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From ethnomathematics to school geometry: Developing an iceberg-based learning trajectory through the Raja Rokan palace

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Abstract

Background: Ethnomathematics has emerged as an effective approach for connecting cultural heritage with school mathematics. Nevertheless, most existing studies focus primarily on identifying mathematical ideas embedded in cultural artefacts without transforming them into systematic instructional designs that support classroom learning.
Aims: This study aimed to develop an Iceberg-based Learning Trajectory (ILT) that translates ethnomathematical concepts embedded in the Raja Rokan Palace into meaningful geometry learning for junior high school students and to design a prototype student worksheet aligned with the proposed trajectory.
Method: A qualitative educational design approach grounded in ethnomathematics was employed. Data were collected through observations, documentation, and previous ethnomathematical findings. The identified mathematical concepts were analysed using content analysis and mapped onto the four levels of the Iceberg Model to construct an ILT comprising learning objectives, learning activities, anticipated students' thinking, instructional scaffolding, and worksheet design.
Results: The findings identified four geometric concepts rectangles, triangles, trapezoids, and semicircles which were systematically transformed into an ILT that supports students' progressive transition from contextual cultural experiences to formal geometric reasoning. The study also produced a culturally responsive worksheet prototype and instructional design principles for developing Iceberg-based learning trajectories.
Conclusion: The proposed ILT provides a transferable framework for integrating cultural heritage into geometry learning and contributes to the advancement of culturally responsive mathematics education. Future studies should examine its classroom implementation and effectiveness.

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INTRODUCTION

Twenty-first century mathematics education is expected to cultivate competencies that extend beyond procedural fluency to include conceptual understanding, mathematical reasoning, problem-solving, communication, and mathematical literacy required to address increasingly complex real-world challenges. The OECD defines mathematical literacy as an individual's capacity to formulate, employ, and interpret mathematics in diverse contexts to make informed and responsible decisions as constructive members of society. Consequently, mathematics learning has shifted from the transmission of abstract procedures toward student-centred approaches that encourage learners to construct mathematical meaning through meaningful experiences and authentic situations (Koskinen & Pitkäniemi, 2022; Lahdenperä et al., 2023; Siligar et al., 2025; Vidic, 2023). This educational transformation is closely aligned with the principles of Realistic Mathematics Education (RME), which positions mathematics as a human activity developed through guided reinvention and progressive mathematization rather than through direct instruction (Van den

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Heuvel-Panhuizen & Drijvers, 2020). Within this perspective, meaningful contexts function not merely as introductory examples but as the foundation from which students progressively construct formal mathematical knowledge. Recent evidence further supports the effectiveness of this approach, with meta-analytic findings demonstrating that RME-based multicontextual learning produces substantial improvements in students' mathematical achievement and conceptual understanding. Similarly, contextual and interactive learning environments grounded in RME have been shown to strengthen students' mathematical modelling abilities and higher-order thinking skills by connecting abstract concepts with authentic experiences (Payadnya et al., 2023; Sutarni et al., 2024). Educational design studies also indicate that students achieve deeper conceptual understanding when learning experiences are organised through coherent learning trajectories that anticipate students' thinking and provide systematic instructional scaffolding (Marleny et al., 2026; Siligar et al., 2025). These findings suggest that meaningful mathematics learning depends not only on the availability of authentic contexts but also on instructional designs that systematically guide students from informal reasoning towards formal mathematical understanding. Therefore, developing structured learning pathways has become an essential priority for improving the quality of mathematics education in the twenty-first century.

Among various contextual approaches, ethnomathematics has emerged as one of the most influential perspectives for connecting school mathematics with students' cultural experiences and local knowledge systems. Ethnomathematics recognises cultural artefacts, traditional architecture, handicrafts, games, ornaments, and indigenous practices as authentic representations of mathematical thinking that can enrich classroom learning (Kabuye Batiibwe, 2024). By positioning mathematics as knowledge embedded within cultural practices, ethnomathematics enables students to perceive mathematical concepts as meaningful, relevant, and closely connected to their everyday experiences. Recent empirical evidence demonstrates that ethnomathematics-based learning significantly improves students' mathematical literacy while simultaneously strengthening cultural awareness and learning engagement (Pratama & Yelken, 2024). Cross-cultural investigations have also revealed that teachers perceive ethnomathematics as an effective approach for contextualising mathematics instruction while preserving local cultural heritage (Acharya et al., 2021; Batiibwe, 2025; Bonney et al., 2026; Payadnya et al., 2024). Furthermore, a recent systematic review indicates that ethnomathematics research has evolved from merely documenting cultural mathematical practices towards developing culturally responsive instructional approaches for mathematics education (Marsigit et al., 2025). These developments reflect an increasing recognition that cultural heritage can function not only as contextual material but also as an important pedagogical resource for supporting conceptual understanding. One example of such cultural heritage is the Raja Rokan Palace, a historical Malay palace in Riau Province whose architectural elements embody diverse geometric concepts relevant to junior high school geometry. Previous investigations have demonstrated that the palace contains various mathematical ideas, including plane figures and spatial structures, indicating its considerable potential as a contextual resource for geometry learning. Consequently, integrating ethnomathematical knowledge derived from cultural heritage into mathematics instruction represents a promising strategy for simultaneously promoting conceptual understanding, cultural appreciation, and meaningful learning experiences.

Despite the growing recognition of ethnomathematics as a valuable educational approach, substantial challenges remain in transforming ethnomathematical findings into systematic instructional practices that can be implemented effectively in mathematics classrooms (Marsigit et al., 2025; Pratama & Yelken, 2024). Existing ethnomathematics studies predominantly focus on identifying mathematical concepts embedded in cultural artefacts and evaluating their educational potential, while providing limited guidance regarding how these concepts should be translated into coherent classroom learning experiences (Marsigit et al., 2025; Pratama & Yelken, 2024). At the same time, studies on learning trajectories within the framework of Realistic Mathematics Education have demonstrated the importance of sequencing learning activities, anticipating students' thinking, and designing instructional scaffolding that supports progressive mathematization (Artigue et al., 2020; Johar et al., 2025; Risdiyanti & Prahmana, 2021). Likewise, recent research has highlighted the pedagogical value of the Iceberg Model in facilitating students' movement from contextual experiences towards increasingly formal mathematical reasoning through interconnected instructional levels (Sari et al., 2025; Van den Heuvel-Panhuizen & Drijvers, 2020). However, these instructional frameworks are generally developed from curriculum content rather than from ethnomathematical exploration grounded in local cultural heritage. Consequently, cultural contexts often function only as supporting illustrations instead of serving as the foundation for students' conceptual development throughout the learning process. This separation creates a pedagogical gap between ethnomathematical exploration and systematic instructional design that remains insufficiently addressed in current mathematics education research. Addressing this challenge requires an instructional framework capable of integrating ethnomathematical knowledge, progressive mathematization, learning trajectories, and instructional scaffolding into a coherent learning design. Such integration is expected to facilitate students' transition from authentic cultural experiences towards formal geometric understanding while preserving the educational value of local cultural heritage. Therefore, developing an Iceberg-based Learning Trajectory grounded in the ethnomathematics of the Raja Rokan Palace represents an important step towards advancing culturally responsive and pedagogically coherent geometry learning.

Recent studies have demonstrated significant progress in integrating ethnomathematics into mathematics education through various perspectives, including literature syntheses on the educational role of ethnomathematics, technology-enhanced learning, learning trajectory development, hypothetical learning trajectory design, Iceberg-based instructional representations, and culturally responsive mathematics education (Abdulrahim & Orosco, 2020; Nursyahidah et al., 2025; Sari et al., 2025; Sukestiyarno et al., 2023; Sunzuma & Umbara, 2025). These studies collectively confirm that cultural contexts can effectively support meaningful mathematics learning and facilitate students' conceptual development. Nevertheless, the existing body of research has largely examined these components independently, with limited attention to how ethnomathematical knowledge can be systematically transformed into a coherent instructional framework that guides students from authentic cultural experiences to formal mathematical understanding. Existing learning trajectory studies primarily emphasise the sequencing of learning activities or the application of Realistic Mathematics Education without explicitly integrating the Iceberg Model as a mechanism for progressive mathematization, while Iceberg-related studies mainly focus on instructional representations rather than on constructing complete learning trajectories grounded in ethnomathematical contexts (Anwar et al., 2023; Sari et al., 2025). Furthermore, previous ethnomathematics research has predominantly concentrated on identifying mathematical concepts embedded in cultural artefacts or developing isolated instructional resources, leaving insufficient guidance for integrating cultural exploration, anticipated students' thinking, instructional scaffolding, and worksheet design within a unified instructional design framework. Therefore, a methodological gap remains in developing an Iceberg-Based Learning Trajectory that systematically operationalises ethnomathematical knowledge into classroom-ready geometry instruction. Addressing this gap, the present study proposes an integrated instructional framework that combines ethnomathematics, the Iceberg Model, and learning trajectory design to facilitate students' progressive transition from culturally situated experiences to formal geometric reasoning through the context of the Raja Rokan Palace.

Building upon the identified research gap, this study aims to develop an Iceberg-based Learning Trajectory (ILT) that systematically transforms the ethnomathematical ideas embedded in the Raja Rokan Palace into meaningful geometry learning for junior high school students. Specifically, the study identifies geometric concepts represented in the architectural elements of the palace and maps these concepts onto the four instructional levels of the Iceberg Model to support students' progressive mathematization. Based on this mapping, the study constructs a coherent learning trajectory that connects contextual cultural experiences with increasingly formal geometric reasoning. The proposed trajectory incorporates clearly defined learning objectives, sequenced learning activities, anticipated students' thinking, instructional scaffolding, and a prototype student worksheet to facilitate classroom implementation. In addition, this study examines how ethnomathematical exploration can function not merely as a source of contextual examples but as the foundation for systematic instructional design. By integrating ethnomathematics, Realistic Mathematics Education (RME), the Iceberg Model, and Learning Trajectory theory, the study proposes a unified framework for culturally responsive geometry instruction. The framework is expected to provide practical guidance for teachers in designing learning experiences that are contextually meaningful, pedagogically coherent, and mathematically rigorous. From a theoretical perspective, the study contributes to the advancement of ethnomathematics research by extending its focus from cultural identification towards instructional design and progressive concept development. From a practical perspective, it offers a transferable instructional framework that can be adapted to other cultural heritage contexts and mathematical topics while preserving local cultural values. Ultimately, this study seeks to strengthen the integration of cultural heritage and school mathematics by demonstrating how ethnomathematical knowledge can be systematically transformed into structured learning that promotes conceptual understanding, mathematical reasoning, and cultural appreciation.

LITERATURE REVIEW

Ethnomathematics has become one of the most influential theoretical perspectives for contextualising mathematics learning by recognising that mathematical ideas are embedded within the cultural practices, traditions, and daily activities of diverse communities. Rather than viewing mathematics as a body of universal knowledge detached from social life, ethnomathematics positions mathematical thinking as knowledge that develops through human interaction with cultural environments and collective experiences (Huancacuri & Melzi, 2026; Opesemowo, 2025; Rodríguez-Nieto & Alsina, 2022). This perspective enables students to connect formal mathematical concepts with familiar cultural contexts, thereby promoting meaningful learning and strengthening conceptual understanding (Acharya et al., 2021; Kurniawan et al., 2023; Leton et al., 2025; Oladejo et al., 2025). Recent studies have demonstrated that ethnomathematics-based instruction significantly improves students' mathematical literacy, learning motivation, and appreciation of cultural heritage while simultaneously supporting the development of higher-order thinking skills (Pratama & Yelken, 2024). Cross-cultural investigations further indicate that teachers perceive ethnomathematics as an effective instructional approach because it contextualises mathematical concepts without diminishing academic rigour while preserving local cultural identity. Furthermore, a recent systematic review concluded that ethnomathematics research has gradually evolved from merely documenting cultural mathematical practices towards developing culturally responsive pedagogical approaches that integrate local wisdom into mathematics education (Marsigit et al., 2025). These developments demonstrate that ethnomathematics is no longer regarded solely as a field of cultural exploration but has become an important pedagogical approach for designing contextual mathematics instruction. Nevertheless, translating ethnomathematical knowledge into coherent classroom learning remains a considerable challenge because teachers require systematic instructional guidance that connects cultural exploration with formal mathematical learning.

Consequently, ethnomathematics should be viewed not only as a source of contextual examples but also as a foundation for developing instructional frameworks capable of facilitating students' conceptual progression towards formal mathematical understanding. Such a perspective provides the theoretical basis for integrating ethnomathematics into structured learning designs that support culturally responsive mathematics education.

Learning trajectory has become a fundamental concept in contemporary mathematics education because it provides a systematic framework for organising students' conceptual development through carefully sequenced learning experiences. A learning trajectory generally consists of interconnected components, including learning goals, instructional tasks, anticipated students' thinking, and pedagogical support that collectively facilitate conceptual growth (Darling-Hammond et al., 2020; Pepin, 2021; Prediger et al., 2022; Wilson & Lehrer, 2021). Rather than presenting mathematical concepts in isolation, learning trajectories organise learning experiences into coherent progressions that enable students to construct increasingly sophisticated mathematical ideas through active engagement. Within the framework of Realistic Mathematics Education, learning trajectories are closely associated with progressive mathematization, whereby students gradually develop formal mathematical understanding from meaningful contextual situations (Van den Heuvel-Panhuizen & Drijvers, 2020). Recent educational design studies have demonstrated that learning trajectories contribute positively to students' conceptual understanding by anticipating learning obstacles and providing appropriate instructional scaffolding throughout the learning process (Johar et al., 2025; Marleny et al., 2026). Research has also shown that culturally contextualised learning trajectories enable students to establish stronger connections between informal reasoning and formal mathematical concepts because learning activities are organised according to students' cognitive development (Siligar et al., 2025). Moreover, adaptive learning trajectories encourage teachers to predict students' responses and prepare instructional interventions that facilitate conceptual refinement during classroom interaction. These characteristics make learning trajectories particularly suitable for supporting inquiry-oriented and student-centred mathematics instruction. However, many existing learning trajectories have been developed primarily from curriculum content rather than from authentic cultural contexts, thereby limiting opportunities to integrate local knowledge into students' conceptual development. Therefore, incorporating ethnomathematical contexts into learning trajectory design represents an important direction for strengthening meaningful and culturally responsive mathematics learning.

The Iceberg Model is recognised as an instructional framework that illustrates students' gradual movement from concrete experiences towards increasingly formal mathematical understanding through progressive levels of abstraction. The model conceptualises mathematical learning as an iceberg in which formal mathematical knowledge visible at the surface is supported by extensive informal reasoning and contextual experiences beneath it. Accordingly, the Iceberg Model organises instruction into four interconnected levels consisting of situational, referential, general, and formal activities that collectively facilitate progressive mathematization (Sari et al., 2025). At the situational level, students engage with authentic contexts that stimulate intuitive reasoning and activate prior knowledge before formal mathematical concepts are introduced. During the referential level, students construct representations and models that bridge concrete experiences with emerging mathematical ideas. The general level encourages students to recognise mathematical relationships, identify patterns, and formulate generalisations that extend beyond specific contextual situations. Finally, the formal level guides students towards symbolic reasoning, mathematical notation, and abstract conceptual understanding that characterise formal school mathematics. Recent studies have demonstrated that the Iceberg Model supports deeper conceptual understanding because it enables students to construct mathematical knowledge through continuous transitions between contextual experiences and formal reasoning (Sari et al., 2025).

Despite its pedagogical advantages, existing applications of the Iceberg Model generally begin from predefined curriculum content instead of ethnomathematical exploration derived from local cultural heritage. Consequently, integrating the Iceberg Model with ethnomathematics and learning trajectory design offers a promising theoretical foundation for developing culturally responsive geometry instruction that systematically guides students from authentic cultural experiences towards formal geometric reasoning.

Integrating ethnomathematics, Learning Trajectory, and the Iceberg Model provides a comprehensive theoretical foundation for designing culturally responsive mathematics instruction that supports students' conceptual development from authentic experiences towards formal mathematical reasoning. These three perspectives are complementary because ethnomathematics supplies meaningful cultural contexts, Learning Trajectory organises students' conceptual progression, and the Iceberg Model operationalises progressive mathematization through structured instructional levels (Sari et al., 2025; Siemon, 2021). Rather than functioning as separate theoretical constructs, they collectively establish an instructional framework that enables students to construct mathematical knowledge through a coherent sequence of contextual exploration, representation, abstraction, and formalisation. Recent educational design studies indicate that contextual learning becomes more effective when authentic situations are systematically connected with students' cognitive development through carefully planned instructional sequences (Johar et al., 2025; Marleny et al., 2026). Likewise, culturally responsive instructional designs have been shown to strengthen students' engagement because learning activities reflect their cultural backgrounds while maintaining the integrity of mathematical concepts (Marsigit et al., 2025; Pratama & Yelken, 2024). Within geometry learning, cultural artefacts provide rich visual and spatial representations that facilitate students' transition from intuitive geometric perception towards formal geometric reasoning.

Consequently, integrating ethnomathematical contexts into an Iceberg-based Learning Trajectory enables students to construct geometric concepts through progressive interactions between cultural experiences and mathematical abstraction. Such an instructional framework also assists teachers in anticipating students' thinking, identifying potential learning obstacles, and providing appropriate instructional scaffolding throughout the learning process. Therefore, integrating these complementary perspectives extends the application of ethnomathematics beyond contextualisation towards a systematic instructional design capable of supporting meaningful and sustainable mathematics learning. This integrated perspective establishes the conceptual foundation for transforming cultural heritage into structured geometry instruction that aligns with contemporary principles of mathematics education.

The Raja Rokan Palace represents a valuable cultural heritage site whose architectural characteristics provide authentic contexts for exploring geometric concepts within school mathematics. Previous ethnomathematical investigations have identified various geometric forms embedded in the palace architecture, including rectangles, triangles, trapezoids, semicircles, and spatial structures that correspond to junior high school geometry content. Nevertheless, these findings have primarily contributed to documenting the mathematical potential of the palace without providing systematic guidance for translating such knowledge into classroom instruction. As a result, teachers still encounter difficulties in utilising ethnomathematical findings as structured learning resources that support students' progressive conceptual development. Developing an instructional framework based on the Raja Rokan Palace therefore represents an important effort to bridge the gap between ethnomathematical exploration and practical classroom implementation (Ramadhani et al., 2026). By integrating the identified cultural mathematical concepts into an Iceberg-based Learning Trajectory, students can be guided from observing authentic architectural forms towards constructing increasingly formal geometric concepts through progressive mathematization. This instructional approach positions cultural exploration not as the final objective of learning but as the starting point for developing conceptual understanding within formal school mathematics. Such a perspective reinforces the role of cultural heritage as a meaningful pedagogical resource capable of enriching mathematics instruction while simultaneously preserving local cultural identity. Furthermore, the proposed framework provides a transferable instructional model that can be adapted to other cultural heritage contexts and mathematical topics without losing its emphasis on conceptual progression and contextual learning. Accordingly, the integration of ethnomathematics, the Iceberg Model, and Learning Trajectory offers a significant theoretical and practical contribution to the advancement of culturally responsive geometry education and forms the conceptual basis for the instructional design proposed in this study.

METHOD

Research Design

This study employed the preliminary design phase of the design research approach Gravemeijer and Cobb (2006) to transform ethnomathematical ideas embedded in the Raja Rokan Palace into a Hypothetical Iceberg-based Learning Trajectory (ILT) for junior high school geometry. This study represents the preliminary design phase (focussing on constructing and conceptually validating an instructional design) grounded in Realistic Mathematics Education (RME), the Iceberg Model, and Learning Trajectory principles.

Research Context and Unit of Analysis

As a qualitative educational design study focused on cultural artifact exploration, the primary unit of analysis was the architectural structure and cultural artifacts of the Raja Rokan Palace in Rokan Hulu Regency, Riau Province, Indonesia. A purposive sampling technique was applied to select this site based on three explicit criteria:

1. its status as a historical Malay cultural heritage site featuring well-preserved architectural components;
2. its established empirical foundation from prior exploratory research, providing a reliable basis for instructional extension; and
3. the high visibility of diverse two-dimensional geometric figures across its roof, doors, windows, and decorative arches that correspond directly to junior high school (SMP) geometry curriculum standards.

Instrumentation

Data were collected using three complementary sources. First, field observations and photographic documentation were conducted to record the architectural elements of the Raja Rokan Palace that potentially embodied geometric concepts. A digital camera was used to capture visual evidence of the building's structural components, including doors, windows, roof gables, roof profiles, and front arches. Second, a structured architectural observation checklist was employed to document the location, characteristics, and geometric properties of each architectural element, including symmetry, parallel sides, angles, and curved forms. Third, a researcher-developed concept-mapping rubric, constructed according to the four levels of the Iceberg Model (situational, referential, general, and formal), was used to systematically map the identified geometric concepts into the proposed Iceberg-based Learning Trajectory. The iterative mapping process between cultural artefacts, geometric concepts, and the four levels of the Iceberg Model was conducted repeatedly until conceptual consistency was achieved across all components of the proposed learning trajectory. Data collection involved direct field observation, photo-documentation, and secondary data synthesis from prior ethnomathematical documentation of the site.

Procedures

The study was conducted in five consecutive stages. The first stage involved field observations and documentation of the architectural components of the Raja Rokan Palace. The second stage consisted of interviews with selected experts to obtain deeper cultural and architectural interpretations of the identified artefacts. In the third stage, the collected data were analyzed to identify mathematical ideas embedded within each architectural element. Subsequently, the identified concepts were analyzed using the Iceberg Model to determine how students could progressively move from contextual cultural experiences toward formal mathematical understanding. Finally, the results of the Iceberg analysis were synthesized into an Iceberg Learning Trajectory, which was then represented through a series of student worksheet activities designed for learning plane geometry.

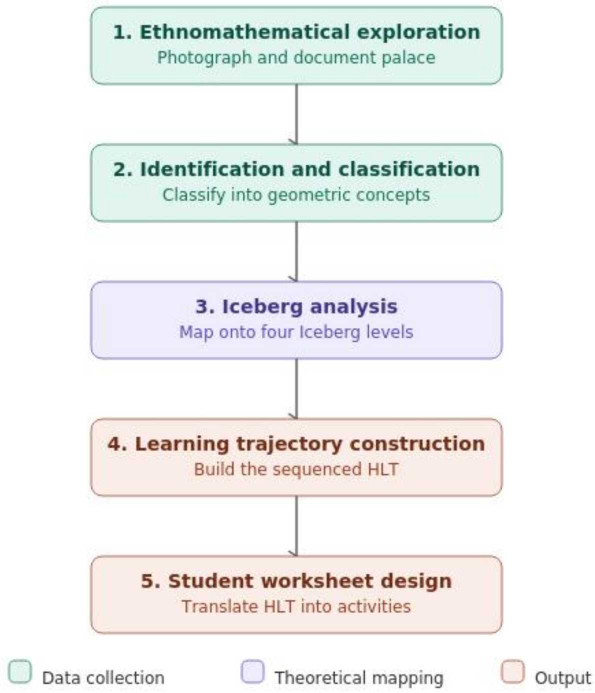


Figure 1. Iceberg-based Learning Trajectory Construction Framework

Data Analysis

Data were analyzed using content analysis and thematic coding supported by constant comparative techniques. The analysis followed a rigorous four-step procedure to ensure analytical depth and eliminate superficial categorization:

1. Identification & Mathematical Classification: Open coding was performed on visual artifacts to extract geometric features, followed by axial coding to classify them into four target junior high school geometry topics: rectangles, triangles, trapezoids, and semicircles.
2. Progressive Iceberg Mapping: Each identified concept was systematically assigned to the four Iceberg Model levels based on operational criteria of progressive mathematization:
   a. Situational Level: Authentic cultural observation of the palace building without formal mathematical language.
   b. Referential Level (Model of): Trace drawings and concrete representations directly tied to specific architectural elements (e.g., tracing a door outline or roof gable).
   c. General Level (Model for): Mathematical property extraction (measuring angles, side lengths, parallel lines, and line symmetry) independent of the specific physical building.

d. Formal Level: Standard mathematical definitions, symbolic formulas (area and perimeter), and contextual problem-solving detached from the physical site.

3. Trajectory & Hypothetical Learning Trajectory Synthesis: The mapped levels were synthesized into a structured Hypothetical Learning Trajectory table articulating the pedagogical flow:

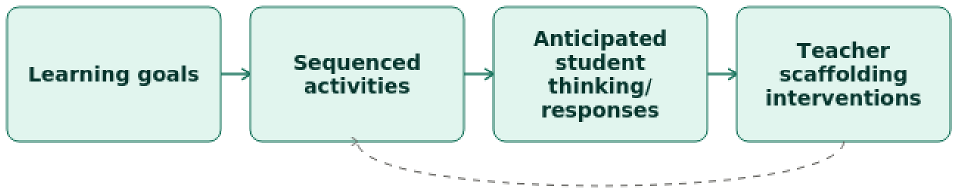


Figure 2. Diagram of the Hypothetical Learning Trajectory (HLT), adapted from Simon (1995)

The dashed line at the bottom of Figure 1 shows that the results of the intervention typically feed back into the design of the next activity — this cycle is iterative, not a one-time process.

4. Operationalization: The Hypothetical Learning Trajectory was operationalized into a 5-stage student worksheet prototype reflecting the Iceberg progression.

Validity and Trustworthiness of the Proposed Design

To ensure the trustworthiness, internal validity, and pedagogical rigor of the proposed Hypothetical ILT constructed without direct classroom implementation, three complementary credibility procedures were implemented as a form of methodological triangulation:

a. Theoretical and Curriculum Cross-Verification: The trajectory's progression and target competencies were systematically cross-examined against national junior high school geometry curriculum standards and the progressive mathematization literature of Realistic Mathematics Education (RME). This procedure aimed to guarantee cognitive appropriateness and the pedagogical feasibility of the designed instructional tasks.
b. Peer Debriefing & Expert Review: The drafted Hypothetical ILT, Iceberg Model level mapping, and student worksheet prototype underwent critical review through peer debriefing and construct validation by specialists in mathematics education. This validation process focused on two primary dimensions: content validity, regarding the mathematical accuracy of the architectural representations, and construct validity, regarding the logical continuity of abstraction levels from situational experience to formal mathematics.
c. Audit Trail and Confirmability: To establish data confirmability and eliminate interpretive bias, all analytical mapping claims connecting architectural features to geometric concepts were directly linked to specific, systematically archived photographic documentation of the Raja Rokan Palace.

RESULTS AND DISCUSSION

Results

Geometric Concepts Identified in the Raja Rokan Palace

Building on the researcher's prior ethnomathematical exploration, which identified rectangles on the doors and windows and a semicircle on the front roof arch, this study identified two additional geometric concepts embedded in the palace's roof structure. The roof's gable—the triangular section situated above the semicircular arch—was found to represent a triangle, while the roof's side profile, formed by the wider, lower section of the two-tiered roof, was found to represent a trapezoid. Table 1 summarizes all four geometric concepts together with their precise architectural location.

Table 1. Geometric concepts identified in the Raja Rokan Palace

Geometric Concept	Architectural Location
Rectangle	Doors and windows
Semicircle	Front roof arch
Triangle	Roof gable (upper tier)
Trapezoid	Roof side profile (lower tier)

Mapping onto the Iceberg Model

These four concepts were then mapped onto the four levels of the Iceberg Model, as illustrated in Figure 2-5. At the situational level, students are presented with the palace itself as an authentic cultural artefact, observing its overall form without yet abstracting any mathematical structure. At the referential level, students represent specific architectural elements—the doors, windows, roof gable, roof profile, and front arch—through drawings or tracings, beginning to notice regularities in shape. At the general level, students identify the defining properties shared across these representations: four right angles and parallel sides for the rectangle, three angles summing to 180° for the triangle, one pair of parallel sides of unequal length for the trapezoid, and a curved boundary equal to half of a full circle for the semicircle. At the formal level, students construct standard mathematical definitions, notations, and formulas (e.g., area and perimeter) for each of the four shapes, independent of the palace context.

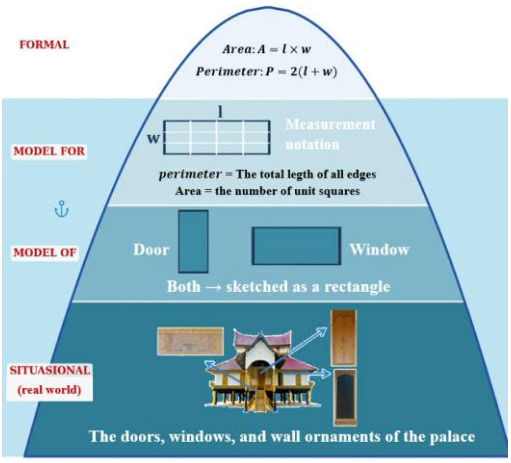


Figure 2. Iceberg mapping of the rectangle concept (doors, windows, and wall ornament)

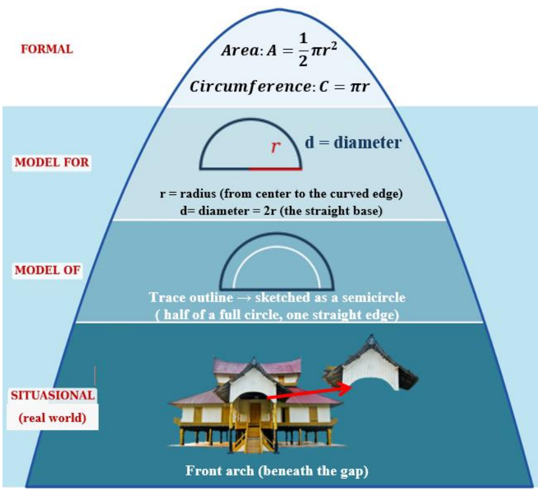


Figure 3. iceberg mapping of the semicircle concept (front roof arch)

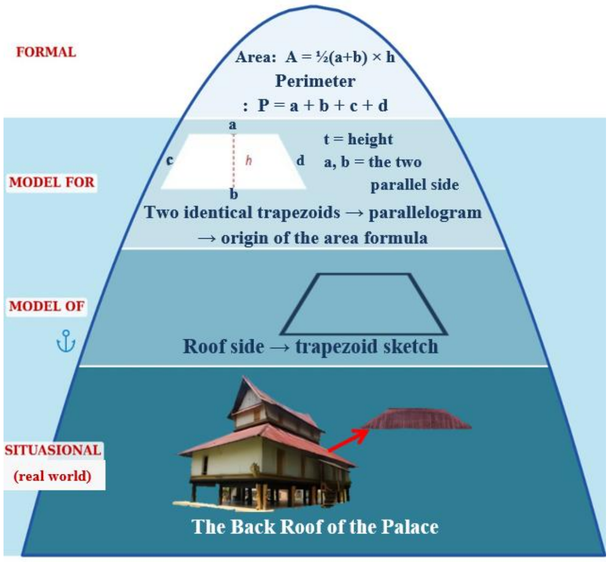


Figure 4. iceberg mapping of the trapezoid concept (roof side profile)

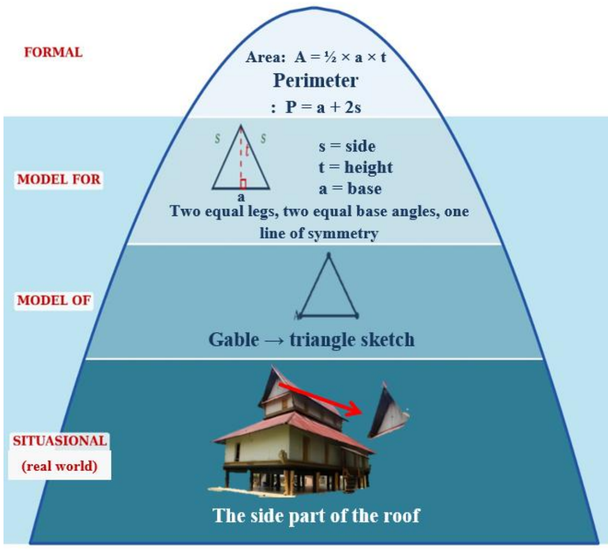


Figure 5. iceberg mapping of the triangle concept (roof gable)

Iceberg-Based Learning Trajectory

From this mapping, an Iceberg Learning Trajectory was constructed, consisting of five components: learning objectives, learning sequence, anticipated student thinking, teacher scaffolding, and expected mathematical thinking (Figure 2). The learning objectives target students' ability to identify, represent, and formally define the four geometric concepts. The learning sequence follows the four iceberg levels in order, moving from observation of the palace, through representational drawing, toward generalization and formalization. Anticipated student thinking includes the expectation that students will initially describe shapes using everyday language (e.g., "the door looks like a long box") before progressing to mathematical terminology. Teacher scaffolding is designed to prompt this progression through guided questioning rather than direct instruction, consistent with the RME principle that teachers act as facilitators rather than sources of ready-made knowledge.

As this study remains at the conceptual and theoretical stage, the Hypothetical Learning Trajectory (HLT) presented below is a *hypothetical* construct in the sense originally proposed by: a reasoned prediction of how students' thinking might develop through a sequence of activities, formulated on the basis of relevant literature on students' geometric thinking and common difficulties in learning geometry, rather than on direct classroom observation. Accordingly, the

"Conjecture of Student Response" presented in each activity table reflects anticipated—not empirically observed—student responses, and the "Teacher's Anticipated Response" represents a planned pedagogical anticipation rather than a documented teaching action. As this HLT has not yet been tested in an actual classroom setting, it should be understood as a design proposal intended to guide future teaching experiments, through which it may subsequently be revised and validated into a Local Instructional Theory (LIT).

Tabel 2. Overview of the Learning Trajectory

Iceberg Level	Activity	Learning Goal
Situational	Activity 1	Exploring the Raja Rokan Palace
Referential (Model of)	Activity 2	Identifying and representing geometric shapes on the palace
General (Model for)	Activity 3	Investigating the properties of each shape
Formal	Activity 4	Constructing formal definitions and formulas

explanation of each activity as follows:

Activity 1 — Exploring the Raja Rokan Palace (Situational Level)

Learning Goal: Students become familiar with the Raja Rokan Palace as a cultural context and begin to notice visually distinct shapes within its architecture, without yet using formal geometric vocabulary.

Description: Students are shown photographs and a short video/text about the Raja Rokan Palace, its history, and its architectural features (doors, windows, roof gable, roof profile, front arch). They are asked to describe, in their own words, what they observe.

Table 3. Conjecture of Student and Teacher Response — Activity 1

Conjecture of Student Response	Teacher's Anticipated Response
Students recognize the palace as a traditional building but do not yet mention any shapes.	The teacher asks guiding questions such as, "What shapes do you notice on the doors? On the roof?" to draw attention toward visual forms
Students spontaneously name everyday shapes (e.g., "the door looks like a long box," "the roof looks pointy").	The teacher gives verbal appreciation and records these informal descriptions to be revisited in Activity 2.
Students focus only on non-geometric aspects (e.g., color, ornament motifs, history).	The teacher acknowledges these observations and redirects attention specifically toward the outline and form of structural elements

Activity 2 — Identifying and Representing Geometric Shapes (Referential Level / Model of)

Learning Goal: Students represent specific architectural elements—the doors and windows, roof gable, roof profile, and front arch—through tracing or drawing, producing a concrete representation ("model of" the situation) that refers directly to the palace.

Description: Using printed images or an interactive worksheet, students trace the outline of four architectural elements: (1) a door or window, (2) the triangular roof gable, (3) the side profile of the lower roof tier, and (4) the arch beneath the front roof. They label each traced outline.

Table 4. Conjecture of Student and Teacher Response — Activity 2

Conjecture of Student Response	Teacher's Anticipated Response
Students can trace the door/window accurately and recognize it resembles a rectangle.	The teacher gives verbal appreciation and asks students to describe the outline in their own words (e.g., "four straight sides, corners like an L").
Students trace the roof gable but describe it only as "pointy" or "like a hat," without recognizing it as a triangle.	The teacher asks, "How many straight sides does this shape have?" to guide students toward counting sides.
Students have difficulty tracing the roof profile because it includes both a slanted upper edge and a flat lower edge, producing an inconsistent outline.	The teacher demonstrates tracing the roof profile step by step, emphasizing the four sides—two parallel of different lengths and two slanted sides.

Students trace the front arch as a full circle rather than a half circle.	The teacher asks students to compare the traced shape with the actual photograph and re-examine where the flat baseline of the arch lies

Activity 3 — Investigating the Properties of Each Shape (General Level / Model for)

Learning Goal: Students move from context-bound representations toward general mathematical properties, identifying shared characteristics such as number of sides, parallel sides, angles, and symmetry that would apply to any instance of the shape—not only the palace's version.

Description: Using their tracings from Activity 2, students measure angles with a protractor, count sides and vertices, and check for parallel or equal sides. They complete a comparison table for the four shapes.

Table 5. Conjecture of Student and Teacher Response — Activity 3

Conjecture of Student Response	Teacher's Anticipated Response
Students correctly identify that the door/window shape has four right angles and two pairs of equal, parallel sides.	The teacher gives verbal appreciation and introduces the term "rectangle" as the mathematical name for this set of properties.
Students measure the roof gable's three angles and notice they sum to approximately 180°, but are unsure whether this always holds true.	The teacher guides students to repeat the measurement using a different-sized triangle drawing and confirms the generalization: the angle sum of a triangle is always 180°.
Students identify that the roof profile shape has one pair of parallel sides of different lengths but struggle to distinguish it from a rectangle.	The teacher asks students to compare the two shapes side by side and highlights that only one pair of sides is parallel in the roof profile shape, unlike the rectangle.
Students measure the front arch and describe it as "half of a circle" but cannot yet state this using the term "semicircle" or explain the 180° arc.	The teacher introduces the term "semicircle" and relates the shape to half of a full 360° circle

Activity 4 — Constructing Formal Definitions and Formulas (Formal Level)

Learning Goal: Students detach from the palace context and construct standard mathematical definitions, notations, and formulas for each shape, demonstrating that abstraction from the cultural context to formal mathematics has been achieved.

Description: Without referring back to the palace images, students write the formal definition of each shape, derive or recall its perimeter and area formulas, and solve contextual numerical problems (e.g., "If the palace's door measures 2 m by 0.8 m, what is its area?").

Table 6. Conjecture of Student and Teacher Response — Activity 4

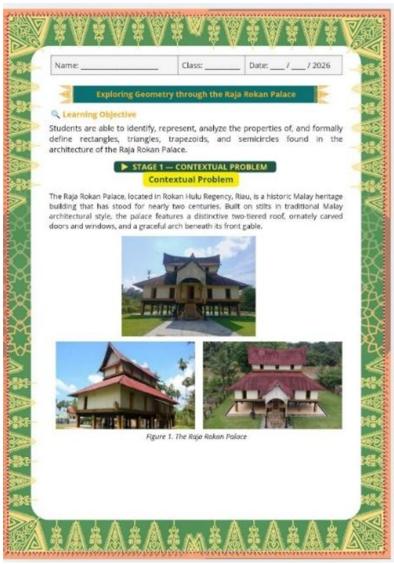
Conjecture of Student Response	Teacher's Anticipated Response
Students can write the definition and area/perimeter formula for a rectangle independently	The teacher gives verbal appreciation and moves to more complex shapes
Students can define a triangle but require assistance recalling the area formula (½ × base × height).	The teacher revisits the connection between the roof gable's shape and the general triangle area formula, using the earlier angle-sum finding as a bridge.
Students confuse the trapezoid's area formula with that of the rectangle	The teacher guides students to derive the trapezoid area formula (½ × (sum of parallel sides) × height) by decomposing the shape into a rectangle and two triangles.
Students can state that a semicircle's area is half that of a full circle but make errors applying the formula ($\frac{1}{2}\pi r^2$).	The teacher works through one worked example with the students, then assigns an independent problem to verify understanding.
Students successfully solve all four formal problems without referring to the palace context.	This indicates the abstraction process from context to formal mathematics has been successfully achieved.

Table 7. Summary Table: Learning Trajectory, Activities, and Iceberg Levels

Student Learning Path	Activity	Geometric Concept	Iceberg Level
Contextual observation	1. Exploring the palace	All four shapes (informal)	Situational
Representation	2.Tracing architectural elements	Rectangle, triangle, trapezoid, semicircle	Referential (Model of)
Generalization	3. Measuring and comparing properties	Sides, angles, parallel lines, symmetry	General (Model for)
Formalization	4. Writing definitions and formulas	Formal names, notations, area/perimeter	Formal

Student Worksheet Design

The learning trajectory was subsequently operationalized into a five-stage student worksheet prototype: (1) a contextual problem introducing a photograph or illustration of the palace and an accompanying real-world question; (2) an exploration task in which students identify and trace geometric shapes within the architectural image; (3) guided questions that scaffold students toward articulating shared properties of each shape; (4) a reflection stage in which students record their own reasoning process; and (5) a formal mathematics stage in which students write standard definitions and solve related numerical problems (e.g., calculating the area of a trapezoidal roof section). This structure mirrors the worksheet designs used in comparable studies employing the iceberg framework.



Name: ________________ Class: ________ Date: ___ / ___ / 2026

Exploring Geometry through the Raja Rokan Palace

Learning Objective

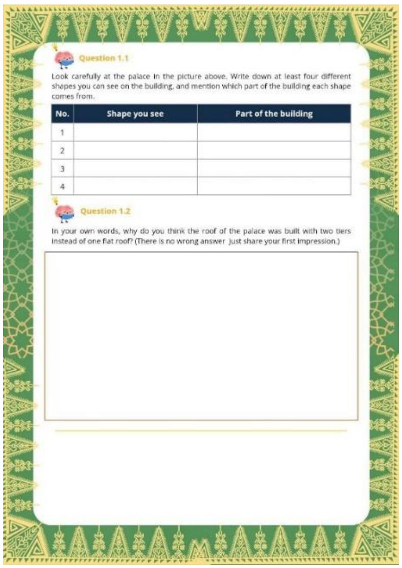
Students are able to identify, represent, analyze the properties of, and formally define rectangles, triangles, trapezoids, and semicircles found in the architecture of the Raja Rokan Palace.

▶ STAGE 1 — CONTEXTUAL PROBLEM

Contextual Problem

The Raja Rokan Palace, located in Rokan Hulu Regency, Riau, is a historic Malay heritage building that has stood for nearly two centuries. Built on stilts in traditional Malay architectural style, the palace features a distinctive two-tiered roof, ornately carved doors and windows, and a graceful arch beneath its front gable.

Figure 1. The Raja Rokan Palace



Question 1.1

Look carefully at the palace in the picture above. Write down at least four different shapes you can see on the building, and mention which part of the building each shape comes from.

No.	Shape you see	Part of the building
1		
2		
3		
4		

Question 1.2

In your own words, why do you think the roof of the palace was built with two tiers instead of one flat roof? (There is no wrong answer just share your first impression.)

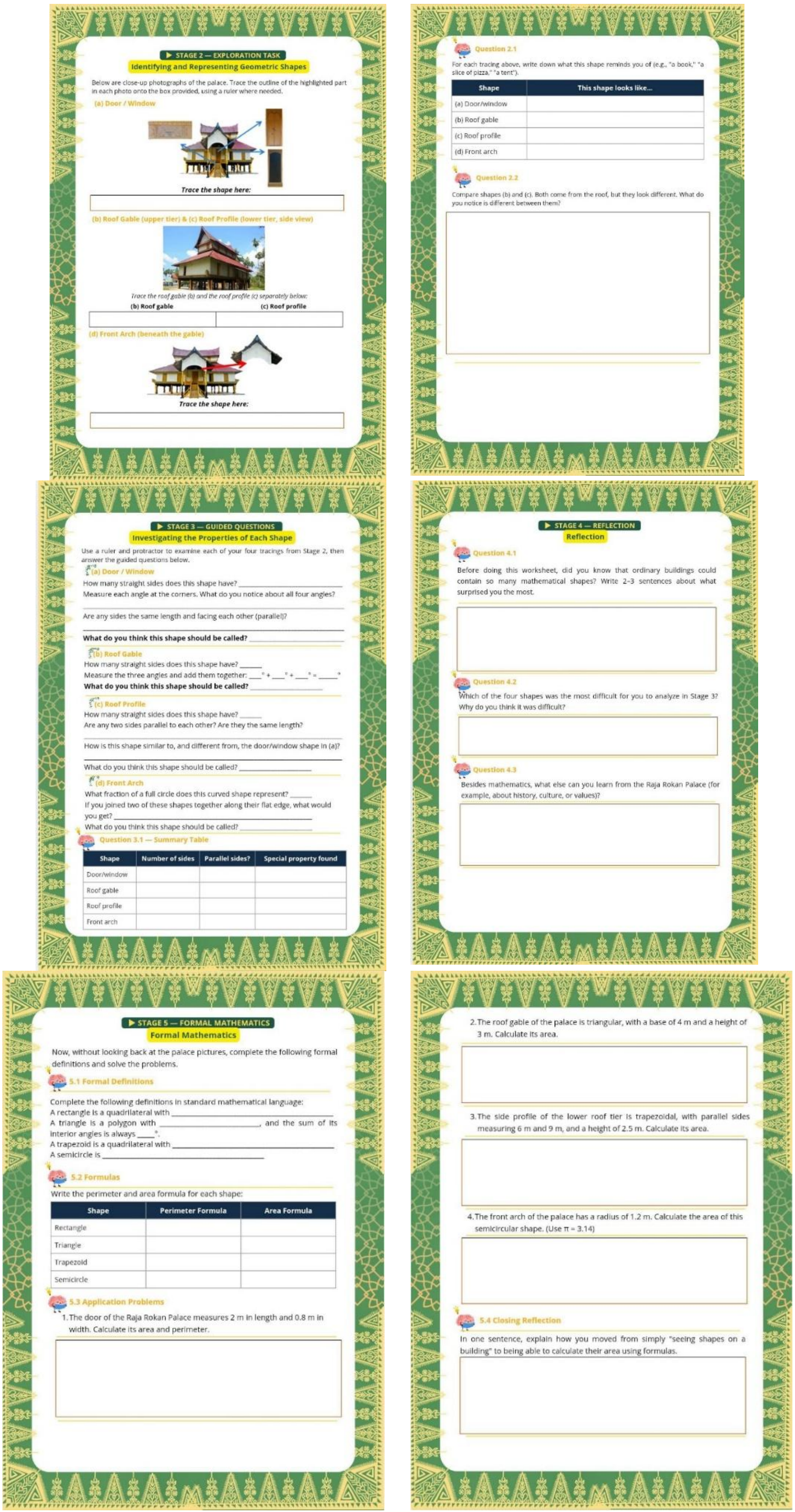


Figure 6. Student Worksheet

Discussion

The findings demonstrate that the Raja Rokan Palace contains multiple architectural elements that can be systematically interpreted as meaningful representations of school geometry, namely rectangles, triangles, trapezoids, and semicircles. These findings extend previous ethnomathematical research on the palace by not only confirming the existence of geometric concepts but also identifying additional mathematical forms embedded within the roof structure, thereby enriching the mathematical potential of the cultural artefact. The identification of these four geometric concepts indicates that traditional architecture provides authentic contexts through which abstract mathematical ideas can be introduced in ways that are meaningful and culturally relevant to students. Rather than functioning merely as historical or aesthetic objects, the architectural components of the palace become pedagogical resources capable of connecting students' everyday cultural experiences with formal mathematical learning. This interpretation is consistent with the fundamental principles of ethnomathematics, which emphasise that mathematical knowledge originates from human cultural activities and should therefore be recognised within local practices and cultural products (Pratama & Yelken, 2024). Likewise, recent studies have reported that integrating local cultural heritage into mathematics instruction enhances students' conceptual understanding, mathematical literacy, and engagement because learning is situated within familiar and meaningful contexts (Marsigit et al., 2025). However, unlike most previous ethnomathematical studies that primarily document mathematical concepts embedded in cultural artefacts, the present study advances the field by transforming those concepts into structured instructional resources suitable for classroom implementation. This transformation addresses an important limitation in previous studies, where ethnomathematical findings frequently remained descriptive and lacked explicit pedagogical implications. Consequently, the present study demonstrates that cultural heritage can contribute not only to preserving local wisdom but also to supporting curriculum-oriented mathematics instruction through systematic instructional design (Yamaguchi, 2025). These findings reinforce the view that ethnomathematics should be understood as a foundation for instructional innovation rather than solely as an approach to documenting cultural mathematical knowledge.

The mapping of the identified geometric concepts onto the four levels of the Iceberg Model illustrates how authentic cultural experiences can be systematically transformed into progressively more abstract mathematical understanding. Beginning with direct observation of the Raja Rokan Palace at the situational level, students are gradually guided towards representation, generalisation, and formal mathematical reasoning through successive instructional stages. Such progression reflects the principle of progressive mathematization proposed within Realistic Mathematics Education, whereby learners construct formal mathematical concepts from meaningful real-world situations rather than receiving abstract definitions directly (Van den Heuvel-Panhuizen & Drijvers, 2020). The findings suggest that each architectural element serves as an instructional bridge connecting intuitive visual perception with increasingly sophisticated geometric reasoning (Cumino et al., 2021; Lee, 2025; Porat & Ceobanu, 2024). This gradual transition is particularly important in geometry learning because students often experience difficulties when abstract concepts are introduced without sufficient contextual support. Previous studies implementing the Iceberg Model have similarly reported that contextual learning environments enable students to develop stronger conceptual understanding by encouraging them to move progressively from informal reasoning to formal mathematical representations (Sari et al., 2025; Wiryanto et al., 2024). Nevertheless, those studies generally employed contextual situations derived from everyday experiences or curriculum-based examples rather than authentic cultural heritage. In contrast, the present study demonstrates that ethnomathematical contexts can effectively function as the situational foundation of the Iceberg Model, thereby enriching the contextual dimension of progressive mathematization while simultaneously preserving cultural identity. The integration achieved in this study therefore extends previous applications of the Iceberg Model by showing that cultural artefacts are capable of supporting each instructional level in a coherent and theoretically grounded manner. This finding contributes to the growing body of literature advocating culturally responsive mathematics education by demonstrating that progressive abstraction can emerge naturally from students' interaction with meaningful cultural environments.

Another important contribution of this study lies in the development of the Iceberg-based Learning Trajectory, which systematically organises ethnomathematical findings into a coherent sequence of learning objectives, learning activities, anticipated student thinking, teacher scaffolding, and expected mathematical understanding. Unlike conventional lesson planning that frequently emphasises instructional content alone, the proposed learning trajectory explicitly anticipates how students' reasoning is expected to develop throughout successive learning activities based on the principles of Hypothetical Learning Trajectory. The inclusion of anticipated student responses and corresponding teacher interventions reflects the view that effective instructional design should consider both conceptual progression and potential learning obstacles before classroom implementation. This characteristic aligns with perspective that instructional design should be viewed as a theoretically informed hypothesis concerning students' mathematical learning rather than as a fixed sequence of teaching activities. Previous studies developing learning trajectories have similarly emphasised the importance of predicting students' thinking in order to facilitate meaningful conceptual development and instructional decision-making (Johar et al., 2025). However, many existing learning trajectories begin with curriculum concepts and subsequently search for contextual examples to illustrate them. The present study adopts the opposite approach by beginning with authentic cultural artefacts and subsequently constructing a learning trajectory that progressively leads students towards formal geometry concepts. This reversal of the instructional design process represents an important methodological contribution because it demonstrates how ethnomathematical exploration can become the primary driver of curriculum development rather than merely functioning as contextual enrichment. Furthermore, the proposed learning trajectory establishes a coherent relationship among ethnomathematics, the Iceberg Model, and Realistic Mathematics Education within a single instructional framework, an integration that remains relatively limited in previous mathematics education research. Consequently, the resulting Hypothetical Iceberg-based Learning Trajectory offers a theoretically grounded design proposal that can guide future classroom teaching experiments and support the development of Local Instructional Theory through empirical validation.

The development of the student worksheet represents the practical realisation of the proposed Iceberg-based Learning Trajectory by translating its theoretical components into structured classroom learning activities. Rather than functioning as a conventional worksheet consisting primarily of procedural exercises, the developed worksheet guides students through successive stages of contextual observation, representation, generalisation, reflection, and formal mathematical reasoning, thereby reflecting the principles of progressive mathematization embedded within the Iceberg Model. The worksheet begins by introducing authentic visual representations of the Raja Rokan Palace, enabling students to explore cultural artefacts before engaging in mathematical abstraction. This instructional sequence reflects the principles of Realistic Mathematics Education, which emphasise that meaningful mathematical understanding emerges when students progressively reinvent formal concepts from realistic situations rather than memorising formulas in isolation (Van den Heuvel-Panhuizen & Drijvers, 2020). The inclusion of guided questions, tracing activities, comparative investigations, reflective prompts, and contextual problem-solving provides multiple opportunities for students to construct their own mathematical understanding while receiving appropriate instructional support. Similar worksheet designs based on the Iceberg framework have been reported to improve students' conceptual understanding because they encourage learners to move gradually from concrete experiences towards abstract reasoning through carefully sequenced learning tasks (Amanda et al., 2024; Wiryanto et al., 2024). However, unlike previous worksheet developments that generally employ everyday situations or textbook contexts, the present study embeds the complete learning sequence within a culturally significant heritage site, thereby strengthening both mathematical meaning and cultural relevance. This integration demonstrates that student worksheets can simultaneously function as instructional media for geometry learning and as educational resources for preserving local cultural heritage (Kazimova et al., 2025; Kurniawan et al., 2023). Furthermore, the proposed worksheet illustrates how cultural exploration, conceptual development, and mathematical formalisation can be organised into a coherent instructional pathway that aligns with students' cognitive development and curriculum objectives . Therefore, the worksheet developed in this study represents not merely a learning medium but an operational representation of the proposed Hypothetical Iceberg-based Learning Trajectory that is ready to be examined through future classroom implementation and refinement.

Although the present study contributes a theoretically grounded instructional framework, its findings should be interpreted within the scope of conceptual educational design rather than empirical classroom effectiveness. The proposed Hypothetical Iceberg-based Learning Trajectory has been developed through systematic ethnomathematical analysis, the principles of the Iceberg Model, and Learning Trajectory theory, yet it has not been validated through teaching experiments involving students and teachers. Consequently, the anticipated student responses, teacher scaffolding, and learning progression described in the instructional framework remain theoretical predictions based on previous literature rather than observations derived from actual classroom practice. This characteristic is consistent with Simon's conception of a Hypothetical Learning Trajectory, which serves as a reasoned instructional hypothesis that requires empirical refinement through iterative implementation before evolving into a Local Instructional Theory. Despite this limitation, the study makes an important theoretical contribution by demonstrating a systematic procedure for transforming ethnomathematical exploration into an instructional design framework that integrates cultural context, progressive mathematization, and anticipated student thinking within a single coherent model. Compared with previous ethnomathematics studies that generally terminate at the identification of mathematical concepts embedded in cultural artefacts, the present research extends the instructional value of ethnomathematics by providing a complete design pathway from cultural exploration to geometry learning. This contribution is particularly significant because it bridges the long-recognised gap between ethnomathematical documentation and classroom-oriented instructional design that has received relatively limited attention in previous mathematics education research. Future research should therefore examine the implementation of the proposed learning trajectory through classroom teaching experiments, evaluate students' conceptual development across the four Iceberg levels, and investigate how teachers adapt the instructional sequence in authentic learning environments. Such empirical investigations would provide opportunities to refine the proposed learning trajectory into a validated Local Instructional Theory while simultaneously generating stronger evidence regarding the effectiveness of culturally responsive geometry instruction. Overall, the present study demonstrates that integrating ethnomathematics, the Iceberg Model, and Learning Trajectory principles offers a promising direction for advancing culturally responsive mathematics education by transforming local cultural heritage into meaningful, systematic, and theoretically grounded geometry learning experiences.

Implications

The findings of this study have important theoretical and practical implications for the advancement of culturally responsive mathematics education by demonstrating a systematic approach to transforming ethnomathematical knowledge into classroom-oriented instructional design. Theoretically, this study extends the scope of ethnomathematics research by moving beyond the identification of mathematical concepts embedded in cultural heritage towards the development of a structured instructional framework that integrates ethnomathematics, the Iceberg Model, and Learning Trajectory principles. This integration contributes to the literature by illustrating how authentic cultural artefacts can function not only as contextual examples but also as foundational elements for designing progressive learning experiences that facilitate students' transition from informal reasoning to formal geometric understanding. Furthermore, the proposed Hypothetical Iceberg-based Learning Trajectory provides an example of how the principles of Realistic Mathematics Education can be operationalised through culturally meaningful instructional sequences that explicitly anticipate students' conceptual development and potential learning difficulties. From a methodological perspective, the study offers a replicable design framework that can be adapted by future researchers seeking to transform other forms of local cultural heritage into mathematics learning trajectories across different mathematical topics and educational contexts. Practically, the instructional framework provides mathematics teachers with an alternative strategy for integrating local culture into geometry instruction without sacrificing curriculum objectives or conceptual rigour. The proposed student worksheet also offers a practical teaching resource that encourages students to engage in observation, representation, generalisation, reflection, and formalisation through meaningful interactions with authentic cultural contexts. In addition, the study supports curriculum developers by providing evidence that local cultural heritage can be systematically incorporated into instructional materials while remaining consistent with contemporary learning theories and competency-based curriculum expectations. For educational policymakers, the findings highlight the potential of integrating cultural heritage into mathematics education as a means of simultaneously strengthening conceptual understanding, preserving local identity, and promoting culturally sustainable education. The study also encourages stronger collaboration among mathematics educators, curriculum designers, cultural heritage experts, and local communities in developing instructional resources that are academically rigorous and culturally authentic. Although the proposed learning trajectory remains hypothetical and requires empirical validation through classroom teaching experiments, it establishes a theoretically grounded foundation upon which future Local Instructional Theory can be developed and refined. Overall, this study demonstrates that transforming ethnomathematical exploration into an Iceberg-based Learning Trajectory represents a promising direction for designing innovative, meaningful, and culturally responsive geometry learning that strengthens the relationship between local cultural heritage and formal school mathematics.

Limitations and Suggestions for Future Research

This study has several limitations that should be acknowledged. First, the study focused on developing a Hypothetical Iceberg-based Learning Trajectory (HILT) and a prototype student worksheet without conducting classroom implementation or teaching experiments. Consequently, the proposed learning trajectory remains a theoretical design that has not yet been empirically validated through students' actual learning experiences. Second, the study was limited to a single cultural heritage site, namely the Raja Rokan Palace, and focused only on four plane geometry concepts: rectangles, triangles, trapezoids, and semicircles. Therefore, the findings should not be generalised to other cultural contexts or mathematical topics without further investigation. Future research should implement the proposed HILT in authentic classroom settings to examine its practicality, effectiveness, and impact on students' geometric understanding. Teaching experiments are also needed to refine the hypothetical learning trajectory into a validated Local Instructional Theory based on students' actual learning processes. In addition, future studies should explore other Indonesian cultural heritage sites and broader mathematical topics to examine the adaptability of the proposed instructional framework. Comparative studies across different cultural contexts would further strengthen the generalisability of ethnomathematics-based instructional design. Such efforts are expected to provide stronger empirical evidence for integrating ethnomathematics, the Iceberg Model, and Learning Trajectory principles into culturally responsive mathematics education while expanding their application across diverse educational settings.

CONCLUSION

This study demonstrates that ethnomathematical concepts embedded in the Raja Rokan Palace can be systematically transformed into a Hypothetical Iceberg-based Learning Trajectory (HILT) for junior high school geometry. Four geometric concepts—rectangles, triangles, trapezoids, and semicircles—were identified and organised into a learning trajectory based on the principles of Realistic Mathematics Education, the Iceberg Model, and Learning Trajectory. The resulting framework illustrates how authentic cultural contexts can support students' progression from informal observations to formal geometric reasoning through progressive mathematization. The study also produced a prototype student worksheet that operationalises the proposed learning trajectory into structured learning activities. Unlike previous ethnomathematics studies that primarily identified mathematical concepts within cultural artefacts, this research provides a systematic framework for translating cultural knowledge into instructional design. Consequently, the proposed HILT contributes both theoretically and practically by integrating ethnomathematics, the Iceberg Model, and Learning Trajectory into a coherent approach to culturally responsive mathematics education. Although the framework has not yet been validated through classroom implementation, it provides a strong foundation for future teaching experiments and the development of Local Instructional Theory. Future research should examine its validity, practicality, and effectiveness in diverse educational settings and across different cultural heritage contexts. Overall, this study highlights the potential of local cultural heritage to enrich geometry learning while strengthening the integration of culture and formal school mathematics.

AUTHOR CONTRIBUTIONS STATEMENT

Nurrahmawati contributed to the conceptualization of the study, ethnomathematical exploration, research design, data collection, data analysis, development of the Iceberg-based Learning Trajectory and student worksheet prototype, interpretation of findings, and preparation of the original manuscript. Lusi Eka Afri contributed to the research design, methodological supervision, interpretation of the results, critical review of the literature, and revision of the manuscript. Arcat contributed to the validation of the instructional framework, interpretation of the findings, academic review, manuscript editing, and final approval of the submitted version. All authors reviewed, revised, and approved the final manuscript and agreed to be accountable for all aspects of the work.

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