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Assessing mathematical communication in elementary fraction learning across different levels of task-specific self-efficacy

Siti Muhdiati*
Universitas Pendidikan Indonesia, INDONESIA

Sufyani Prabawanto
Universitas Pendidikan Indonesia, INDONESIA

Sandi Budi Iriawan
Universitas Pendidikan Indonesia, INDONESIA

Effy Mulyasari
Universitas Pendidikan Indonesia, INDONESIA

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Abstract

Background: Mathematical communication is a fundamental competency in elementary mathematics because it enables students to represent, explain, and justify mathematical ideas through visual, verbal, and symbolic forms. In fraction learning, these communication processes require students to integrate multiple representations, yet their performance may differ according to task-specific self-efficacy.

Aims: This study aimed to assess elementary students' mathematical communication in fraction learning across different levels of task-specific self-efficacy and to examine the association between self-efficacy and mathematical communication ability.

Method: An ex post facto quantitative design was employed involving 176 fifth-grade students from four elementary schools in West Java, Indonesia. Data were collected using a mathematical communication test and a task-specific mathematics self-efficacy questionnaire. Students were classified into low, moderate, and high self-efficacy groups using tertile categorization. Data were analyzed through descriptive statistics, the Kruskal–Wallis test, Bonferroni-adjusted pairwise comparisons, epsilon squared, and Spearman's rank correlation.

Results: Mathematical communication differed significantly across self-efficacy levels ($H = 86.635$, $p < 0.001$, $\varepsilon^2 = 0.49$). Students with higher self-efficacy consistently achieved superior performance across drawing, written explanation, and mathematical expression indicators. Pairwise comparisons confirmed significant differences among all groups, while Spearman's analysis revealed a strong positive association between self-efficacy and mathematical communication ability ($\rho = 0.729$, $p < 0.001$).

Conclusion: Task-specific self-efficacy is strongly associated with elementary students' mathematical communication in fraction learning. Students with higher self-efficacy demonstrated more effective communication across multiple mathematical representations, highlighting the importance of integrating affective factors into mathematics assessment and instruction.

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INTRODUCTION

Mathematical communication is widely recognized as one of the essential competencies that should be developed from the early stages of mathematics education because it enables students to organize, express, interpret, and justify mathematical ideas systematically. Through mathematical communication, students learn to transform abstract concepts into meaningful explanations using

***Corresponding author:**
Siti Muhdiati, Universitas Pendidikan Indonesia, Indonesia
sitimuhdiati@upi.edu

written language, visual representations, mathematical symbols, diagrams, tables, and other forms of representation (Kusmaryono et al., 2024). This competency extends beyond obtaining correct numerical answers because it requires learners to explain the reasoning underlying their solutions and defend the appropriateness of the strategies they employ. Meaningful mathematical communication also encourages students to clarify their thinking, refine conceptual understanding, and engage in constructive discussions with teachers and peers during classroom activities (Fadzil & Osman, 2025; Siswantari et al., 2025; Webb et al., 2023). The process of expressing mathematical ideas contributes to deeper conceptual learning by helping students connect concrete experiences with increasingly abstract mathematical concepts. As students communicate their reasoning, they become more capable of identifying misconceptions, evaluating alternative strategies, and constructing coherent mathematical arguments. Consequently, mathematical communication serves as a bridge between conceptual understanding, mathematical reasoning, and problem-solving ability rather than functioning as an isolated learning outcome (Aljura et al., 2025; Szabo et al., 2020). The growing emphasis on competency-based education further reinforces the importance of communication as an integral component of mathematics learning. Developing this competency from elementary school is particularly important because it establishes the foundation for more advanced mathematical thinking in subsequent educational levels. Therefore, strengthening mathematical communication should be regarded as a central objective of elementary mathematics education rather than merely a complementary learning activity.

The growing importance of mathematical communication is also closely associated with the educational demands of the twenty-first century, where learners are expected to communicate ideas effectively while solving authentic and increasingly complex problems. Modern mathematics education encourages students not only to understand mathematical procedures but also to explain, justify, and evaluate mathematical reasoning in collaborative learning environments (Aksu & Zengin, 2022; Demir & Zengin, 2023; Fraihat et al., 2022; Nilimaa, 2023). Communication enables learners to transform mathematical knowledge into meaningful discussions that support critical thinking, creativity, collaboration, and reflective learning (Chasanah et al., 2020; Rehman et al., 2024). These competencies are becoming increasingly important because mathematical literacy requires students to explain how mathematical ideas are generated and applied in various contexts rather than simply performing calculations accurately. Nevertheless, international assessments continue to indicate that many students experience considerable difficulty in demonstrating higher-order mathematical competencies that involve reasoning, explanation, and interpretation (Aydin & Birgili, 2023; Azid et al., 2022). Such findings suggest that students still encounter challenges in translating mathematical thinking into coherent written, visual, and symbolic forms. These challenges often originate during elementary education, where foundational communication skills are expected to develop alongside conceptual understanding. If students do not acquire sufficient opportunities to communicate mathematical ideas at this stage, these limitations may persist and become more pronounced as mathematical concepts become increasingly abstract. Consequently, improving mathematical communication during elementary education is an important strategy for strengthening students' long-term mathematical competence and preparing them for more advanced learning experiences. This perspective highlights the need to understand not only students' mathematical performance but also the factors associated with their ability to communicate mathematical ideas effectively.

Among the topics taught in elementary mathematics, fraction learning represents one of the most demanding contexts for developing mathematical communication because it requires students to coordinate multiple forms of representation simultaneously. Students are expected to interpret contextual situations, construct visual models, formulate mathematical expressions, and explain their reasoning in ways that accurately reflect fractional relationships (Barbosa & Vale, 2021; Lee &

Yeo, 2026; Marmur et al., 2020). Unlike whole numbers, fractions require students to develop flexible conceptual understanding while integrating symbolic, visual, and verbal representations into meaningful mathematical explanations (Barbieri et al., 2020; Barbosa & Vale, 2021; Herreros-Torres et al., 2026; Tossavainen & Helenius, 2024). These representational demands often make fraction learning particularly difficult for elementary students, resulting in misconceptions, incomplete reasoning, and ineffective mathematical communication. Previous classroom observations have shown that many students are able to perform computational procedures but experience difficulties when asked to explain why a particular strategy works or to justify their solutions using mathematical language. Such conditions indicate that successful fraction learning depends not only on procedural fluency but also on students' capacity to communicate mathematical ideas clearly and logically. Besides cognitive competence, affective characteristics may also influence students' willingness and confidence to express mathematical reasoning. One important affective characteristic is task-specific self-efficacy, which reflects students' confidence in completing particular mathematical tasks successfully. Students who possess stronger confidence are generally more willing to attempt challenging problems, communicate their reasoning openly, and persist when encountering conceptual difficulties, whereas students with weaker confidence may hesitate to explain their ideas even when they possess adequate conceptual knowledge. Understanding how mathematical communication varies across different levels of task-specific self-efficacy therefore becomes essential for designing mathematics instruction that simultaneously supports students' conceptual development, communication competence, and affective readiness.

Although previous studies have substantially enriched the literature on mathematics education, several important gaps remain. Research on mathematical communication has primarily focused on the effectiveness of instructional approaches, digital technologies, blended learning environments, and language-based pedagogies for enhancing students' communication skills rather than examining the learner characteristics associated with communication performance (Attard & Holmes, 2022; Pasani & Amelia, 2025). Likewise, studies on fraction learning have predominantly investigated conceptual understanding, multiple representations, adaptive learning environments, learning obstacles, and technology-supported interventions without explicitly considering students' affective characteristics that may influence how they communicate mathematical ideas (Oppmann et al., 2025; Reinhold et al., 2020; Sari et al., 2024; Schadl & Ufer, 2023). Meanwhile, research on task-specific self-efficacy has consistently demonstrated its predictive role in mathematical problem posing, problem solving, and instructional quality; however, these studies have rarely explored its relationship with mathematical communication, particularly in elementary mathematics classrooms (Cai et al., 2026; Z. Liu et al., 2020; Lu et al., 2023; Siefer et al., 2021). Furthermore, existing studies generally examine mathematical communication as a single construct, with limited attention to its multidimensional nature encompassing drawing, written explanation, and mathematical expression, especially within the context of fraction learning among elementary students. Consequently, empirical evidence explaining how mathematical communication varies across different levels of task-specific self-efficacy in elementary fraction learning remains scarce. Addressing this gap is expected to provide a more comprehensive understanding of the interplay between cognitive and affective factors in mathematics learning while extending current evidence on mathematical communication by integrating task-specific self-efficacy into the assessment of elementary students' communication performance.

Considering the importance of mathematical communication in elementary mathematics and the potential role of task-specific self-efficacy in supporting students' learning, further investigation is needed to understand how these two constructs are related within the context of fraction learning. Existing evidence has largely examined mathematical communication through instructional

interventions or learning innovations, while comparatively less attention has been given to differences associated with students' confidence in completing specific mathematical tasks. Similarly, studies concerning self-efficacy have predominantly emphasized mathematical achievement and problem-solving performance rather than mathematical communication. Consequently, empirical evidence explaining how students with different levels of task-specific self-efficacy demonstrate mathematical communication remains limited, particularly in elementary fraction learning. Addressing this issue may contribute to a more comprehensive understanding of the interaction between cognitive and affective aspects of mathematics learning. Such evidence is important because communication represents an observable manifestation of students' conceptual understanding as well as their confidence in expressing mathematical reasoning. The present study therefore focuses on elementary students' mathematical communication within the context of fraction learning while considering differences in task-specific self-efficacy. Specifically, the study assesses mathematical communication through three complementary dimensions, namely drawing, written explanation, and mathematical expression. It also examines whether meaningful differences exist among students with low, moderate, and high levels of task-specific self-efficacy and investigates the association between self-efficacy and mathematical communication performance. The findings are expected to enrich the literature on elementary mathematics education by providing empirical evidence that supports the integration of affective and cognitive perspectives in the assessment and improvement of mathematical communication.

LITERATURE REVIEW

Mathematical communication is regarded as one of the central competencies in mathematics education because it enables learners to express, interpret, and justify mathematical ideas through various forms of representation, including written language, mathematical symbols, diagrams, graphs, and visual models. Rather than functioning solely as a means of presenting answers, mathematical communication supports the construction of conceptual understanding by encouraging students to explain their reasoning, evaluate alternative strategies, and engage in meaningful mathematical discourse (Yimam & Dagnew Kelkay, 2022). The process of communicating mathematical thinking also facilitates knowledge construction because students refine their understanding while organizing and articulating mathematical ideas. Effective mathematical communication requires students to integrate conceptual knowledge, procedural understanding, and mathematical reasoning into coherent explanations that can be interpreted by others (Chinofunga et al., 2025; Ncube & Luneta, n.d.; Siswantari et al., 2025; Tong et al., 2021). Consequently, communication is considered an essential process through which mathematical understanding becomes observable during learning activities. Mathematics classrooms that encourage discussion, questioning, and explanation provide students with greater opportunities to develop deeper conceptual understanding and more flexible reasoning. Through regular communication activities, students become more capable of identifying misconceptions, defending mathematical arguments, and evaluating different problem-solving approaches. These experiences contribute not only to mathematical achievement but also to students' confidence in expressing mathematical ideas within collaborative learning environments. For this reason, mathematical communication has become an important indicator in assessing students' mathematical proficiency across different educational levels. Therefore, strengthening mathematical communication should be considered a fundamental objective of mathematics instruction because it supports the development of comprehensive mathematical competence.

Fraction learning represents one of the most conceptually demanding topics in elementary mathematics because it requires students to coordinate numerical, symbolic, visual, and contextual representations simultaneously. Unlike whole-number operations, fraction concepts require students to understand proportional relationships, equivalence, magnitude, and part-whole reasoning before they can communicate mathematical ideas accurately (Pedersen & Bjerre, 2021). Students are expected to interpret contextual situations, construct appropriate mathematical models, explain computational procedures, and justify their solutions using mathematical language that accurately reflects fractional relationships (Barbieri et al., 2020; Govender & Machingura, 2023; Manfreda Kolar & Hodnik, 2021). These representational demands make mathematical communication an inseparable component of successful fraction learning rather than merely an accompanying skill. However, numerous studies have shown that elementary students frequently experience misconceptions when interpreting fractions, particularly when translating between visual representations, symbolic notation, and verbal explanations (Birgin & Eryılmaz, 2025; Kusmaryono et al., 2024; Mohamed et al., 2021). Such misconceptions often result in incomplete mathematical reasoning, incorrect representations, and difficulties in explaining solution strategies coherently. Learning activities that encourage students to represent fractions through multiple forms of communication have been shown to improve conceptual understanding and reduce representational errors. Opportunities to discuss fraction concepts, compare alternative solution strategies, and justify mathematical reasoning also contribute to more meaningful mathematical learning experiences. Consequently, assessing mathematical communication within fraction learning provides valuable information about both students' conceptual understanding and their ability to communicate mathematical thinking effectively. This perspective suggests that mathematical communication should be viewed as an integral component of fraction learning assessment because it reflects students' understanding beyond procedural accuracy.

Besides cognitive competence, affective characteristics play an important role in shaping students' engagement and performance during mathematics learning. One of the most influential affective constructs is task-specific self-efficacy, which refers to learners' confidence in their capability to successfully perform particular mathematical tasks under specific learning conditions (Granello et al., 2025; Jameson et al., 2022; Q. Liu et al., 2020; Zakariya, 2022; Zuo et al., 2024). Unlike general academic self-efficacy, task-specific self-efficacy focuses on students' beliefs regarding their ability to accomplish clearly defined mathematical activities, making it more sensitive to variations across learning contexts. Students who possess stronger task-specific self-efficacy generally demonstrate greater persistence, invest more effort, and exhibit higher resilience when encountering cognitively demanding mathematical problems. Conversely, students with lower levels of confidence often hesitate to express their reasoning, avoid participating in mathematical discussions, and become discouraged when facing challenging tasks. Since mathematical communication requires students to externalize their thinking through explanations, representations, and mathematical expressions, confidence in completing specific tasks may influence the quality of communication they produce (Ata Baran & Kabael, 2021; Pantaleon et al., 2023; Tong et al., 2021). Students who believe they can successfully solve mathematical problems are generally more willing to explain solution strategies, justify mathematical arguments, and revise their reasoning based on feedback. This relationship suggests that mathematical communication is influenced not only by conceptual understanding but also by students' beliefs about their own capabilities when completing mathematical tasks. Understanding how task-specific self-efficacy is associated with mathematical communication may therefore provide a more comprehensive explanation of differences in students' communication performance during fraction learning. Consequently, integrating cognitive and

affective perspectives offers a stronger theoretical foundation for assessing mathematical communication in elementary mathematics education.

The relationship between task-specific self-efficacy and mathematical communication can be explained through the interaction between cognitive engagement and students' beliefs about their own capabilities. Students with stronger confidence in completing mathematical tasks are generally more willing to explain their reasoning, construct mathematical representations, and communicate solution strategies even when faced with challenging problems. Higher levels of task-specific self-efficacy also encourage learners to persist when their initial strategies are unsuccessful, enabling them to revise explanations and improve the quality of their mathematical communication (Bjerke, 2026; DiNapoli, 2023; Halmo et al., 2024; Zakariya, 2022) . In contrast, students with lower confidence often experience anxiety, hesitate to express mathematical ideas, and avoid participating in classroom discussions because they fear making mistakes. These behavioral differences suggest that self-efficacy influences not only students' motivation to learn mathematics but also their willingness to externalize mathematical thinking through written explanations, diagrams, and symbolic representations. Previous empirical studies have consistently reported positive associations between mathematics self-efficacy and academic achievement, problem-solving ability, mathematical reasoning, and learning persistence across different educational levels (Holenstein et al., 2022; Wang et al., 2024; Zuo et al., 2024). Nevertheless, considerably fewer studies have specifically examined whether task-specific self-efficacy is associated with students' mathematical communication, particularly within elementary fraction learning. Moreover, existing investigations have generally emphasized overall communication performance without examining its multidimensional components, including drawing, written explanation, and mathematical expression. Understanding these relationships is important because mathematical communication represents one of the most observable manifestations of students' conceptual understanding during mathematics learning. Therefore, investigating mathematical communication across different levels of task-specific self-efficacy may provide more comprehensive evidence regarding the combined influence of affective and cognitive factors on students' mathematical performance.

The present study is grounded in the assumption that mathematical communication develops through the interaction between conceptual understanding, representational competence, and learners' confidence in performing mathematical tasks. Within this perspective, mathematical communication is viewed as a multidimensional construct encompassing drawing, written explanation, and mathematical expression, each representing different ways in which students externalize mathematical thinking (Genç et al., 2025; Teledahl et al., 2025). Fraction learning provides an appropriate context for examining these dimensions because students are required to move flexibly among visual, verbal, and symbolic representations while solving contextual mathematical problems (Barbosa & Vale, 2021; Herreros-Torres et al., 2026; Merkel et al., 2025; Mohamed et al., 2021). At the same time, task-specific self-efficacy is expected to influence students' willingness to engage in these communication processes because confidence affects effort, persistence, and participation during mathematical activities (Kim & Bong, 2023; Lu et al., 2023; Nuutila et al., 2020; Su & Li, 2026). Students with higher task-specific self-efficacy are theoretically more likely to communicate mathematical ideas clearly because they perceive challenging tasks as manageable rather than threatening. Conversely, students with lower confidence may possess partial conceptual understanding but still struggle to communicate their reasoning due to hesitation and reduced engagement during problem solving. This theoretical perspective suggests that differences in mathematical communication may reflect variations in both cognitive competence and affective readiness rather than conceptual understanding alone. Consequently, assessing mathematical communication across different levels of task-specific self-efficacy may provide a more

comprehensive evaluation of students' mathematical performance in elementary fraction learning. Such an approach also extends current mathematics education research by integrating cognitive and affective perspectives within a single analytical framework. Based on this theoretical foundation, the present study proposes that students with higher levels of task-specific self-efficacy will demonstrate significantly stronger mathematical communication than students with lower levels of task-specific self-efficacy, particularly in the dimensions of drawing, written explanation, and mathematical expression.

METHOD

Research Design

This study employed a quantitative research approach using an ex post facto design to examine differences in elementary students' mathematical communication across different levels of task-specific self-efficacy. The ex post facto approach was selected because the study investigated naturally existing differences among students without manipulating the independent variable or providing any instructional intervention. In this design, task-specific self-efficacy functioned as the grouping variable, whereas mathematical communication served as the dependent variable. The study focused on identifying whether students with different levels of confidence in completing fraction-related mathematical tasks demonstrated different levels of mathematical communication performance. Since the research sought to describe existing conditions rather than establish causal relationships, the findings were interpreted as differences and associations among naturally occurring groups. This design was considered appropriate because students had already completed instruction on fraction addition and subtraction before the data collection process began. Consequently, the study examined authentic variations in students' mathematical communication that emerged from their existing learning experiences.

Participants

The participants consisted of 176 fifth-grade students enrolled in four public elementary schools located in West Java, Indonesia. The participating schools represented four different administrative regions, namely West Bandung Regency, Cimahi City, Bandung City, and Garut Regency, providing broader representation of elementary mathematics learning contexts. All participants had completed the curriculum on fraction addition and subtraction before participating in the study. Students were selected using a combination of cluster sampling and purposive sampling techniques. Cluster sampling was employed because the research involved schools from different geographical regions, whereas purposive sampling ensured that all participants possessed similar learning experiences regarding fraction operations. Every available fifth-grade student within the selected classrooms participated in the study to minimize sampling bias. Participation was voluntary and conducted during regular mathematics learning activities after obtaining permission from the respective schools. Students' identities were anonymized throughout the study to maintain confidentiality, and all collected data were used exclusively for research purposes.

Instruments and Data Collection

Two research instruments were employed to collect the data. The first instrument was a mathematical communication test designed to measure students' ability to communicate mathematical ideas in fraction learning. The test consisted of ten open-ended questions focusing on fraction addition and subtraction. The items were developed to assess three dimensions of

mathematical communication, namely drawing, written explanation, and mathematical expression. The drawing dimension evaluated students' ability to represent mathematical ideas using diagrams, figures, or other visual representations. The written explanation dimension measured students' ability to explain mathematical reasoning logically and coherently using written language. The mathematical expression dimension assessed students' ability to formulate mathematical models and represent contextual situations using appropriate mathematical symbols and expressions. Students' responses were evaluated using an analytic scoring rubric ranging from 0 to 4 for each item. A score of four represented a complete, accurate, and well-organized response, whereas lower scores reflected decreasing levels of completeness, conceptual understanding, procedural accuracy, and communication quality. The total scores obtained from all items were subsequently converted to a scale ranging from 0 to 100 to facilitate interpretation and statistical analysis.

The second instrument was a task-specific mathematics self-efficacy questionnaire designed to measure students' confidence in completing mathematical communication tasks involving fraction operations. The questionnaire consisted of ten task-oriented statements corresponding to the mathematical communication activities included in the test. Students indicated their confidence using percentage-based responses that reflected how certain they were that they could successfully complete each task. The overall self-efficacy score was calculated by averaging students' confidence ratings across all questionnaire items. Prior to the main study, both instruments underwent expert validation to ensure content appropriateness, clarity of language, and alignment with the intended constructs. The mathematical communication test was subsequently administered to a pilot sample to evaluate empirical validity, internal consistency, item discrimination, and item difficulty. The results indicated that all items demonstrated satisfactory psychometric properties and were therefore retained for the main study. Data collection was conducted after students had completed classroom instruction on fraction addition and subtraction. The task-specific self-efficacy questionnaire was administered before the mathematical communication test to capture students' confidence prior to performing the mathematical tasks. After completing the questionnaire, students worked individually on the mathematical communication test under standardized classroom conditions. This sequence was intended to prevent students' confidence ratings from being influenced by their experiences while completing the communication assessment.

Data Analysis

The collected data were analyzed using both descriptive and inferential statistical techniques. Descriptive statistics were first calculated to summarize students' mathematical communication scores and task-specific self-efficacy levels, including the mean, standard deviation, minimum score, maximum score, median, and score distribution. Students' self-efficacy scores were then categorized into low, moderate, and high groups using tertile classification. This categorization enabled comparisons of mathematical communication across different levels of task-specific self-efficacy while maintaining relatively balanced group sizes. Before conducting inferential analyses, assumption testing was performed to determine the suitability of the statistical procedures. Data normality was examined using the Kolmogorov-Smirnov and Shapiro-Wilk tests, whereas homogeneity of variance was evaluated using Levene's test. Since the data violated both normality and homogeneity assumptions, non-parametric statistical procedures were selected. Differences in mathematical communication among the three self-efficacy groups were examined using the Kruskal-Wallis test. Following significant overall differences, pairwise comparisons with Bonferroni adjustment were conducted to identify specific group differences while controlling for Type I error. To evaluate the practical significance of the observed differences, epsilon squared (ε^2) was calculated as an effect size measure. Finally, Spearman's rank correlation analysis was employed to examine the strength and direction of the association between students' original task-specific self-efficacy scores

and their mathematical communication scores. Statistical significance for all inferential analyses was established at the 0.05 level.

Research Procedure

The study was conducted through six sequential stages to ensure methodological consistency throughout the research process. The first stage involved reviewing theories and previous empirical studies related to mathematical communication, fraction learning, and task-specific self-efficacy to establish the theoretical foundation of the study. During this stage, the research objectives, variables, communication indicators, and analytical framework were clearly defined. The second stage focused on developing the research instruments. Mathematical communication test items, the analytic scoring rubric, and the task-specific self-efficacy questionnaire were constructed based on the conceptual framework. These instruments were subsequently evaluated through expert validation and revised according to the feedback received to improve their clarity, relevance, and suitability for elementary school students. The third stage consisted of pilot testing the mathematical communication instrument. The pilot study was conducted to examine empirical validity, reliability, item discrimination, and item difficulty. The evaluation results confirmed that all items met acceptable psychometric standards and were suitable for administration in the main study.

The fourth stage involved collecting data from the participating schools. Students first completed the task-specific self-efficacy questionnaire, followed by the mathematical communication assessment. All assessments were administered individually under standardized classroom conditions to ensure consistency across schools. The fifth stage comprised scoring and organizing the collected data. Students' responses were evaluated using the analytic rubric, and communication scores were calculated for each indicator as well as for the overall mathematical communication performance. Self-efficacy scores were computed and categorized into low, moderate, and high levels using tertile classification before being entered into the statistical software. The final stage involved statistical analysis and interpretation of the findings. Descriptive and inferential analyses were performed to examine differences in mathematical communication across self-efficacy groups and to investigate the association between task-specific self-efficacy and mathematical communication. The findings were subsequently interpreted in relation to the research objectives and theoretical framework to draw conclusions regarding the role of task-specific self-efficacy in elementary students' mathematical communication during fraction learning.

Figure 1. illustrates the overall research procedure adopted in this study, comprising seven sequential stages: literature review, instrument development, validation and pilot testing, participant selection, data collection, data processing and analysis, and interpretation of the findings. This systematic workflow was designed to ensure methodological rigor, transparency, and the validity of the research outcomes. Beginning with the establishment of the theoretical framework, the procedure progressed through the development and validation of research instruments, standardized data collection, and appropriate statistical analyses, culminating in the interpretation of the results and the formulation of conclusions and educational implications. Overall, the framework provides a clear and coherent overview of the research process undertaken to examine the relationship between task-specific self-efficacy and elementary students' mathematical communication in fraction learning.

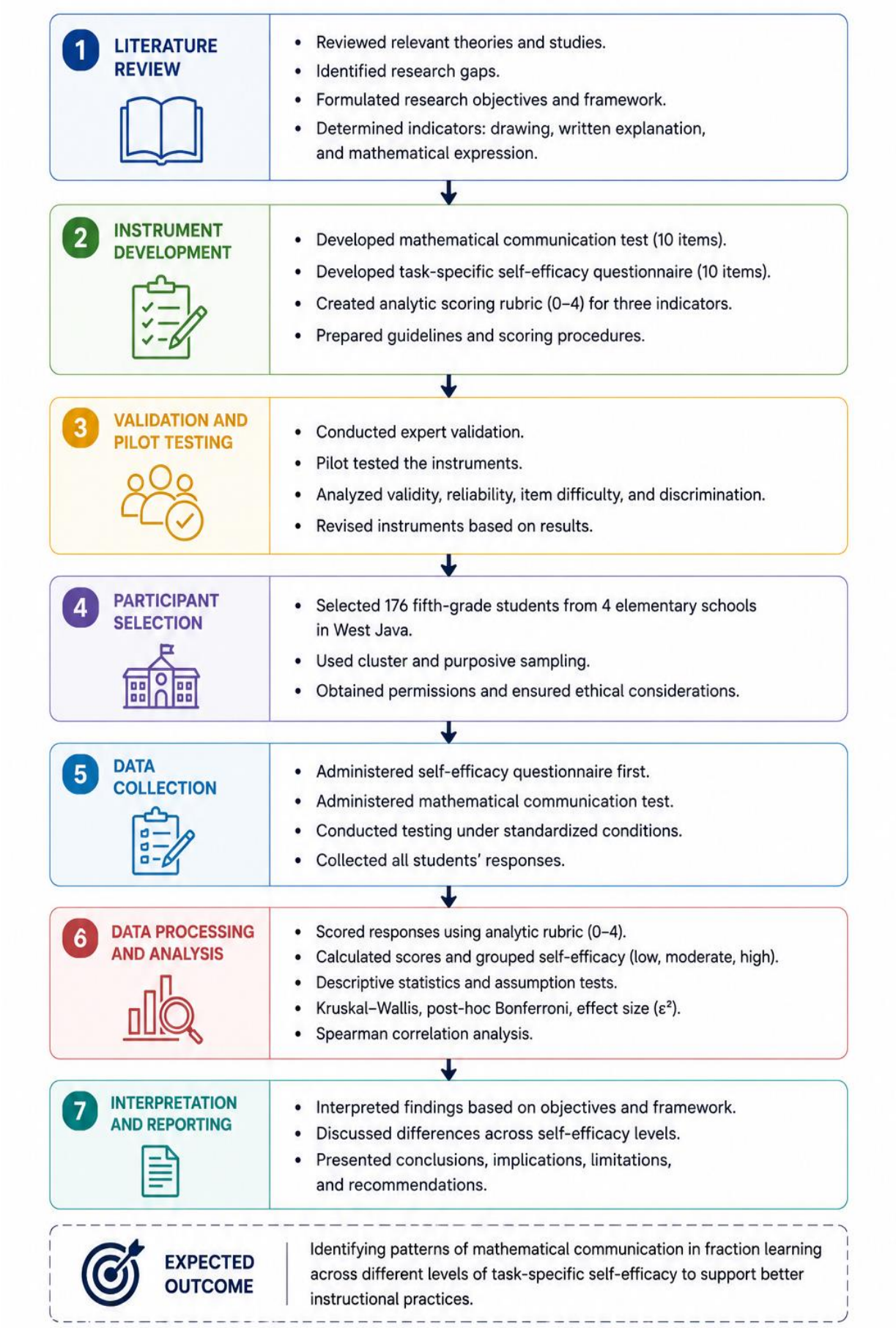


Figure 1. Research Method Flow

RESULTS AND DISCUSSION

Results

The results showed that students' mathematical communication ability in fraction learning differed across self-efficacy levels. This study involved 176 fifth-grade elementary school students. Students' self-efficacy scores were obtained from the average percentage of confidence on the self-efficacy scale items. The scores were then classified into three levels using tertile-based categorization. Tertile-based categorization is a data grouping technique that divides ordered scores into three groups based on the data distribution, so that the resulting groups represent the lower, middle, and upper parts of the score distribution. Explained that continuous data may be categorized into tertiles, quartiles, or quintiles for statistical analysis under certain conditions. Stated that continuous or ordinal data can be divided into groups with an equal number of observations, and that categorizing a variable into three groups requires two cut points, known as tertiles. However, the categorization of continuous variables should be conducted carefully and transparently because it may reduce information if the category boundaries are not clearly explained.

In this study, students' self-efficacy scores were sorted from the lowest to the highest and then divided into three groups. Since several students had identical scores at the category boundaries, students with the same boundary scores were placed in the same category to avoid assigning identical self-efficacy scores to different groups. Based on this procedure, students with self-efficacy scores of 34% or lower were categorized as having low self-efficacy, students with scores between 35% and 62% were categorized as having moderate self-efficacy, and students with scores of 63% or higher were categorized as having high self-efficacy. This classification was used to compare students' mathematical communication ability across the three groups. The descriptive statistics of students' mathematical communication ability based on self-efficacy levels are presented in Table 1.

Table 1. Descriptive Statistics of Mathematical Communication Ability Based on Self-Efficacy Levels

Self-Efficacy Level	N	Mean	Std. Deviation	Minimum	Maximum
Low	60	24.65	11.918	3	85
Moderate	56	51.45	21.471	8	88
High	60	70.35	20.447	10	88
Total	176	48.76	26.366	3	88

Table 1 shows that students with high self-efficacy obtained the highest mean score in mathematical communication ability, with a mean of 70.35 and a standard deviation of 20.447. Students with moderate self-efficacy obtained a mean score of 51.45 with a standard deviation of 21.471. Meanwhile, students with low self-efficacy obtained the lowest mean score, with a mean of 24.65 and a standard deviation of 11.918. These results indicate a clear pattern in which students with higher self-efficacy tend to have higher mathematical communication ability. The descriptive data also show that the score range varied across groups. In the low self-efficacy group, the minimum score was 3 and the maximum score was 85. In the moderate self-efficacy group, the scores ranged from 8 to 88. In the high self-efficacy group, the scores ranged from 10 to 88. Although some students in the low self-efficacy group were able to obtain relatively high scores, the average score of this group remained substantially lower than those of the moderate and high self-efficacy groups. This finding suggests that self-efficacy may be associated with students' ability to explain mathematical ideas, represent fraction problems, and express mathematical models in written form. To provide a more detailed description of students' mathematical communication ability, the scores were also analyzed based on three indicators: drawing, written text, and mathematical expression. Since each indicator consisted of a different number of items, the scores for each indicator were converted into

a 0–100 scale. The descriptive statistics of students' mathematical communication ability by indicator and self-efficacy level are presented in Table 2.

Table 2. Descriptive Statistics of Mathematical Communication Ability by Indicators and Self-Efficacy Levels

Indicator	Self-Efficacy Level	N	Mean	Std. Deviation	Median	Minimum	Maximum
Drawing	Low	60	35.05	10.38	33.00	0	67
Drawing	Moderate	56	46.32	18.69	42.00	0	75
Drawing	High	60	60.43	20.83	67.00	17	83
Written Text	Low	60	21.80	15.94	25.00	0	83
Written Text	Moderate	56	54.93	21.03	54.00	8	100
Written Text	High	60	66.28	16.24	75.00	8	83
Mathematical Expression	Low	60	18.57	15.91	25.00	0	100
Mathematical Expression	Moderate	56	52.13	31.44	56.00	0	100
Mathematical Expression	High	60	80.42	27.72	94.00	0	100

Table 2 indicates that students' mathematical communication scores tended to increase along with their self-efficacy levels across the three indicators. For the drawing indicator, the average score rose from 35.05 among students with low self-efficacy to 46.32 among those with moderate self-efficacy, and reached 60.43 in the high self-efficacy group. A comparable trend was observed in the written text indicator, with mean scores of 21.80, 54.93, and 66.28 for the low, moderate, and high self-efficacy groups, respectively. The largest gap was found in the mathematical expression indicator. Students in the low self-efficacy group achieved an average score of 18.57, whereas those in the moderate group obtained 52.13. Meanwhile, students with high self-efficacy recorded the highest mean score, reaching 80.42. These results suggest that variations in students' mathematical communication ability were evident not only in the overall score but also consistently reflected in each indicator, including drawing, written text, and mathematical expression. Overall, the descriptive results suggest that students with stronger self-efficacy were more able to represent fraction problems visually, explain mathematical reasoning in written form, and construct mathematical expressions. This pattern supports the view that self-efficacy is associated with students' ability to communicate mathematical ideas in fraction learning. Prior to hypothesis testing, the distribution of the data was examined through a normality test. This analysis was carried out to determine whether the data met the assumption of normal distribution. The Kolmogorov-Smirnov and Shapiro-Wilk tests were used for this purpose, and the results are displayed in Table 3.

Table 3. Normality Test Results

Variable	Kolmogorov-Smirnov Statistic	df	Sig.	Shapiro-Wilk Statistic	df	Sig.
Mathematical communication ability	0.145	176	< 0.001	0.901	176	< 0.001
Self-efficacy score	0.109	176	< 0.001	0.950	176	< 0.001

Table 3 indicates that the significance values for both mathematical communication ability and self-efficacy were below 0.05 in the Kolmogorov-Smirnov and Shapiro-Wilk tests. These results suggest that the data did not follow a normal distribution, meaning that the normality assumption was not satisfied. Consequently, hypothesis testing was carried out using a non-parametric

procedure, specifically the Kruskal-Wallis test. A homogeneity test was also conducted to examine whether the variance of mathematical communication ability scores was equal across the low, moderate, and high self-efficacy groups. Levene's test was used to assess the homogeneity of variance, and the results are presented in Table 4 below.

Table 4. Homogeneity Test Results

Variable	Levene Statistic	df1	df2	Sig.	Decision
Mathematical communication ability	10.513	2	173	< 0.001	Not homogeneous

Table 4 presents the result of the homogeneity test for mathematical communication ability, with a significance value of < 0.001. Since this value is below 0.05, it indicates that the variance of mathematical communication scores among students with low, moderate, and high self-efficacy was unequal. Thus, the assumption of homogeneity was not fulfilled. Together with the normality test results, this finding shows that the data did not satisfy the requirements for parametric analysis. For this reason, hypothesis testing was performed using a non-parametric method, namely the Kruskal-Wallis test. The Kruskal-Wallis test was applied to determine whether students' mathematical communication ability differed significantly across self-efficacy levels. This procedure was selected because the preliminary assumption tests showed that the data were neither normally distributed nor homogeneous. Prior to reporting the significance value of the Kruskal-Wallis test, the mean rank of mathematical communication ability for each self-efficacy group is shown in Table 5.

Table 5. Mean Rank of Mathematical Communication Ability Based on Self-Efficacy Levels

Self-Efficacy Level	N	Mean Rank
Low	60	41.83
Moderate	56	97.17
High	60	127.08
Total	176	-

Table 5 demonstrates that the mean rank of students' mathematical communication ability rose as their self-efficacy level increased. The highest mean rank was achieved by students in the high self-efficacy group, with a value of 127.08. Meanwhile, students with moderate self-efficacy reached a mean rank of 97.17, whereas those with low self-efficacy had the lowest mean rank, at 41.83. This trend suggests that students who possessed stronger self-efficacy generally showed better mathematical communication ability compared with students in the moderate and low self-efficacy groups. To further examine whether these differences in mean rank were statistically significant, the Kruskal-Wallis test was performed. The results of this analysis are provided in Table 6.

Table 6. Kruskal-Wallis Test Results of Mathematical Communication Ability Based on Self-Efficacy Levels

Variable	Kruskal-Wallis H	df	Sig.	Effect Size (ε^2)	Decision
Mathematical communication ability	86.635	2	< 0.001	0.49	Significant

Table 6 shows that the Kruskal-Wallis H value was 86.635 with 2 degrees of freedom and a significance value of < 0.001. Since the significance value was lower than 0.05, the null hypothesis was rejected and the alternative hypothesis was accepted. Therefore, there was a significant difference in students' mathematical communication ability based on self-efficacy levels. In addition to statistical significance, the effect size was calculated using epsilon squared. The effect size value was 0.49, indicating a large effect. This result shows that the difference in students' mathematical communication ability across self-efficacy levels was not only statistically significant, but also had a

substantial magnitude. Thus, self-efficacy level was substantially associated with differences in students' mathematical communication ability in fraction learning. Since the Kruskal-Wallis test showed a significant difference, pairwise comparisons with Bonferroni adjustment were conducted to identify which self-efficacy groups differed significantly in mathematical communication ability. The results of the pairwise comparisons are presented in Table 7 below.

Table 7. Pairwise Comparisons of Mathematical Communication Ability Across Self-Efficacy Levels

Pairwise Comparison	Test Statistic	Std. Error	Std. Test Statistic	Sig.	Adjusted Sig.	Decision
Low–Moderate	-55.345	9.453	-5.855	< 0.001	< 0.001	Significant
Low–High	-85.258	9.289	-9.179	< 0.001	< 0.001	Significant
Moderate–High	-29.914	9.453	-3.164	0.002	0.005	Significant

Table 7 shows that all pairwise comparisons among the self-efficacy groups were statistically significant. The low self-efficacy group differed significantly from the moderate self-efficacy group, with an adjusted significance value of < 0.001. The low self-efficacy group also differed significantly from the high self-efficacy group, with an adjusted significance value of < 0.001. In addition, the moderate self-efficacy group differed significantly from the high self-efficacy group, with an adjusted significance value of 0.005. These results indicate that students' mathematical communication ability differed significantly across all self-efficacy levels. To provide a visual summary of the pairwise comparison results, the mean rank differences among the self-efficacy groups are presented in Figure 1.

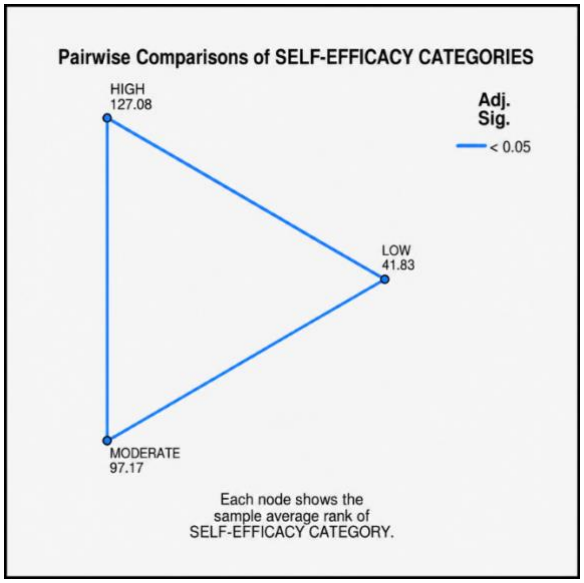


Figure 2. Pairwise Comparisons of Mathematical Communication Ability Based on Self-Efficacy Levels

Figure 1 shows that the high self-efficacy group had the highest mean rank, namely 127.08, followed by the moderate self-efficacy group with a mean rank of 97.17, while the low self-efficacy group had the lowest mean rank, namely 41.83. The connecting lines in the figure indicate that all pairwise comparisons were statistically significant based on the adjusted significance value of < 0.05. Thus, the results presented in Tables 6 and 7 and Figure 1 strengthen the finding that the higher the students' self-efficacy, the better their mathematical communication ability in fraction learning. The pairwise comparison results further support the descriptive analysis and the Kruskal-Wallis test findings. Students in the high self-efficacy group demonstrated the strongest mathematical communication ability, followed by those in the moderate self-efficacy group, while students with

low self-efficacy showed the lowest performance. These results indicate that stronger self-efficacy is related to better mathematical communication ability in fraction learning.

Table 8. Spearman's Rank Correlation Between Self-Efficacy and Mathematical Communication Ability

Variables	Spearman's rho	Sig.	N	Decision
Self-efficacy score and mathematical communication ability	0.729	< 0.001	176	Significant

Spearman's rank correlation was also applied to provide additional support for the analysis by examining the relationship between students' initial self-efficacy scores and their mathematical communication ability scores. The analysis revealed a significant positive correlation between self-efficacy and mathematical communication ability, with $\rho = 0.729$ and $p < 0.001$. This result suggests that students who had higher self-efficacy scores were more likely to achieve higher scores in mathematical communication ability. Accordingly, the correlation analysis reinforces the Kruskal-Wallis results, which indicated significant differences in mathematical communication ability among students with low, moderate, and high self-efficacy levels.

Discussion

The findings of this study show that students' mathematical communication ability in fraction learning differed significantly across self-efficacy levels. Students with high self-efficacy obtained the highest mathematical communication scores, followed by students with moderate self-efficacy, while students with low self-efficacy obtained the lowest scores. This pattern was consistently reflected in the descriptive statistics, mean rank results, Kruskal-Wallis test, and pairwise comparisons. These results indicate that self-efficacy is associated with students' ability to communicate mathematical ideas in fraction learning. However, since this study used an ex post facto design, the findings should not be interpreted as evidence of a causal effect. Rather, the results show that students with different levels of self-efficacy also demonstrated different levels of mathematical communication ability. The supplementary Spearman's rank correlation analysis further supports this finding by showing that students' original self-efficacy scores were positively and significantly associated with their mathematical communication ability scores. This result is important because it confirms that the association between self-efficacy and mathematical communication ability was not only observed after categorizing students into low, moderate, and high groups, but also appeared when self-efficacy was analyzed as an original continuous score. Therefore, the correlation result strengthens the interpretation that higher self-efficacy is associated with better mathematical communication ability in fraction learning.

This finding can be understood through self-efficacy theory, which explains that individuals' beliefs about their capabilities influence how they approach tasks, how much effort they invest, how persistent they are when facing difficulties, and how they respond to failure. In mathematics learning, students who believe that they are capable of solving a problem are more likely to attempt the task, explore strategies, monitor their work, and revise their answers when needed (Jiang et al., 2021; Nilimaa, 2023; Oudman et al., 2022; Williams & Williams, 2021). Conversely, students with low self-efficacy may stop working more quickly, avoid complex representations, or hesitate to write explanations because they doubt the correctness of their ideas. Thus, self-efficacy does not only relate to students' emotional readiness but also to their willingness to engage with the cognitive processes required in mathematical communication. Mathematical communication is a complex ability because it requires students to express mathematical thinking through different forms of representation. In this study, mathematical communication was measured through three indicators: drawing, written text, and mathematical expression. These indicators show that mathematical communication is not

limited to finding the correct answer but also involves representing problems visually, explaining solution processes in written language, and constructing mathematical models using symbols or mathematical sentences. This perspective suggests that mathematical communication involves translating mathematical information into language, symbols, and representations while using multiple representations to explain mathematical ideas and solve problems. The three indicators are also consistent with contemporary mathematics education frameworks that conceptualize mathematical communication through visual representations, written explanations, and mathematical expressions. Therefore, students' success in mathematical communication depends not only on conceptual understanding but also on their confidence to make their thinking visible through explanations, diagrams, and mathematical expressions.

The relationship between self-efficacy and mathematical communication can also be explained by the nature of communication tasks themselves. Tasks that require students to draw representations, write explanations, and construct mathematical expressions often demand more than procedural fluency. Students need to interpret the situation, choose appropriate representations, connect representations with mathematical symbols, and justify their reasoning. These processes may be particularly challenging for elementary school students when mathematical problems are presented in contextual forms. Students with higher self-efficacy are more likely to perceive such tasks as manageable challenges, whereas students with lower self-efficacy may perceive them as threatening or excessively difficult. Previous evidence suggests that mathematics self-efficacy develops through mastery experiences, observational learning, verbal encouragement, and learners' emotional or psychological conditions (Lao et al., 2025; Zakariya, 2022). These experiences may influence whether students feel sufficiently confident to explain their reasoning and persist when their initial solutions are incomplete or incorrect. The findings of this study are consistent with previous research demonstrating a positive relationship between self-efficacy and mathematical communication ability (Holenstein et al., 2022; Özdemir et al., 2021; Pérez-Fuentes et al., 2020). Previous studies have shown that students with stronger confidence in mathematics tend to communicate mathematical ideas more effectively than students with lower confidence. Other studies have likewise reported that mathematics self-efficacy is positively associated with students' mathematics achievement across different educational levels (Özcan & Kültür, 2021; Pérez-Fuentes et al., 2020; Sakellariou, 2022). Research on mathematical communication has also emphasized the importance of expressing mathematical ideas through written explanations, visual representations, and mathematical symbols. The present study extends this body of knowledge by showing that differences in mathematical communication ability can already be observed among fifth-grade elementary school students when they are grouped according to task-specific self-efficacy in fraction learning.

The context of fraction learning provides an important explanation for the pattern found in this study. Fractions are widely recognized as one of the most challenging topics in elementary mathematics because they require students to understand part-whole relationships, compare quantities, perform operations involving different denominators, and move flexibly among visual, verbal, and symbolic representations. Recent evidence indicates that fraction learning represents a fundamental component of mathematical development while simultaneously posing substantial conceptual challenges for elementary students. In fraction tasks, students are expected not only to calculate accurately but also to represent situations, justify procedures, and construct mathematical expressions that correspond to contextual situations. Studies have further suggested that combining verbal, symbolic, pictorial, and contextual representations supports students' conceptual understanding and performance in fraction learning. Students with higher self-efficacy may therefore be more willing to engage with these representational demands because they believe they

are capable of understanding the problem and completing the task. This may explain why students with higher self-efficacy in the present study demonstrated stronger mathematical communication ability during fraction learning. These findings also suggest that mathematical communication and self-efficacy should not be viewed as separate aspects of mathematics learning. Mathematical communication requires students to take intellectual risks by presenting ideas, explaining strategies, and justifying answers that may still be incomplete. Such behaviors are more likely to emerge when students possess confidence in their ability to learn and improve. Therefore, strengthening mathematical communication may require learning environments that simultaneously support students' cognitive development and affective readiness. Teachers should provide opportunities for students to use multiple representations, discuss solution strategies, receive constructive feedback, and experience success while solving mathematical problems. Through these learning experiences, students may gradually develop stronger confidence to communicate mathematical ideas, particularly in mathematical topics such as fractions that require flexible reasoning and multiple representations.

Implications

The findings of this study imply that self-efficacy should be considered an important affective factor in supporting students' mathematical communication ability, especially in fraction learning. Students with higher self-efficacy showed better ability to represent fraction problems, explain mathematical reasoning, and construct mathematical expressions. Therefore, mathematics instruction in elementary schools should not only focus on conceptual understanding and procedural skills, but also on building students' confidence to communicate their mathematical thinking. Teachers need to create learning situations that encourage students to try, explain, revise, and share their ideas without fear of making mistakes. These findings also have implications for the design of fraction learning activities. Fraction tasks should provide opportunities for students to use multiple representations, such as pictures, diagrams, written explanations, and mathematical symbols. Since mathematical communication involves drawing, written text, and mathematical expression, teachers can design tasks that explicitly ask students to show their thinking in these different forms. At the same time, scaffolding, guided questioning, peer discussion, and constructive feedback can be used to help students develop both confidence and communication ability. Such learning support is important because students with low self-efficacy may understand part of the concept but hesitate to express their reasoning clearly. In addition, the results suggest that teachers and researchers should pay attention to task-specific self-efficacy when assessing students' mathematical learning. Measuring students' confidence in completing specific mathematical communication tasks can provide useful information about their readiness to engage with challenging problems. This information can help teachers identify students who need additional support, not only in understanding fraction concepts but also in building confidence to represent, explain, and model mathematical situations. Thus, integrating self-efficacy support into mathematics instruction may contribute to more meaningful mathematical communication practices in elementary classrooms.

Limitations and Suggestions for Future Research

This study has several limitations that should be considered when interpreting the findings. First, the use of an ex post facto design allows the identification of differences and associations between task-specific self-efficacy and mathematical communication but does not permit causal conclusions. Second, the participants were limited to 176 fifth-grade students from four elementary schools in West Java, which may restrict the generalizability of the findings to other grade levels, regions, and educational contexts. Third, mathematical communication was assessed only through written tasks involving fraction addition and subtraction, focusing on drawing, written explanation, and mathematical expression, without capturing students' oral communication or classroom

interactions. Fourth, the categorization of self-efficacy into low, moderate, and high groups was based on the score distribution within the sample and should therefore be interpreted as relative rather than universal. Future studies are encouraged to involve larger and more diverse samples representing different geographical regions and school characteristics to improve external validity. Employing experimental or longitudinal designs would also provide stronger evidence regarding the development of task-specific self-efficacy and its relationship with mathematical communication over time. Future research may further incorporate mixed-method approaches to obtain deeper insights into students' cognitive and affective learning processes. In addition, assessing oral mathematical communication, collaborative discussions, and classroom discourse would provide a more comprehensive understanding of students' communication competence. Expanding the investigation to other mathematical topics beyond fractions may also determine whether the observed patterns are consistent across different conceptual domains. Researchers are further encouraged to examine additional learner characteristics, such as mathematics anxiety, motivation, metacognitive awareness, and self-regulated learning, that may interact with task-specific self-efficacy in shaping mathematical communication. Collectively, these directions will contribute to a more comprehensive understanding of the cognitive and affective factors that support mathematical communication in elementary mathematics education.

CONCLUSION

This study aimed to examine students' mathematical communication ability in fraction learning based on their self-efficacy levels. The findings showed that students' mathematical communication ability differed significantly across low, moderate, and high self-efficacy groups. Students with high self-efficacy demonstrated the highest mathematical communication ability, followed by students with moderate self-efficacy, while students with low self-efficacy showed the lowest ability. The Kruskal-Wallis test, effect size analysis, and pairwise comparisons confirmed that the differences among all self-efficacy groups were statistically significant and had a substantial magnitude. The supplementary Spearman's rank correlation analysis also showed a positive and significant association between students' original self-efficacy scores and their mathematical communication ability scores, supporting the findings obtained from the group comparison analysis. The differences were also reflected in the three indicators of mathematical communication ability, namely drawing, written text, and mathematical expression. Students with higher self-efficacy tended to obtain higher scores in representing fraction problems visually, explaining mathematical reasoning in written form, and constructing mathematical expressions. These findings indicate that self-efficacy is an important affective factor associated with students' mathematical communication ability in fraction learning.

The results of this study suggest that mathematics learning in elementary schools should provide opportunities for students to develop both mathematical communication ability and self-efficacy. Teachers can support students by designing fraction tasks that involve multiple representations, guided explanations, peer discussion, and constructive feedback. Such learning experiences may support students' confidence in communicating mathematical ideas and help them engage more actively with challenging mathematical tasks. Since this study used an ex post facto design, the findings should be interpreted as differences in mathematical communication ability across self-efficacy levels, not as evidence of a causal relationship. Future studies are recommended to involve broader samples, different grade levels, and various school contexts to strengthen the generalizability of the findings. Further research may also use experimental, longitudinal, or mixed-method designs to explore more deeply how self-efficacy develops and how it relates to students'

mathematical communication over time. In addition, future studies can examine oral mathematical communication and classroom interaction to obtain a more comprehensive understanding of students' mathematical communication in fraction learning.

AUTHOR CONTRIBUTIONS STATEMENT

Siti Muhdiati contributed to the conceptualization of the study, instrument development, data collection, data analysis, interpretation of the findings, and manuscript drafting. Sufyani Prabawanto contributed to research supervision, methodological guidance, data interpretation, and critical review of the manuscript. Sandi Budi Iriawan contributed to the theoretical framework, validation of the research direction, and manuscript revision. Effy Mulyasari provided academic supervision, guided the overall research process, and critically reviewed and refined the manuscript. All authors reviewed, approved, and agreed to the published version of the manuscript.

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