opportunities for reflection—such as through reflective questioning, discussions on solution outcomes, or comparisons of alternative strategies—thereby allowing evaluation skills to develop fully.

Conceptual understanding provides the foundation for students to engage in metacognitive activities. In the context of integration techniques, students must understand the characteristics of various function forms, the relationship between derivatives and integrals, and the principles governing the use of each integration technique. Students with a solid grasp of concepts find it easier to recognize the nature of a problem and connect it to their existing knowledge. This serves as the foundation for the emergence of awareness, as students become conscious of what they know, what questions to ask, and which concepts are relevant to solving the problem. Your article also demonstrates that a weak understanding of integral concepts can make it difficult for students to determine the appropriate steps and integration techniques.

In integration techniques, students must determine whether to use substitution, integration by parts, partial fractions, trigonometric identities, or other methods. The process of selecting a strategy constitutes a form of **awareness**. As students check the suitability of the chosen strategy while working through the problem, they are engaging in **monitoring**. Once a result is obtained, verifying the correctness of the answer and the effectiveness of the strategy constitutes **evaluation**. Therefore, the use of problem-solving strategies provides a context for students to practice three metacognitive activities: awareness, monitoring, and evaluation.

Scaffolding plays a facilitative role by assisting students in performing metacognitive activities they cannot yet execute independently. Currently, scaffolding has not contributed significantly to the aspects of awareness and monitoring; regarding evaluation, its contribution has also been limited because students often feel confident that their solution is correct. In this regard, scaffolding could be more effective by explicitly prompting students to reflect on their work after completing the problem.

Discussion

The findings reveal a differentiated pattern of metacognitive processes across the three participants, with problem-solving performance broadly corresponding to the consistency with which awareness and monitoring were demonstrated. R1 exhibited the most stable metacognitive profile, successfully recognizing problem structures and selecting appropriate techniques across substitution, integration by parts, partial fractions, and trigonometric integration tasks. R2 also demonstrated relatively strong awareness, particularly when identifying substitution and combined substitution-integration-by-parts strategies, but monitoring became less consistent as the problems demanded greater conceptual and procedural coordination. In contrast, R3 frequently recognized potentially relevant integration concepts but experienced difficulty connecting those concepts to strategies that were mathematically appropriate for the structure of each problem. This pattern suggests that possessing knowledge of integration techniques is distinguishable from knowing when and how those techniques should be strategically employed. Comparable evidence was who found that students' metacognitive knowledge in integral calculus could be further developed, particularly in relation to identifying appropriate solution strategies and checking problem-solving outcomes. Integration-technique problem solving as involving multiple methods, including substitution, trigonometric substitution, integration by parts, and rational-function integration, illustrating the strategic diversity students must coordinate in integral calculus. The present findings extend this perspective by showing that differences among students were visible not merely in whether they knew particular techniques but also in their capacity to regulate the selection and execution of those techniques. In particular, R3's inappropriate use of integration by parts, incomplete variable substitution, and incorrect partial-fraction manipulation illustrate how insufficient coordination between conceptual knowledge and metacognitive awareness can disrupt subsequent problem-solving activity. Thus, metacognitive awareness in integration problem solving should be understood as strategic recognition of the relationship between problem characteristics, relevant mathematical knowledge, and an appropriate course of action rather than simply familiarity with available integration formulas.

Monitoring emerged as a second important dimension distinguishing the participants, particularly when the selected strategy required several interconnected transformations. R1 consistently maintained relationships among variables, differentials, limits, algebraic transformations, and integration procedures, indicating that monitoring operated throughout rather than only at the beginning of problem solving. R2 displayed comparable monitoring on relatively successful solutions but showed substantial weaknesses when determining partial-fraction constants and performing a trigonometric substitution that left incompatible variables within the

same integral expression. R3 demonstrated still greater difficulty, continuing solution procedures even after mathematical inconsistencies had emerged, including unchanged integration limits, incomplete changes of variables, and invalid algebraic or integral manipulations. These findings indicate that monitoring involves more than following an ordered sequence of procedures; it requires continuous judgments about whether each operation remains mathematically compatible with previous decisions. Emphasize metacognitive regulation as a central component of problem solving and demonstrate the importance of examining regulatory activity through measures capable of capturing what learners actually do while solving problems (Zepeda & Nokes-Malach, 2023). The current results provide a calculus-specific illustration of this distinction because procedural continuation was sometimes observed even when effective monitoring was absent. In other words, students could remain behaviorally engaged in solving an integral while failing to recognize that their mathematical representation had become internally inconsistent. This distinction is particularly evident for R2 and R3, whose errors were not solely failures of computation but failures to use emerging information as feedback for reconsidering the ongoing strategy. Accordingly, effective monitoring in integral calculus appears to depend on students treating each transformation as information to be checked rather than as a procedural step that merely needs to be completed.

The most consistent finding across all three cases was the weakness of evaluation, including for R1, whose solutions were generally mathematically successful. None of the participants consistently demonstrated explicit verification through differentiation, systematic rechecking of procedures, comparison with alternative methods, or reflection on whether the selected strategy was efficient. This pattern indicates that obtaining a final answer did not necessarily trigger a second level of judgment concerning the validity or quality of the completed solution. The finding closely who identified checking the outcomes of integral-calculus problem solving as an area in which students' metacognitive knowledge required further development. A similar difficulty was observed whose study of metacognitively scaffolded problem solving found that students increasingly demonstrated structured problem-solving elements over time but continued to experience particular difficulty fully demonstrating evaluation (Vo et al., 2024). The convergence is noteworthy because the latter study was conducted in chemistry rather than integral calculus, suggesting that evaluation may represent a demanding component of metacognitive problem solving across quantitatively oriented domains. Further describe evaluation within their scaffold as checking and troubleshooting, reinforcing the importance of deliberately incorporating verification into structured problem-solving activity. In the present study, this weakness was consequential not only because students omitted reflective statements but because evaluation could have exposed substantive mathematical errors, particularly in R2 and R3. Differentiating an antiderivative, reconsidering variable consistency, or comparing the form of a result with expected integral properties could have revealed several errors before the solutions were finalized. Therefore, evaluation should not be treated as an optional concluding activity but as a regulatory mechanism capable of feeding information back into monitoring and prompting revision of an earlier strategy.

The findings also clarify the role of scaffolding in the observed metacognitive processes, although the qualitative design does not support a causal claim that scaffolding produced improvements in metacognitive ability. Within the scaffolded problem-solving context, R1 and R2 were generally able to recognize relevant problem characteristics and organize several integration procedures, whereas R3 displayed partial awareness and less stable procedural regulation. This variation indicates that the availability of instructional support does not guarantee equivalent metacognitive engagement among students with different levels of mathematical understanding and problem-solving performance (Hidayat et al., 2025; Prastiti et al., 2026; Susanna et al., 2026; Wakhata et al., 2023; W. Zhang & Lian, 2024). Rather, scaffolding appears to provide a context in which students' existing strengths and limitations in awareness, monitoring, and evaluation become visible

during mathematical activity. Similarly showed that a metacognitive problem-solving scaffold can structure activities around understanding, analysis, planning, implementation, and evaluation, while also finding that some elements remain more difficult for students to demonstrate consistently (Vo et al., 2024). Contemporary instructional frameworks likewise distinguish metacognitive scaffolding from purely procedural support, emphasizing reflection questions and checklists that prompt learners to plan, monitor, and evaluate their problem-solving processes. This distinction is directly relevant to the present results because step-by-step assistance may be sufficient to orient students toward a procedure without necessarily prompting them to question the validity of the procedure or independently verify its outcome. The repeated absence of evaluation therefore suggests that scaffolding for integration problems may need to extend beyond prompts about what step should be performed next. Prompts such as asking students to differentiate the obtained antiderivative, justify why a particular integration technique is appropriate, locate a possible error, or compare two feasible strategies could make evaluation an explicit component of the problem-solving sequence. Hence, the results support a shift from predominantly procedural scaffolding toward more deliberately metacognitive scaffolding that encourages students to explain, inspect, verify, and revise their mathematical reasoning.

Taken together, the findings suggest that metacognitive development in integration problem solving is better represented as a differentiated regulatory process than as a single general ability. Awareness was comparatively visible across participants, monitoring varied more substantially with performance and task demands, and evaluation remained consistently underdeveloped regardless of whether the final answer was correct. This pattern contributes to earlier integral-calculus research by moving beyond the identification of conceptual and procedural difficulties toward an explanation of how students regulate their knowledge while selecting and executing integration strategies. That integral-calculus research had historically concentrated more heavily on conceptual and procedural knowledge than on students' metacognition, while their findings highlighted strategy identification and outcome checking as areas requiring further development. The present results add a process-oriented perspective by illustrating how weaknesses can occur at different regulatory points: R3 frequently struggled with initial strategic alignment, R2 encountered breakdowns during procedural monitoring, and all three participants showed limited post-solution evaluation. This interpretation also accords with broader work on metacognitive scaffolding showing that structured support can make problem-solving processes more explicit while reflective and evaluative components remain challenging for learners (Vo et al., 2024). Pedagogically, the findings indicate that integration instruction should not end when students identify a suitable technique and successfully perform its associated calculations. Students should also be required to articulate why a technique fits the integrand, monitor whether transformations preserve mathematical consistency, and provide explicit evidence that the resulting antiderivative or definite-integral value is reasonable. Such practices may transform verification from an optional final check into an integral component of mathematical reasoning and encourage students to use errors as signals for strategic reconsideration. Overall, the study suggests that scaffolded integration problem solving is a useful context for revealing differences in metacognitive regulation, while the persistent weakness in evaluation identifies a specific dimension that warrants more explicit attention in calculus instruction and future research.

Implications

The findings imply that calculus instruction should develop not only students' procedural competence but also their awareness, monitoring, and evaluation during integration problem solving. Lecturers should incorporate metacognitive prompts that encourage students to justify strategy selection, monitor the consistency of solution steps, and verify final answers. Particular

attention should be given to evaluation because this dimension remained weak across all performance levels. Verification activities, such as differentiating antiderivatives, reviewing calculations, and comparing alternative strategies, should therefore become explicit components of learning. Scaffolding should also be adapted to students' different problem-solving needs rather than focusing solely on procedural guidance. Such an approach can support students in identifying errors and developing greater control over their mathematical reasoning. Assessment should likewise consider strategy justification, procedural monitoring, and solution verification in addition to final-answer accuracy. Overall, integrating metacognitive scaffolding into calculus instruction can promote more reflective, systematic, and independent integration problem solving.

Limitations and Suggestions for Future Research

This study is limited by its small qualitative sample of three participants representing different problem-solving performance levels within a single Mathematics Education context. The focus on integration techniques also limits the transferability of the findings to other mathematical topics. Moreover, written solutions and follow-up interviews may not fully capture students' metacognitive processes as they occur during problem solving. Because the study did not employ an experimental or comparative design, the findings should not be interpreted as evidence of a causal effect of scaffolding on metacognitive development. Future research should involve larger and more diverse samples across institutions and mathematical topics. Think-aloud protocols, classroom observations, or stimulated-recall interviews could also provide richer evidence of awareness, monitoring, and evaluation. Experimental or longitudinal studies are recommended to examine how different forms of scaffolding relate to metacognitive development over time. Particular attention should be given to evaluative scaffolding that explicitly encourages students to verify solutions, compare strategies, and reflect on their mathematical reasoning.

CONCLUSION

This study concludes that Mathematics Education students primarily demonstrate metacognitive processes through awareness and monitoring, while evaluation remains comparatively underdeveloped in solving integration problems. The findings indicate a metacognitive sequence of conceptual understanding → problem recognition → strategy selection → strategy implementation → monitoring → evaluation. Conceptual understanding supports students in recognizing problem structures and selecting appropriate integration techniques, while monitoring helps maintain the accuracy and consistency of solution procedures. Scaffolding can facilitate these processes through guiding questions that activate prior knowledge and encourage students to reconsider their strategies. However, evaluation requires more explicit support because students rarely verify their final solutions independently. Therefore, scaffolding should incorporate verification prompts, such as checking antiderivatives through differentiation and reviewing solution procedures. Overall, integrating awareness, monitoring, and evaluation within scaffolded instruction can support more systematic, reflective, and independent integration problem solving.

AUTHOR CONTRIBUTIONS STATEMENT

Siti Maslihah contributed to the conceptualization, research design, instrument development, data collection, data analysis, interpretation of the findings, and preparation of the original manuscript. Kartono contributed to the development of the theoretical framework, research

methodology, supervision, interpretation of the findings, and critical revision of the manuscript. Wardono contributed to methodological refinement, validation of the research instruments, data analysis, interpretation of the results, and manuscript review. Kharisudin contributed to data validation, interpretation of the findings, manuscript editing, and critical review for academic quality and coherence. All authors reviewed and approved the final version of the manuscript for publication.

REFERENCES

- Azizah, R., Tahmir, S., Asdar, A., Nurhasanah, N., & Hirpan, H. (2025). Scaffolding metakognitif dalam pemecahan masalah geometri ditinjau dari gaya belajar auditorial. *Issues in Mathematics Education (IMED)*, 9(1), 72–83. <https://doi.org/10.35580/imed.v9i1.4619>
- Braithwaite, D. W., & Sprague, L. (2021). Conceptual knowledge, procedural knowledge, and metacognition in routine and nonroutine problem solving. *Cognitive Science*, 45(10), e13048. <https://doi.org/10.1111/cogs.13048>
- Cervantes-Barraza, J. A., Rodríguez-Nieto, C. A., Damian-Mojica, A., & Morales-Carballo, A. (2025). Promoting the use of the Python programming language to analyze contextualized situations on derivatives and integrals considering the fundamental theorem of calculus. *Eurasia Journal of Mathematics, Science and Technology Education*, 21(9), em2705. <https://doi.org/10.29333/ejmste/16885>
- Ferreira, P. D. C., Ferreira, A. I., Simão, A. M. V. D. V., Prada, R., Paulino, A. P., & Rodrigues, R. (2022). Self-regulated learning and working memory determine problem-solving accuracy in math. *The Spanish Journal of Psychology*, 25, e23. <https://doi.org/10.1017/SJP.2022.19>
- Gao, Z., Yan, H., Liu, J., Cui, C., Wang, J., Zhang, X., Sun, X., & Feng, J. (n.d.). Untangling complexity of competency evolution: Multi-channel learning trajectories of score, engagement and problem-solving efficiency. *British Journal of Educational Technology*, .. <https://doi.org/10.1111/bjet.70017>
- Garte, S., Marshall, P., & Kauffman, S. (2025). The reasonable ineffectiveness of mathematics in the biological sciences. *Entropy*, 27(3).. <https://doi.org/10.3390/e27030280>
- Greengrass, C. J. (2026). Transforming clinical reasoning—The role of AI in supporting human cognitive limitations. *Frontiers in Digital Health*, 7.. <https://doi.org/10.3389/fdgth.2025.1715440>
- Hegedus, S. J., & Otálora, Y. (2023). Mathematical strategies and emergence of socially mediated metacognition within a multi-touch Dynamic Geometry Environment. *Educational Studies in Mathematics*, 112(2), 289–307. <https://doi.org/10.1007/s10649-022-10170-4>
- Hidayat, R., Mohd Saad, M. R., & Wewe, M. (2025). A meta-analysis of the effect of metacognitive instruction on mathematics achievement. *Cogent Education*, 12(1), 2517510. <https://doi.org/10.1080/2331186X.2025.2517510>
- Im, Y., Choe, J., & Shin, Y. (2026). Capturing the temporal interplay of knowledge construction and group-level regulation in collaborative problem solving. *Metacognition and Learning*, 21(1), 29. <https://doi.org/10.1007/s11409-026-09473-1>
- Ke, F., & Clark, K. M. (2020). Game-based multimodal representations and mathematical problem solving. *International Journal of Science and Mathematics Education*, 18(1), 103–122. <https://doi.org/10.1007/s10763-018-9938-3>
- Khalid, M., Saad, S., Hamid, S. R. A., Abdullah, M. R., Ibrahim, H., & Shahrill, M. (2020). Enhancing creativity and problem solving skills through creative problem solving in teaching mathematics. *Creativity Studies*, 13(2), 270–291. <https://doi.org/10.3846/cs.2020.11027>
- Kholid, M. N., Mahmudah, M. H., Ishartono, N., Putra, F. G., & Forthmann, B. (2024). Classification of students' creative thinking for non-routine mathematical problems. *Cogent Education*, 11(1), 2394738. <https://doi.org/10.1080/2331186X.2024.2394738>
- Liu, J., Keane, T., Sun, D., Yang, Y., & Yang, Y. (2025). Design and evaluation of ChatGPT-MWPS: An AI-enhanced learning system for improving primary students' mathematical word problem solving. *Smart Learning Environments*, 12(1), 68. <https://doi.org/10.1186/s40561-025-00419-9>

- Malenkov, Y., Kapustina, I., Kudryavtseva, G., Shishkin, V. V., & Shishkin, V. I. (2021). Digitalization and strategic transformation of retail chain stores: Trends, impacts, prospects. *Journal of Open Innovation: Technology, Market, and Complexity*, 7(2). <https://doi.org/10.3390/joitmc7020108>
- Mannan, A. M. K., Bajuri, M. R., Abdul Rahim, S. S., & Abdullah, N. A. (2026). How do digital tools support metacognition in mathematics learning? A systematic review of research on GeoGebra and problem-solving. *Educational Research Review*, 52, 100827. <https://doi.org/10.1016/j.edurev.2026.100827>
- Mannan, A. M. K., Bajuri, M. R., Abdullah, N. A., & Redzuan, R. S. (2025). Exploring metacognitive processes in calculus problem-solving: A case study using GeoGebra. *Journal of Mathematics, Science and Technology Education*, 2(2), 103–115. <https://doi.org/10.12973/jmste.2.2.103>
- Maries, A., & Singh, C. (2023). Helping students become proficient problem solvers part I: A brief review. *Education Sciences*, 13(2). <https://doi.org/10.3390/educsci13020156>
- Mas', Badolo, U., Malik, M. A., Rafi', Nur, A., & Latifa, A. (2025). The impact of metacognitive strategy training on higher-order thinking skills (HOTS) in high school mathematics: A quasi-experimental study. *International Journal of Environment, Engineering and Education*, 7(2), 146–157. <https://doi.org/10.55151/ijeedu.v7i2.302>
- McDonald, P. A., & Smith, J. M. (2020). Improving mathematical learning in Scotland's Curriculum for Excellence through problem posing: An integrative review. *The Curriculum Journal*, 31(3), 398–435. <https://doi.org/10.1002/curj.15>
- Michalsky, T. (2024). Metacognitive scaffolding for preservice teachers' self-regulated design of higher order thinking tasks. *Heliyon*, 10(2). <https://doi.org/10.1016/j.heliyon.2024.e24280>
- Msomi, A. M., & Bansilal, S. (2022). Analysis of students' errors and misconceptions in solving linear ordinary differential equations using the method of Laplace transform. *International Electronic Journal of Mathematics Education*, 17(1). <https://doi.org/10.29333/iejme/11474>
- Ncube, M., & Luneta, K. (n.d.). Concept-based instruction: Improving learner performance in mathematics through conceptual understanding. *Pythagoras*, 46(1), 815. <https://doi.org/10.4102/pythagoras.v46i1.815>
- Prastiti, T. D., Ahmadah, I. F., & Siswono, T. Y. E. (2026). Metacognitive regulation in collaborative math problem-solving among heterogeneous secondary students. *Eurasia Journal of Mathematics, Science and Technology Education*, 22(5), em2834. <https://doi.org/10.29333/ejmste/18518>
- Riva, G. (2026). The architecture of cognitive amplification: Enhanced cognitive scaffolding as a resolution to the comfort-growth paradox in AI-supported learning. *Cognitive Systems Research*, 99, 101500. <https://doi.org/10.1016/j.cogsys.2026.101500>
- Rivas, S. F., Saiz, C., & Ossa, C. (2022). Metacognitive strategies and development of critical thinking in higher education. *Frontiers in Psychology*, 13. <https://doi.org/10.3389/fpsyg.2022.913219>
- Rivers, M. L., Dunlosky, J., & Persky, A. M. (2020). Measuring metacognitive knowledge, monitoring, and control in the pharmacy classroom and experiential settings. *American Journal of Pharmaceutical Education*, 84(5), 7730. <https://doi.org/10.5688/ajpe7730>
- Su, C.-Y. (2026). Integrating metacognitive scaffolding into a physics simulation for elementary school students' conceptual understanding of force and motion. *Journal of Science Education and Technology*, 35(4), 917–932. <https://doi.org/10.1007/s10956-025-10288-z>
- Susanna, T., Lasse, E., Henrik, N. J., & Sari, H.-N. (2026). From problem-solving to reflection: Activating diverse metacognitive skills in mathematics. *International Journal of Science and Mathematics Education*, 24(2), 10. <https://doi.org/10.1007/s10763-025-10643-x>
- Szabo, Z. K., Körtesi, P., Guncaga, J., Szabo, D., & Neag, R. (2020). Examples of problem-solving strategies in mathematics education supporting the sustainability of 21st-century skills. *Sustainability*, 12(23). <https://doi.org/10.3390/su122310113>
- Tanreso, N. F., Irmawati, I., Musliha, M., Aldasafitri, J., Ain, N., Yusuf, M., & Hijra, H. (2025). Analisis literatur: Peran metakognisi dalam pemecahan masalah matematika siswa. *Pedagogy: Jurnal Pendidikan Matematika*, 10(3), 1287–1295. <https://doi.org/10.30605/pedagogy.v10i3.6992>
- Toikka, S., Eronen, L., Atjonen, P., & Havu-Nuutinen, S. (2024). Combined conceptualisations of metacognitive knowledge to understand students' mathematical problem-solving. *Cogent Education*, 11(1), 2357901. <https://doi.org/10.1080/2331186X.2024.2357901>

- Verschaffel, L., Schukajlow, S., Star, J., & Van Dooren, W. (2020). Word problems in mathematics education: A survey. *ZDM*, 52(1), 1–16. <https://doi.org/10.1007/s11858-020-01130-4>
- Vo, K., Sarkar, M., White, P. J., & Yuriev, E. (2024). Development of problem-solving skills supported by metacognitive scaffolding: Insights from students' written work. *Chemistry Education Research and Practice*, 25(4), 1197–1209. <https://doi.org/10.1039/D3RP00284E>
- Wakhata, R., Mutarutinya, V., & Balimuttajjo, S. (2023). Exploring the impact of Stein et al.'s levels of cognitive demand in supporting students' mathematics heuristic problem-solving abilities. *Frontiers in Education*, 8. <https://doi.org/10.3389/feduc.2023.949988>
- Wang, C.-Y., & Huang, M.-Y. (2026). Strengthening novice diagnostic problem solving through explicit metacognitive scaffolding in emergency medicine education. *Medical Education Online*, 31(1), 2662714. <https://doi.org/10.1080/10872981.2026.2662714>
- Wang, W.-S., Lin, C.-J., Lee, H.-Y., Huang, Y.-M., & Wu, T.-T. (2025). Enhancing self-regulated learning and higher-order thinking skills in virtual reality: The impact of ChatGPT-integrated feedback aids. *Education and Information Technologies*, 30(14), 19419–19445. <https://doi.org/10.1007/s10639-025-13557-x>
- Weldeana, H. N., Sbhatu, D. B., & Berhe, G. T. (2023). Freshman STEM students' misconceptions in a basic limit theorem of $\lim x \rightarrow 0 \left(\sin x / x \right)$. *Heliyon*, 9(12). <https://doi.org/10.1016/j.heliyon.2023.e22359>
- Wilson, J., & Clarke, D. (2004). Towards the modelling of mathematical metacognition. *Mathematics Education Research Journal*, 16(2), 25–48. <https://doi.org/10.1007/BF03217394>
- Wu, J., Wang, J., Lei, S., Wu, F., & Gao, X. (2025). The impact of metacognitive scaffolding on deep learning in a GenAI-supported learning environment. *Interactive Learning Environments*, 33(9), 5166–5183. <https://doi.org/10.1080/10494820.2025.2479162>
- Zepeda, C. D., & Nokes-Malach, T. J. (2023). Assessing metacognitive regulation during problem solving: A comparison of three measures. *Journal of Intelligence*, 11(1). <https://doi.org/10.3390/jintelligence11010016>
- Zhang, J., Yin, J., & Wang, R. (2020). Basic framework and main methods of uncertainty quantification. *Mathematical Problems in Engineering*, 2020(1), 6068203. <https://doi.org/10.1155/2020/6068203>
- Zhang, W., & Lian, R. (2024). The impact of reading metacognitive strategies on mathematics learning efficiency and performance: An analysis using PISA 2018 data in China. *Acta Psychologica*, 246, 104247. <https://doi.org/10.1016/j.actpsy.2024.104247>
- Zhao, Y., & Saleh, S. (2025). Metacognition and mathematical problem-solving: An empirical investigation into series problems with junior high school students. *Mathematics Education Journal*, 9(1), 90–101. <https://doi.org/10.22219/mej.v9i1.39924>
- Zhong, X., & Zhan, Z. (2024). An intelligent tutoring system for programming education based on informative tutoring feedback: System development, algorithm design, and empirical study. *Interactive Technology and Smart Education*, 22(1), 3–24. <https://doi.org/10.1108/ITSE-09-2023-0182>
- Özgeldi, M., & Aydın, U. (2021). Identifying competency demands in calculus textbook examples: The case of integrals. *International Journal of Science and Mathematics Education*, 19(1), 171–191. <https://doi.org/10.1007/s10763-019-10046-9>