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Exploring learning obstacles in circle and chord geometry through diagnostic assessment: Evidence from indonesian senior high school students

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Abstract

Background: Learning obstacles remain a major challenge in mathematics education, particularly in circle and chord geometry, where students frequently encounter difficulties in conceptual understanding, formula application, and interpreting geometric representations. These difficulties often result in errors in mathematical problem-solving and indicate the need for a comprehensive analysis of students' learning obstacles.
Aim: This study aims to identify and classify students' learning obstacles in circle and chord geometry based on Brousseau's learning obstacle theory, encompassing ontogenic, didactical, and epistemological obstacles.
Methods: This study employed a descriptive qualitative approach involving three Grade 11 students from SMA Negeri 1 Kedamean, Indonesia, who were purposively selected based on the lowest diagnostic test scores and teacher recommendations. Data were collected through five open-ended diagnostic test items and semi-structured interviews and analyzed using data reduction, data display, and conclusion drawing, supported by methodological triangulation to ensure data credibility.
Results: The findings indicate that students experienced conceptual difficulties characterized by procedural formula application without conceptual reasoning, inadequate explanation of solution processes, dependence on familiar problem patterns, and limited understanding of problem statements.
Conclusion: The study identified epistemological, ontogenic, and combined epistemological–ontogenic learning obstacles, suggesting that students' academic achievement does not necessarily reflect their conceptual understanding of circle and chord geometry.

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INTRODUCTION

Mathematics is a fundamental subject in school education because it equips students with logical reasoning, analytical thinking, problem-solving skills, and the ability to make informed decisions in various academic and real-life contexts (Sari et al., 2025). Beyond developing computational competence, mathematics contributes to students' intellectual growth by fostering critical thinking, creativity, and systematic reasoning, which are essential competencies for lifelong learning (Fitriatien et al., 2025; Siswanto & Meiliasari, 2024; Yanti et al., 2022). Consequently, mathematics instruction should emphasize meaningful learning experiences that enable students to construct conceptual knowledge rather than merely memorize mathematical procedures. Such an approach encourages students to actively participate in the learning process, develop mathematical reasoning, and apply their knowledge flexibly to solve unfamiliar problems (Szabo et al., 2020; Torres-Peña et al., 2025). However, mathematics classrooms continue to be dominated by teacher-

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centered instruction, where knowledge is primarily transmitted from teachers to students, limiting opportunities for learners to construct their own mathematical understanding (Maharani et al., 2022). This instructional practice frequently results in superficial learning because students tend to imitate solution procedures without understanding the underlying mathematical concepts. As a consequence, students often encounter learning obstacles that hinder their ability to interpret mathematical ideas, connect related concepts, and apply appropriate reasoning when solving problems. These learning obstacles reduce students' capacity to transfer previously acquired knowledge into new mathematical situations, leading to repeated misconceptions and procedural errors. Therefore, identifying the factors that contribute to learning obstacles has become an important concern in mathematics education research because it provides valuable information for improving instructional practices and students' conceptual understanding. Understanding the nature of students' learning obstacles is therefore essential for developing more effective mathematics instruction that supports meaningful learning and sustainable conceptual development.

Learning obstacles refer to barriers that prevent students from achieving expected learning outcomes despite having participated in formal instruction and are frequently reflected in low academic achievement and persistent conceptual difficulties. According to learning obstacles emerge when students experience limitations in constructing and applying mathematical concepts appropriately across different learning situations. These obstacles may arise from students' cognitive readiness, instructional design, or the characteristics of mathematical knowledge itself, resulting in ontogenic, didactical, and epistemological obstacles that influence learning performance (Ario et al., 2025; Hendriyanto et al., 2024; Meika et al., 2025). Numerous studies have shown that students' errors in solving mathematical problems often provide meaningful evidence of underlying conceptual difficulties rather than merely indicating computational mistakes. Consequently, identifying learning obstacles is an important prerequisite for designing instructional strategies that effectively address students' learning needs and improve conceptual understanding. This perspective aligns with the implementation of Indonesia's Merdeka Curriculum, which emphasizes student-centered learning and the development of twenty-first-century competencies, including critical thinking, problem-solving, communication, and collaboration (Fitriatien et al., 2021). The Merdeka Curriculum encourages students to engage actively in meaningful learning experiences through inquiry, exploration, and contextual problem-solving activities rather than relying solely on procedural knowledge. Such educational principles are also consistent with UNESCO's vision of future education, which advocates learning environments that promote curiosity, conceptual exploration, creativity, and lifelong learning competencies. In the context of mathematics education, particularly in learning circle and chord geometry, these principles require students to develop conceptual understanding that enables them to explain mathematical relationships, interpret geometric representations, and justify solution strategies instead of depending exclusively on memorized formulas. Therefore, investigating students' learning obstacles in geometry has become increasingly important for ensuring that curriculum implementation effectively promotes meaningful mathematical understanding and improves students' learning outcomes.

Previous studies have extensively investigated students' learning obstacles in mathematics to identify factors contributing to limited conceptual understanding and persistent learning difficulties. Research conducted demonstrated that learning obstacles frequently emerge when students have not adequately mastered prerequisite knowledge and fundamental mathematical concepts, while reported that conceptual deficiencies systematically hinder students' ability to construct new mathematical understanding (Magfiroh et al., 2024). Similarly, that limited conceptual understanding constitutes the primary source of learning obstacles in trigonometry, whereas emphasized that learning obstacles are influenced not only by students' conceptual limitations but also by instructional practices that fail to facilitate meaningful knowledge construction (Ferede et al., 2026). Collectively, these studies indicate that learning obstacles consistently arise when mathematics instruction remains predominantly procedural, causing students to memorize algorithms without fully understanding the underlying concepts and relationships among mathematical ideas. In geometry education, particularly in circle and chord topics, students are expected to identify geometric elements accurately, explain the relationships among chords, arcs, central angles, and distances from the center, and apply these concepts in solving contextual mathematical problems. However, preliminary classroom observations conducted in Grade 11 at SMA Negeri 1 Kedamean revealed that many students experienced considerable difficulty in interpreting geometric representations, understanding the relationships among circle elements, selecting appropriate mathematical concepts, and explaining the rationale behind the formulas they applied. These findings suggest that students' understanding remains largely procedural rather than conceptual, despite having received formal instruction on circle and chord geometry. Although previous studies have examined learning obstacles in mathematical topics such as algebra, trigonometry, and other domains, relatively limited attention has been devoted to comprehensively investigating learning obstacles in circle and chord geometry at the senior high school level using a qualitative approach grounded in Brousseau's Learning Obstacle Theory. Furthermore, existing studies have generally focused on identifying students' errors or evaluating instructional interventions, whereas limited evidence is available regarding the integration of diagnostic assessment and qualitative interviews to explain the cognitive characteristics underlying students' learning obstacles in geometry. Therefore, a comprehensive investigation is required to identify and classify students' ontogenic, didactical, and epistemological learning obstacles in circle and chord geometry, thereby providing empirical evidence to support the development of instructional strategies that strengthen conceptual understanding and improve geometry learning in secondary mathematics education.

Although previous studies have substantially advanced the understanding of students' learning obstacles and conceptual difficulties in mathematics, several important research gaps remain. Existing studies have predominantly investigated learning obstacles in mathematical domains such as set theory, arithmetic, algebraic thinking, fractions, and probability, demonstrating the significance of epistemological, didactical, and ontogenic obstacles in shaping students' mathematical learning processes (Sidik et al., 2021; Wijaya et al., 2021). Meanwhile, research in geometry education has largely focused on enhancing students' conceptual understanding through instructional innovations, including GeoGebra-assisted learning, problem-based learning, virtual and augmented reality, collaborative learning environments, and technology integration, rather than examining the cognitive characteristics underlying students' learning obstacles (Gurmu et al., 2024; Su et al., 2022; Viberg et al., 2023). Furthermore, studies on diagnostic assessment have primarily emphasized the development of diagnostic models, assessment frameworks, and measurement instruments for identifying students' reasoning difficulties and misconceptions, with limited attention given to integrating diagnostic evidence with qualitative exploration to explain why learning obstacles emerge during mathematics learning (Alfageh et al., 2024; Säfström et al., 2024; Wu et al., 2020). Although several studies have reported students' difficulties in learning geometry, these investigations generally describe learning difficulties or evaluate instructional effectiveness without systematically classifying the obstacles according to Brousseau's Learning Obstacle Theory, particularly in the context of circle and chord geometry among senior high school students (Alghadari et al., 2020; Birgin & Topuz, 2021; Jablonski & Ludwig, 2023). Therefore, this study addresses these gaps by integrating diagnostic assessment and qualitative interviews to comprehensively identify and classify students' ontogenic, didactical, and epistemological learning obstacles in circle and chord geometry based on Brousseau's Learning Obstacle Theory, thereby providing empirical evidence to support more effective concept-oriented geometry instruction.

This study aims to explore the learning obstacles experienced by Indonesian senior high school students in learning circle and chord geometry through diagnostic assessment. Specifically, the study seeks to identify the characteristics of students' learning obstacles when solving geometry problems involving circles and chords. It further aims to classify these obstacles into ontogenic, didactical, and epistemological categories based on Brousseau's Learning Obstacle Theory. The study also intends to investigate the underlying cognitive factors that contribute to students' conceptual and procedural difficulties in geometry learning. By integrating diagnostic test results with semi-structured interview data, this research provides a more comprehensive understanding of students' mathematical thinking processes than could be achieved through written responses alone. Furthermore, the study examines how different types of learning obstacles influence students' ability to interpret geometric representations, select appropriate solution strategies, and justify mathematical reasoning. The findings are expected to enrich the existing literature on learning obstacles in mathematics education, particularly in the context of geometry learning. This study also seeks to provide empirical evidence regarding the usefulness of diagnostic assessment for identifying students' conceptual difficulties before instructional intervention. In addition, the findings are expected to assist mathematics teachers in designing learning experiences that address students' specific learning needs and promote deeper conceptual understanding. Ultimately, this research contributes to the development of evidence-based instructional strategies that support meaningful geometry learning and improve mathematics education in secondary schools.

LITERATURE REVIEW

Learning obstacle has become one of the most widely discussed theoretical perspectives in mathematics education because it provides a comprehensive explanation of the factors that hinder students from developing meaningful conceptual understanding during the learning process. According to Brousseau's Learning Obstacle Theory, students' learning difficulties cannot be interpreted solely as a consequence of low academic ability but should also be understood in relation to the characteristics of mathematical knowledge, instructional practices, and learners' cognitive development. Numerous studies have demonstrated that learning obstacles are closely associated with students' limited conceptual understanding and their inability to connect previously acquired knowledge with new mathematical concepts. Epistemological obstacles frequently emerge because students fail to establish coherent prerequisite conceptual structures, resulting in persistent errors when solving more advanced mathematical problems. Learning obstacles have also been shown to originate not only from students' conceptual limitations but also from instructional materials and learning designs that are insufficient to facilitate meaningful conceptual knowledge construction. Similar patterns have been identified in sequence and series learning, where students experience considerable difficulty relating mathematical representations to abstract concepts and consequently rely on procedural algorithms without understanding their mathematical meaning (Acosta et al., 2024; Becker et al., 2023). Students likewise frequently commit errors in solving trigonometric equations because of inadequate mastery of fundamental concepts, incorrect formula selection, and inappropriate algebraic manipulation (Khairunnisa et al., 2022). Furthermore, learning obstacles in statistics are influenced by students' learning readiness, conceptual misconceptions, and instructional strategies that have not fully supported meaningful knowledge construction. Collectively, these findings indicate that learning obstacles consistently occur across various mathematical topics and are predominantly associated with weaknesses in conceptual understanding rather than merely procedural performance. Therefore, understanding the characteristics and underlying causes of learning obstacles is essential for developing instructional approaches that effectively promote conceptual understanding and improve students' mathematical reasoning abilities.

Despite the substantial body of research on learning obstacles in mathematics, previous investigations have primarily concentrated on topics such as trigonometric derivatives, inverse functions, sequences and series, trigonometric equations, and statistics, while relatively little attention has been devoted to circle and chord geometry at the senior high school level. This imbalance is noteworthy because learning circle and chord geometry requires students to integrate visual representations, geometric reasoning, spatial visualization, and conceptual understanding simultaneously, making it one of the more cognitively demanding topics in school mathematics. Existing studies have largely identified learning obstacles through written tests and interviews, providing valuable information regarding students' errors but offering limited insight into the multidimensional factors underlying these difficulties (Saha et al., 2024; Taye & Mengesha, 2024). In contrast, integrating diagnostic assessment, learning obstacle questionnaires, and semi-structured interviews through methodological triangulation enables a more comprehensive investigation of students' cognitive processes and the origins of their learning obstacles. Such an approach provides richer evidence for distinguishing whether students' difficulties originate from cognitive readiness, instructional factors, or limitations in conceptual knowledge according to Brousseau's theoretical framework. Moreover, only a limited number of studies have examined learning obstacles within the implementation of Indonesia's Merdeka Curriculum, which emphasizes conceptual understanding, problem-solving, and student-centered learning through instructional approaches such as Problem-Based Learning (PBL). Since the curriculum encourages students to actively construct mathematical knowledge rather than memorize formulas, identifying learning obstacles has become increasingly important for evaluating the effectiveness of contemporary instructional practices. Consequently, investigating learning obstacles in circle and chord geometry extends the existing literature beyond merely identifying students' mathematical errors by providing a deeper understanding of the cognitive and instructional factors that shape conceptual learning. Such knowledge is expected to support teachers in designing more responsive instructional interventions that address students' specific learning needs while strengthening conceptual understanding in geometry. Therefore, the present study contributes to mathematics education by expanding the application of Brousseau's Learning Obstacle Theory to circle and chord geometry through an integrated diagnostic assessment approach within the context of Indonesian secondary education.

Learning obstacles in geometry deserve particular attention because geometry learning requires students to coordinate multiple forms of mathematical representation, including verbal explanations, symbolic expressions, numerical calculations, and visual interpretations simultaneously. Unlike many other mathematical topics that primarily emphasize symbolic manipulation, geometry requires students to construct logical relationships among geometric properties before selecting appropriate solution strategies. Specifically, students are expected to understand the relationships among chords, arcs, central angles, perpendicular bisectors, and distances from the center while applying these concepts to solve both routine and non-routine problems. These expectations indicate that successful learning depends not only on procedural fluency but also on students' ability to integrate conceptual understanding with spatial reasoning and mathematical justification (Fitriatien, 2020). Nevertheless, many students continue to experience difficulties in interpreting geometric diagrams, connecting visual representations with mathematical concepts, and explaining the rationale behind the formulas they apply. These difficulties frequently lead students to memorize formulas without understanding the conceptual relationships that govern geometric properties, making them vulnerable to errors when encountering unfamiliar problem situations. From the perspective of Brousseau's Learning Obstacle Theory, these recurring misconceptions indicate epistemological obstacles because students' existing knowledge cannot be appropriately transferred to different mathematical contexts. In addition, insufficient cognitive readiness may simultaneously generate ontogenic obstacles that limit students' ability to organize mathematical reasoning during problem-solving activities. Consequently, investigating learning obstacles in geometry requires a comprehensive approach that considers both students' conceptual understanding and the cognitive processes underlying their mathematical reasoning. Such an approach provides a stronger theoretical foundation for improving geometry instruction and promoting deeper conceptual learning in secondary mathematics education.

Diagnostic assessment has increasingly been recognized as an effective approach for identifying students' conceptual understanding and revealing misconceptions before they become persistent learning barriers. Unlike conventional achievement tests that primarily measure learning outcomes, diagnostic assessment is specifically designed to identify the sources of students' conceptual difficulties and provide detailed information regarding their learning needs. The information generated through diagnostic assessment enables teachers to distinguish whether students' errors originate from misconceptions, insufficient prerequisite knowledge, inappropriate reasoning strategies, or limitations in cognitive readiness. When combined with qualitative methods such as semi-structured interviews, diagnostic assessment provides richer evidence regarding students' mathematical thinking processes because written responses alone often fail to explain why particular errors occur. Furthermore, integrating multiple data collection techniques allows researchers to validate students' responses through methodological triangulation, thereby improving the credibility and trustworthiness of qualitative findings (Biddix & Bourke, 2026; Oranga, 2025). Such triangulation is particularly valuable in mathematics education because learning obstacles frequently emerge from complex interactions among conceptual understanding, instructional experiences, and cognitive development that cannot be adequately explained using a single source of evidence. Consequently, integrating diagnostic tests, learning obstacle questionnaires, and interviews provides a more comprehensive framework for identifying students' learning obstacles than relying exclusively on achievement scores or written tests. This integrated approach also enables researchers to classify learning obstacles more accurately according to Brousseau's theoretical categories while simultaneously explaining the cognitive characteristics associated with each obstacle type. Therefore, diagnostic assessment should be viewed not merely as an evaluation instrument but also as an important pedagogical tool for informing instructional decision-making and improving mathematics learning. Such comprehensive identification is particularly relevant for geometry learning, where conceptual understanding and visual reasoning play central roles in successful mathematical problem-solving.

The literature reviewed above demonstrates that learning obstacles constitute a multidimensional phenomenon influenced by students' conceptual understanding, cognitive readiness, instructional practices, and the characteristics of mathematical knowledge itself. Previous studies have consistently confirmed the existence of learning obstacles across diverse mathematical topics while emphasizing the importance of strengthening conceptual understanding through more effective instructional approaches (Khairunnisa et al., 2022). Nevertheless, the existing literature provides relatively limited evidence regarding the characteristics of learning obstacles encountered by senior high school students when learning circle and chord geometry, particularly within the context of Indonesian mathematics education. Moreover, previous investigations have generally emphasized identifying students' errors or evaluating instructional innovations, whereas fewer studies have attempted to explain the underlying cognitive mechanisms responsible for those errors through an integrated qualitative diagnostic approach. The present study addresses this limitation by combining diagnostic assessment, learning obstacle questionnaires, and semi-structured interviews to investigate students' learning obstacles comprehensively. Guided by Brousseau's

Learning Obstacle Theory, the study examines how ontogenic, didactical, and epistemological obstacles manifest during students' engagement with circle and chord geometry problems. This theoretical perspective provides a systematic framework for interpreting students' mathematical thinking beyond the identification of procedural mistakes alone. The findings are expected to enrich the literature on learning obstacles in mathematics education while extending the application of Brousseau's theoretical framework to geometry learning at the secondary school level. Furthermore, the study provides empirical evidence that may support teachers in designing instructional strategies capable of reducing conceptual difficulties and promoting meaningful mathematical understanding. Accordingly, this literature review establishes the theoretical foundation for examining students' learning obstacles through diagnostic assessment and supports the methodological framework employed in the present study.

METHOD

Research Design

This study employed a descriptive qualitative research design to obtain an in-depth understanding of the learning obstacles experienced by senior high school students when learning circle and chord geometry. A qualitative approach was considered appropriate because the study sought to explore students' conceptual understanding, mathematical reasoning, and cognitive processes rather than examine causal relationships among variables. The study was grounded in Brousseau's Learning Obstacle Theory, which classifies learning obstacles into ontogenic, didactical, and epistemological categories. This theoretical framework enabled the researchers to interpret students' learning difficulties from multiple perspectives, including cognitive readiness, instructional practices, and conceptual knowledge construction. Qualitative evidence was collected through diagnostic assessment, learning obstacle questionnaires, and semi-structured interviews to provide a comprehensive understanding of students' learning experiences. The integration of multiple qualitative data sources allowed the researchers to examine learning obstacles beyond students' written responses and explore the reasoning underlying their mathematical decisions. Consequently, the research emphasized describing and interpreting students' learning obstacles rather than testing hypotheses or measuring statistical relationships. This design provides an appropriate methodological framework for investigating conceptual difficulties in geometry learning and generating practical implications for mathematics instruction.

Participants

The study was conducted at SMA Negeri 1 Kedamean, Indonesia, during the 2025/2026 academic year. The initial participants consisted of thirty-two Grade 11 students from class XI-5 who completed the diagnostic assessment on circle and chord geometry. Participant selection followed a purposive sampling strategy because qualitative research requires information-rich participants capable of providing detailed insights into the investigated phenomenon. Three selection criteria were established, namely low diagnostic test performance, indications of learning obstacles based on the learning obstacle questionnaire, and recommendations from the mathematics teacher regarding persistent conceptual difficulties. Based on these criteria, three students demonstrating different characteristics of learning obstacles were selected as the research participants and were coded as S1, S2, and S3 to preserve anonymity. The selected participants represented diverse conceptual difficulties, enabling a comprehensive exploration of students' learning obstacles. Prior to participation, permission to conduct the study was obtained from the school administration, while students participated voluntarily throughout the research process. The confidentiality of all participants was maintained by replacing students' identities with research codes during data analysis and reporting.

Instruments

Three complementary research instruments were employed, including a diagnostic test, a learning obstacle questionnaire, and semi-structured interview guidelines. The diagnostic assessment consisted of five open-ended questions specifically designed to evaluate students' conceptual understanding and identify learning obstacles in circle and chord geometry. The questions required students to interpret geometric representations, explain mathematical relationships, justify solution procedures, and solve contextual geometry problems rather than simply perform numerical calculations. A learning obstacle questionnaire containing thirty statements was subsequently administered to identify students' perceptions of their conceptual difficulties based on the indicators of ontogenic, didactical, and epistemological obstacles proposed by Brousseau. The questionnaire adopted a four-point Likert scale consisting of Strongly Agree, Agree, Disagree, and Strongly Disagree to facilitate descriptive analysis of students' responses. Semi-structured interviews were then conducted with the selected participants to clarify diagnostic test responses, investigate students' reasoning processes, and explore the cognitive factors underlying their learning obstacles. Prior to implementation, all research instruments underwent content validation by two experts consisting of a mathematics education lecturer from Universitas PGRI Adi Buana Surabaya and an experienced mathematics teacher from SMA Negeri 1 Kedamean. The validation process confirmed that the instruments were appropriate for identifying students' learning obstacles and collecting qualitative evidence. Data collection was carried out sequentially through diagnostic testing, questionnaire administration, participant selection, and individual interviews, allowing methodological triangulation to strengthen the credibility of the findings.

Data Analysis

Qualitative data were analyzed using the interactive analysis model developed by Miles and Huberman, which consists of data reduction, data display, and conclusion drawing. During the data reduction stage, students' written responses, questionnaire results, and interview transcripts were carefully reviewed to identify meaningful information related to learning obstacles. Relevant data were subsequently coded and classified into ontogenic, didactical, and epistemological obstacles according to Brousseau's Learning Obstacle Theory. The categorized findings were then organized into descriptive narratives and supporting tables to facilitate systematic interpretation. Cross-case comparisons among participants were conducted to identify similarities and differences in the characteristics of students' learning obstacles. Finally, conclusions were drawn by integrating evidence obtained from the diagnostic assessment, questionnaire responses, and interview findings. To enhance the trustworthiness of the study, methodological triangulation was applied by comparing information obtained from the three data sources, thereby increasing the credibility and consistency of the qualitative interpretations.

Research Procedure

The research was implemented in three consecutive phases: preparation, data collection, and data analysis. During the preparation phase, preliminary classroom observations were conducted while students participated in Problem-Based Learning (PBL) activities to identify potential learning obstacles in circle and chord geometry. Based on these observations, the researchers developed the diagnostic test, learning obstacle questionnaire, and semi-structured interview protocol, all of which were validated by two experts before implementation. After obtaining approval from the school, the diagnostic assessment was administered to thirty-two Grade 11 students, followed by the administration of the learning obstacle questionnaire. The results of these instruments, together with recommendations from the mathematics teacher, were used to select three participants through purposive sampling. Semi-structured interviews were subsequently conducted individually to obtain deeper explanations regarding students' conceptual understanding, reasoning strategies, and learning difficulties. Following data collection, all qualitative evidence was analyzed using the Miles and Huberman interactive model, while methodological triangulation was employed to verify the consistency of the findings across the three research instruments. The entire research procedure ensured that the identified learning obstacles were supported by multiple sources of qualitative evidence, thereby improving the validity, credibility, and trustworthiness of the study.

Figure 1. Research Method Flow

RESULTS AND DISCUSSION

Results

Diagnostic Assessment Performance and Participant Selection

A diagnostic assessment consisting of five open-ended questions on circle and chord geometry was administered to 32 Grade 11 students to identify their conceptual understanding and potential learning obstacles. The assessment results revealed considerable variation in students' performance, with scores ranging from 5 to 74, indicating substantial differences in conceptual mastery among participants. Most students obtained scores between 50 and 65, suggesting that although they were able to solve some procedural tasks, many still experienced conceptual difficulties when interpreting geometric relationships and selecting appropriate solution strategies. Conversely, several students achieved extremely low scores, indicating significant challenges in understanding the fundamental concepts of circle and chord geometry. These findings suggest that students' mathematical performance was not homogeneous and that varying levels of conceptual understanding existed within the same classroom. Therefore, the diagnostic assessment provided an appropriate basis for identifying students who required further qualitative investigation. In addition to the diagnostic assessment, a learning obstacle questionnaire was administered to all participants to obtain complementary information regarding students' perceived learning difficulties. The questionnaire findings were subsequently compared with the diagnostic assessment results and mathematics teacher recommendations to strengthen participant selection through methodological triangulation. The integration of these three sources of evidence enabled the researchers to identify students who consistently demonstrated conceptual difficulties across different forms of assessment. Consequently, three participants representing different characteristics of learning obstacles were purposively selected for detailed qualitative analysis.

Table 1. Descriptive Statistics of Diagnostic Test Scores

Variable	Value
Number of students	32
Highest score	74
Lowest score	5
Score range	69
Dominant score interval	50–65
Number of selected participants	3

Note: The diagnostic assessment consisted of five open-ended questions designed to evaluate students' conceptual understanding of circle and chord geometry.

To ensure that the selected participants represented the characteristics of students experiencing learning obstacles, participant selection was conducted using three complementary criteria, namely diagnostic test performance, learning obstacle questionnaire results, and recommendations provided by the mathematics teacher. The diagnostic assessment served to identify students' conceptual understanding, while the questionnaire captured students' perceptions regarding the difficulties they experienced during mathematics learning. Teacher recommendations provided additional evidence based on classroom observations of students' learning behaviors and academic performance. Combining these three sources of information reduced the possibility of selecting participants solely on the basis of test scores and provided a more comprehensive representation of students' learning obstacles. This triangulation strategy strengthened the credibility of participant selection because each participant consistently demonstrated learning difficulties across multiple sources of evidence. Based on these criteria, three students were selected as research participants and assigned the codes S1, S2, and S3 to maintain confidentiality. The selected participants exhibited different patterns of conceptual difficulties, enabling a comprehensive exploration of learning obstacles from multiple perspectives. This variation was considered important because qualitative research aims to investigate the characteristics of a phenomenon rather than to generalize statistical findings. The profiles of the selected participants are presented in Table 2.

Table 2. Characteristics of Selected Participants

Participant	Initial	PSAS Score	Diagnostic Test Score	Selection Basis
S1	ENCS	7	5	Low diagnostic score, questionnaire results, and teacher recommendation
S2	RDA	10	10	Low diagnostic score, questionnaire results, and teacher recommendation
S3	EMP	21	48	Low diagnostic score, questionnaire results, and teacher recommendation

Classification of Learning Obstacles

The integration of diagnostic assessment results, learning obstacle questionnaire responses, and semi-structured interview findings revealed that the three selected participants demonstrated distinct patterns of learning obstacles in circle and chord geometry. Although all participants experienced conceptual difficulties, the characteristics and underlying causes of these difficulties differed considerably. The qualitative analysis showed that students' learning obstacles were not limited to procedural errors but also involved conceptual misunderstandings, inadequate cognitive readiness, and limited mathematical reasoning. Furthermore, the findings indicate that similar classroom instruction may produce different learning responses depending on students' prior knowledge, conceptual understanding, and cognitive preparedness. These results support Brousseau's proposition that learning obstacles originate from multiple dimensions rather than from students' academic achievement alone. Consequently, each participant was classified according to the dominant characteristics of learning obstacles identified through triangulation of diagnostic tests, questionnaires, and interview data. The classification demonstrates that learning obstacles manifested as epistemological obstacles, ontogenic obstacles, or a combination of both categories. Interestingly, no participant exhibited didactical obstacles as the dominant category because the observed learning difficulties were primarily associated with conceptual understanding and cognitive readiness rather than instructional design. This finding suggests that students' conceptual development remains the primary challenge in learning circle and chord geometry. The classification of learning obstacles for each participant is summarized in Table 3.

Table 3. Classification of Learning Obstacles

Participant	Diagnostic Test Score	Dominant Findings	Learning Obstacle Category
S1	5	Unable to identify mathematical information, concepts, and solution strategies; relied on trial-and-error calculations; demonstrated low cognitive readiness	Epistemological and Ontogenic
S2	10	Solved only the first problem; discontinued subsequent problems because of confusion and low confidence	Ontogenic
S3	48	Selected appropriate formulas but relied on memorized procedures without conceptual justification	Epistemological

The classification presented in Table 3 indicates that each participant demonstrated unique characteristics of learning obstacles despite studying the same mathematical topic. Participant S1 exhibited the most complex learning profile because conceptual misconceptions were accompanied by inadequate cognitive readiness, preventing systematic problem-solving. In contrast, Participant S2 primarily experienced ontogenic obstacles, as reflected in the inability to continue solving unfamiliar problems despite demonstrating limited procedural knowledge at the beginning of the assessment. Participant S3 demonstrated a different pattern by successfully recalling mathematical formulas but experiencing difficulty explaining the conceptual relationships underlying the solution procedures. These findings indicate that students' mathematical errors should not be interpreted solely as evidence of inadequate procedural skills but should also be understood from the perspectives of conceptual understanding and cognitive development. Moreover, the differences observed among participants suggest that students with relatively similar academic performance may experience fundamentally different learning obstacles. Therefore, effective mathematics instruction should be designed to accommodate the specific characteristics of students' conceptual difficulties rather than applying uniform remediation strategies. The variation identified in this study further highlights the importance of diagnostic assessment in distinguishing the nature of students' learning obstacles before instructional intervention is implemented. These findings provide the foundation for the individual case analyses presented in the following sections, which explain the characteristics of each learning obstacle category in greater detail.

Individual Case Analysis

Case 1: Participant S1 (Epistemological and Ontogenic Obstacles)

The diagnostic assessment and interview findings consistently demonstrated that Participant S1 experienced the most severe learning obstacles among the three participants. Across all five diagnostic problems, the participant was unable to identify relevant mathematical information, determine appropriate concepts, or formulate systematic solution strategies. Rather than applying geometric principles, the participant attempted various arithmetic operations using the numerical values presented in the questions without understanding their mathematical relationships. This trial-and-error strategy indicates that the participant lacked a conceptual framework for interpreting circle and chord geometry. Interview data further revealed that the participant experienced confusion immediately after reading each problem and was unable to recall previously learned concepts related to chords, central angles, arcs, and the Pythagorean theorem. The participant also reported uncertainty regarding how to begin solving the problems and admitted to guessing possible operations because no clear solution strategy could be identified. These findings suggest that the participant's difficulties extended beyond conceptual misconceptions and included insufficient cognitive readiness to organize mathematical reasoning. According to Brousseau's Learning Obstacle Theory, such characteristics indicate the simultaneous presence of epistemological and ontogenic obstacles. Consequently, Participant S1 demonstrated the most comprehensive pattern of learning obstacles among all research participants.

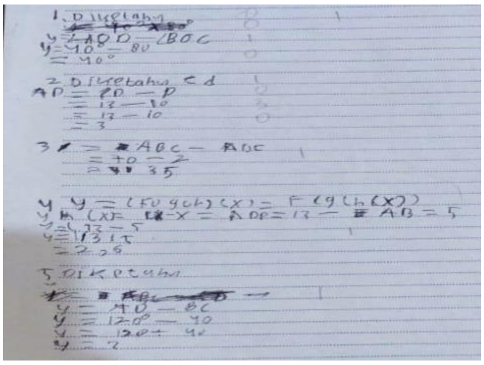

Figure 2. Representative diagnostic test responses of Participant S1 illustrating combined epistemological and ontogenic learning obstacles.

Case Analysis of Participant S2

Figure 3. Representative Diagnostic Test Responses of Participant S2

Participant S2 demonstrated a learning pattern that differed substantially from that of Participant S1. The diagnostic assessment showed that the participant successfully completed only the first problem using an appropriate mathematical formula, although no conceptual explanation was provided to justify the selected solution strategy. For the remaining four questions, the participant left the answer sheets blank and did not attempt any written solution. The interview findings revealed that the participant experienced confusion after encountering the first problem and perceived the subsequent questions as considerably more difficult. Rather than attempting alternative solution strategies, the participant decided to discontinue the assessment because of uncertainty regarding the appropriate mathematical concepts to apply. The participant further explained that low self-confidence and fear of producing incorrect answers discouraged further attempts to solve the remaining problems. Consequently, the absence of written responses limited the opportunity to observe the participant's mathematical reasoning directly through the diagnostic test. Nevertheless, the interview provided clear evidence that the primary difficulty was associated with cognitive readiness and affective responses rather than conceptual misconceptions or procedural errors. According to Brousseau's Learning Obstacle Theory, these characteristics are consistent with **ontogenic obstacles**, in which students' cognitive preparedness and psychological readiness hinder meaningful engagement with mathematical problem-solving. Therefore, Participant S2 was classified as predominantly experiencing **ontogenic learning obstacles**, characterized by low learning readiness, uncertainty, and reduced confidence when confronted with unfamiliar geometry problems.

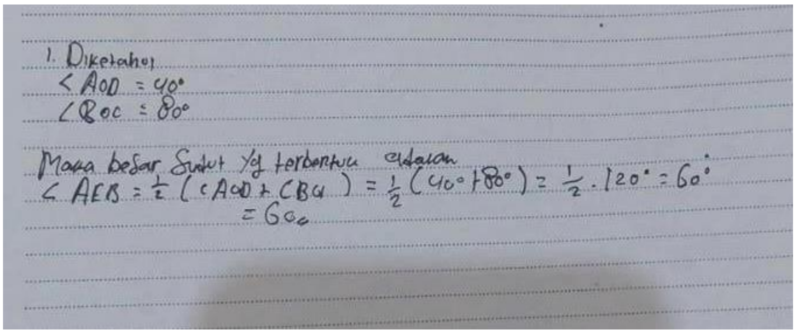

Figure 3. Representative diagnostic test responses of Participant S2 illustrating dominant ontogenic learning obstacles.

Case Analysis of Participant S3

Figure 4. Representative Diagnostic Test Responses of Participant S3

Compared with the other participants, Participant S3 demonstrated relatively better procedural performance during the diagnostic assessment. The participant correctly selected appropriate formulas and successfully completed several computational procedures, resulting in a substantially higher diagnostic test score. However, detailed analysis of the written responses and interview transcripts indicated that successful problem solving was primarily based on memorized procedures rather than conceptual understanding. When asked to explain the mathematical relationships among central angles, chords, arcs, and the application of the Pythagorean theorem, the participant was unable to provide coherent conceptual justifications. Instead, the participant reported relying on previously encountered examples and applying formulas that appeared similar to those used in classroom exercises. This strategy was effective for familiar problems but became ineffective when the mathematical structure differed from examples encountered during instruction. Consequently, several inconsistencies appeared in the participant's solution procedures, including incomplete reasoning and inaccurate mathematical interpretations despite correct formula selection. These findings indicate that the participant possessed procedural knowledge but lacked sufficient conceptual understanding to explain why particular mathematical procedures were appropriate. From the perspective of Brousseau's Learning Obstacle Theory, such characteristics represent epistemological obstacles, where previously acquired knowledge is insufficient for solving mathematical problems presented in new contexts. Therefore, Participant S3 was classified as predominantly experiencing epistemological learning obstacles, reflecting limitations in conceptual understanding rather than deficiencies in learning readiness.

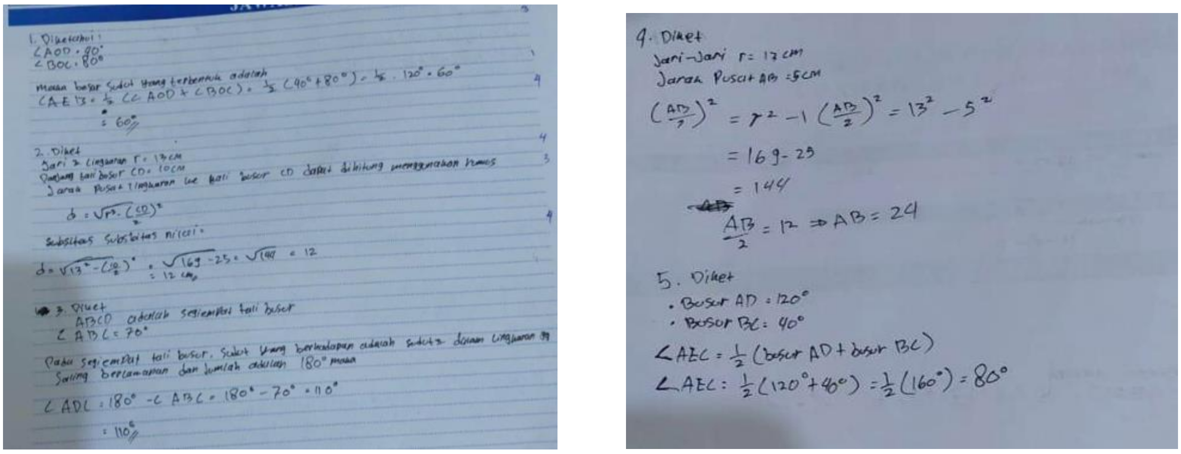

Figure 4. Representative diagnostic test responses of Participant S3 illustrating dominant epistemological learning obstacles.

Cross-Case Analysis of Learning Obstacles

To obtain a comprehensive understanding of students' learning obstacles, a cross-case analysis was conducted by comparing the diagnostic assessment results, questionnaire responses, and interview findings across the three participants. Although all participants experienced difficulties in learning circle and chord geometry, the characteristics and underlying causes of these difficulties differed considerably. Participant S1 demonstrated the most comprehensive learning obstacle profile because conceptual misconceptions were accompanied by insufficient cognitive readiness, preventing the participant from identifying mathematical information, selecting appropriate concepts, and organizing systematic solution strategies. In contrast, Participant S2 primarily experienced ontogenic obstacles, as reflected by the inability to continue solving unfamiliar problems despite demonstrating limited procedural understanding during the initial stage of the assessment. Participant S3 exhibited a different pattern by successfully recalling mathematical formulas while experiencing considerable difficulty explaining the conceptual relationships underlying the solution procedures. These findings indicate that students with similar learning experiences may demonstrate substantially different cognitive characteristics during mathematical problem solving. Consequently, learning obstacles should not be interpreted solely from students' achievement scores but should also consider conceptual understanding, cognitive readiness, and mathematical reasoning. The variation observed among participants further demonstrates that diagnostic assessment combined with qualitative interviews provides richer information than written tests alone for identifying students' learning obstacles. Overall, the findings confirm that epistemological and ontogenic obstacles were the dominant categories identified in this study, whereas didactical obstacles were not observed as the primary source of students' conceptual difficulties.

Table 4. Cross-Case Comparison of Learning Obstacles

Aspect	S1	S2	S3
Understanding problem statements	Low	Low	Moderate
Conceptual understanding	Very Low	Low	Moderate
Formula selection	Incorrect	Limited	Appropriate
Conceptual justification	Very Low	Not demonstrated	Limited
Problem-solving strategy	Trial and error	No strategy	Procedural
Cognitive readiness	Low	Very Low	Moderate
Mathematical reasoning	Very Low	Low	Moderate
Dominant learning obstacle	Epistemological + Ontogenic	Ontogenic	Epistemological

Triangulation of Diagnostic Test, Questionnaire, and Interview Findings

To improve the credibility of the qualitative findings, methodological triangulation was performed by comparing information obtained from the diagnostic assessment, learning obstacle questionnaire, and semi-structured interviews for each participant. The triangulation process demonstrated a high level of consistency among the three sources of evidence in identifying students' learning obstacles. The diagnostic assessment revealed students' observable performance in solving circle and chord geometry problems, whereas the questionnaire provided additional information regarding students' perceptions of their learning difficulties. Furthermore, the interview findings clarified the reasoning processes underlying students' written responses and explained why particular errors occurred during problem solving. For Participant S1, all three instruments consistently indicated severe conceptual difficulties accompanied by low cognitive readiness, resulting in combined epistemological and ontogenic obstacles. Similarly, Participant S2 consistently demonstrated limited learning readiness, uncertainty, and low confidence, confirming the dominance of ontogenic obstacles across all data sources. In contrast, Participant S3 showed relatively better procedural performance on the diagnostic assessment; however, both the questionnaire and interviews revealed that conceptual understanding remained insufficient, supporting the classification of epistemological obstacles. The consistency observed across the three instruments indicates that no single source of evidence alone was sufficient to explain students' learning obstacles comprehensively. Instead, integrating diagnostic assessment, questionnaire responses, and interview data provided a more complete understanding of the characteristics and underlying causes of students' conceptual difficulties. Therefore, methodological triangulation strengthened the trustworthiness of the findings and increased confidence in the learning obstacle classifications presented in this study.

Participant	Diagnostic Test	Questionnaire	Interview	Final Classification
S1	Very low conceptual understanding	High indication of conceptual and cognitive difficulties	Unable to explain concepts or solution strategies	Epistemological + Ontogenic
S2	Incomplete responses	High indication of low learning readiness	Low confidence and inability to continue solving problems	Ontogenic
S3	Procedural success	Moderate conceptual difficulties	Unable to justify mathematical concepts	Epistemological

Summary of Major Findings

The present study identified three distinct patterns of learning obstacles among senior high school students learning circle and chord geometry. First, students experienced epistemological obstacles when previously acquired mathematical knowledge was insufficient for interpreting new geometric situations and explaining conceptual relationships. These obstacles were characterized by inappropriate formula selection, fragmented conceptual understanding, and reliance on memorized procedures without mathematical justification. Second, ontogenic obstacles emerged when students demonstrated inadequate cognitive readiness and lacked confidence in engaging with unfamiliar mathematical problems, resulting in incomplete or abandoned solution attempts. Third, one participant exhibited a combination of epistemological and ontogenic obstacles, indicating that conceptual deficiencies and cognitive readiness may simultaneously influence students' mathematical performance. Across all participants, conceptual understanding was found to be a more influential factor than procedural ability in determining successful problem solving. The findings further demonstrate that students with comparable classroom learning experiences may develop substantially different learning obstacle profiles depending on their cognitive characteristics and prior conceptual knowledge. Moreover, diagnostic assessment alone was insufficient to explain these differences comprehensively, whereas integrating questionnaires and interviews provided richer evidence regarding students' reasoning processes and conceptual development. The absence of dominant didactical obstacles in this study suggests that the observed learning difficulties were primarily associated with students' conceptual understanding and cognitive readiness rather than deficiencies in classroom instruction. These findings reinforce the importance of employing multiple assessment methods to diagnose students' learning difficulties before designing instructional interventions. Overall, the results provide empirical evidence that learning obstacles in circle and chord geometry are multidimensional and require instructional strategies that address both conceptual understanding and students' cognitive preparedness.

Discussion

The present study demonstrates that learning obstacles in circle and chord geometry extend beyond procedural mistakes and primarily originate from limitations in students' conceptual understanding and cognitive readiness. The integration of diagnostic assessments, learning obstacle questionnaires, and semi-structured interviews revealed that students who learned the same mathematical content under identical classroom conditions developed different patterns of learning obstacles, indicating that mathematical understanding is influenced by individual cognitive characteristics rather than instructional exposure alone. These findings support Brousseau's Learning Obstacle Theory, which proposes that learning obstacles arise from interactions among students' cognitive development, prior knowledge, and the nature of mathematical concepts rather than representing isolated errors in problem solving. The results further indicate that students' written responses alone cannot adequately explain the origins of their mathematical difficulties because similar errors may emerge from different cognitive processes. The triangulation of multiple qualitative data sources provided richer evidence for understanding why students selected particular solution strategies and how they interpreted mathematical concepts during problem solving. Such findings reinforce the importance of diagnostic assessment as a means of revealing students' conceptual thinking rather than merely evaluating procedural performance (Kruit et al., 2020; Prediger et al., 2023; Tirado-Olivares et al., 2024). From a mathematics education perspective, conceptual understanding represents the foundation of meaningful learning because students who understand relationships among mathematical concepts are more likely to adapt their knowledge when encountering unfamiliar situations (Irshid et al., 2023; Ncube & Luneta, n.d.; Nilimaa, 2023). Conversely, students who rely primarily on memorized procedures tend to experience persistent misconceptions when solving problems requiring conceptual reasoning and mathematical justification. Therefore, the findings of this study suggest that evaluating students' achievement solely through written test scores may overlook important differences in conceptual understanding and cognitive readiness. Consequently, identifying students' learning obstacles through multiple sources of evidence provides a stronger basis for designing instructional interventions that promote meaningful geometry learning.

Participant S1 demonstrated the most complex pattern of learning obstacles because epistemological and ontogenic obstacles emerged simultaneously throughout the diagnostic assessment and interview sessions. The participant consistently failed to identify relevant mathematical information, establish conceptual relationships among geometric elements, and determine appropriate solution strategies, indicating that previously acquired knowledge had not been integrated into a coherent conceptual framework. Similar findings were reported who observed that students with fragmented conceptual understanding frequently performed arbitrary numerical operations instead of applying meaningful mathematical reasoning when solving geometry problems (Rohimah & Prabawanto, 2020). According this condition represents epistemological obstacles because students' existing knowledge becomes inadequate when mathematical problems are presented in unfamiliar contexts, while further argued that incomplete conceptual structures prevent learners from adapting previously acquired knowledge to increasingly complex situations. The interview findings additionally revealed that Participant S1 experienced confusion immediately after reading the problems and was unable to determine an appropriate starting point, suggesting that conceptual limitations were accompanied by insufficient cognitive readiness. This interpretation is consistent with who explained that ontogenic obstacles emerge when students' cognitive maturity, previous learning experiences, and mental preparedness are insufficient to support effective mathematical reasoning (Wahyuningrum et al., 2019). The learning obstacle questionnaire also indicated substantial difficulties in conceptual understanding and independent problem solving, confirming that both epistemological and ontogenic obstacles occurred simultaneously. These findings suggest that conceptual deficiencies become more pronounced when students lack confidence and cognitive readiness to organize mathematical reasoning systematically, thereby preventing them from constructing meaningful relationships among geometric concepts. Consequently, improving conceptual understanding alone may not sufficiently reduce learning obstacles unless mathematics instruction simultaneously strengthens students' cognitive readiness, self-confidence, and reasoning abilities during problem-solving activities. Therefore, the findings concerning Participant S1 provide empirical evidence that learning obstacles should be interpreted as interacting cognitive phenomena rather than independent categories, thereby extending the practical application of Brousseau's Learning Obstacle Theory within secondary geometry education.

Unlike Participant S1, Participant S2 predominantly experienced ontogenic obstacles, indicating that the primary source of difficulty was associated with cognitive readiness rather than conceptual misconceptions. Although the participant correctly selected an appropriate formula in the first diagnostic problem, the inability to continue solving subsequent problems demonstrated limited confidence and insufficient preparedness for engaging with increasingly complex mathematical tasks. This finding is consistent with explanation that ontogenic obstacles arise when students' developmental readiness and cognitive capacity are insufficient to support successful learning despite exposure to the same instructional experiences. Similar observations were who found that students often possess partial procedural knowledge but fail to sustain mathematical reasoning when confronted with unfamiliar problems requiring higher-order thinking. Furthermore, the absence of written responses for most diagnostic items indicated that the participant's difficulties were not necessarily caused by misconceptions regarding circle and chord geometry but rather by uncertainty in initiating and regulating the problem-solving process. The learning obstacle questionnaire strengthened this interpretation by revealing low levels of learning readiness and self-confidence, while the interview findings confirmed that the participant abandoned problem solving because of perceived inability rather than lack of effort. Similarly argued that students' beliefs about their own mathematical ability substantially influence their willingness to engage with challenging mathematical tasks (Odiri, 2021; Skilling et al., 2020). These findings imply that cognitive readiness and affective factors are closely interconnected during mathematical problem solving, particularly when students encounter problems requiring conceptual reasoning instead of routine procedural application. Consequently, mathematics instruction should not only emphasize conceptual mastery but should also cultivate students' confidence, persistence, and self-regulated learning to reduce the occurrence of ontogenic learning obstacles. The present findings therefore highlight that effective geometry instruction requires equal attention to conceptual development and students' psychological preparedness, particularly within student-centered learning environments promoted by the Merdeka Curriculum.

The findings related to Participant S3 further demonstrate that procedural success should not be interpreted as evidence of adequate conceptual understanding because correct mathematical procedures may still be performed through memorization rather than meaningful reasoning. Although Participant S3 successfully selected appropriate formulas and obtained several correct answers, the participant was unable to explain the conceptual relationships among central angles, chords, arcs, and the application of the Pythagorean theorem during the interview. This finding indicates that the participant's knowledge remained procedural and context-dependent, preventing the flexible application of mathematical concepts to unfamiliar situations. Such characteristics are consistent with Brousseau's description of epistemological obstacles, in which previously acquired knowledge becomes insufficient when learners encounter problems requiring conceptual adaptation rather than procedural repetition. Similar observations were who found that students frequently relied on memorized algorithms without understanding the mathematical principles governing those procedures (Gonda et al., 2022; Han et al., 2026; Siregar et al., 2025), while emphasized that procedural fluency often masks conceptual weaknesses when assessment focuses primarily on obtaining correct answers. Likewise, demonstrated that students commonly transfer solution procedures from previously encountered examples without first analyzing the conceptual characteristics of new mathematical problems (Gulvara et al., 2023). The present study extends these findings by showing that epistemological obstacles remain evident even when students appear capable of solving routine problems correctly because conceptual justification and mathematical reasoning remain incomplete. This finding suggests that procedural achievement should be interpreted cautiously, particularly in geometry learning where understanding the relationships among mathematical concepts is more important than recalling isolated formulas. Consequently, mathematics instruction should provide students with opportunities to explain, justify, and reflect upon their mathematical reasoning rather than emphasizing procedural accuracy alone. Such instructional practices are expected to strengthen conceptual understanding, reduce epistemological obstacles, and promote more flexible mathematical thinking across different problem contexts.

Overall, the present study demonstrates that learning obstacles in circle and chord geometry are multidimensional phenomena resulting from interactions among students' conceptual understanding, cognitive readiness, and mathematical reasoning rather than from a single source of learning difficulty. The triangulation of diagnostic assessment, learning obstacle questionnaires, and semi-structured interviews enabled a more comprehensive identification of students' learning obstacles than could be achieved through written assessments alone, highlighting the importance of integrating multiple sources of qualitative evidence when investigating students' mathematical thinking. Unlike many previous studies that primarily identified students' errors or examined learning obstacles within algebraic or trigonometric (Khairunnisa et al., 2022), the present study specifically explores learning obstacles in circle and chord geometry through the integration of diagnostic assessment, questionnaires, and interviews. This integrated approach provides deeper insight into the cognitive mechanisms underlying students' mathematical difficulties and demonstrates that different obstacle categories may coexist even among students learning under identical instructional conditions. Another important finding is that didactical obstacles were not identified as the dominant source of students' learning difficulties because the evidence consistently indicated that conceptual understanding and cognitive readiness played more influential roles in determining students' mathematical performance. This result does not imply that instructional practices are unimportant; rather, it suggests that the implementation of Problem-Based Learning in the observed classroom may have provided sufficient learning opportunities, whereas students' individual conceptual development remained the primary challenge. These findings contribute to the existing literature by extending the practical application of Brousseau's Learning Obstacle Theory to circle and chord geometry at the senior high school level and by demonstrating the value of diagnostic assessment for identifying students' conceptual difficulties before instructional intervention. From a pedagogical perspective, the results indicate that mathematics teachers should complement routine achievement tests with diagnostic assessments capable of revealing students' conceptual reasoning, cognitive readiness, and misconceptions before designing remedial instruction. Such an approach is highly relevant to the implementation of the Merdeka Curriculum, which emphasizes meaningful learning, conceptual understanding, and the development of higher-order thinking skills through student-centered instructional practices. Therefore, integrating diagnostic assessment into regular classroom practice has considerable potential to support more responsive, evidence-based, and concept-oriented mathematics instruction while reducing the occurrence of learning obstacles in geometry learning.

Implications

The findings of this study have important theoretical, pedagogical, and practical implications for mathematics education, particularly in teaching circle and chord geometry. The identification of epistemological and ontogenic learning obstacles indicates that students' mathematical difficulties cannot be addressed through procedural instruction alone but require learning approaches that strengthen conceptual understanding and cognitive readiness. Theoretically, this study extends the application of Brousseau's Learning Obstacle Theory by demonstrating that multiple categories of learning obstacles may coexist and influence students' mathematical reasoning. Pedagogically, the findings highlight the importance of integrating diagnostic assessment with qualitative inquiry to identify students' misconceptions and learning needs before instruction. Mathematics teachers are therefore encouraged to emphasize conceptual reasoning, mathematical justification, and interpretation of geometric representations rather than formula memorization. In addition, instructional strategies should be differentiated according to the characteristics of students' learning obstacles to provide more targeted learning support. From a practical perspective, schools and curriculum developers can utilize these findings to design concept-oriented learning resources and assessment practices aligned with the objectives of the Merdeka Curriculum. Teacher professional development should also strengthen competencies in diagnosing learning obstacles and using assessment evidence to guide instructional decisions. Furthermore, incorporating diagnostic assessment into routine classroom practice may facilitate the early identification of conceptual difficulties and enable timely instructional intervention. Overall, this study demonstrates that understanding students' learning obstacles provides a valuable foundation for developing more adaptive, evidence-based, and student-centered mathematics instruction that enhances conceptual understanding and mathematical reasoning.

Limitations and Suggestions for Future Research

This study has several limitations that should be considered when interpreting its findings. First, the study involved only three purposively selected participants from a single senior high school, limiting the transferability of the findings to broader educational contexts. Second, the investigation focused exclusively on circle and chord geometry and therefore may not fully represent learning obstacles encountered in other mathematical topics. Third, although diagnostic tests, learning obstacle questionnaires, and semi-structured interviews provided comprehensive qualitative evidence, the study did not evaluate the effectiveness of instructional interventions for reducing the identified learning obstacles. Future research is therefore recommended to include larger and more diverse samples across different educational settings and mathematical domains to enhance the generalizability of the findings. In addition, longitudinal, mixed-methods, or experimental studies are needed to examine the development of learning obstacles over time and evaluate the effectiveness of instructional approaches such as Problem-Based Learning, inquiry-based learning, and technology-enhanced mathematics instruction. Future studies should also integrate classroom observations and learning analytics to provide a more comprehensive understanding of students’ mathematical reasoning processes. Furthermore, developing and validating diagnostic assessment models that enable early identification of learning obstacles and personalized instructional support would provide valuable contributions to evidence-based mathematics education and strengthen the implementation of the Merdeka Curriculum.

CONCLUSION

This study explored the learning obstacles experienced by Indonesian senior high school students in learning circle and chord geometry through the integration of diagnostic assessment, learning obstacle questionnaires, and semi-structured interviews based on Brousseau’s Learning Obstacle Theory. The findings demonstrate that students experienced different categories of learning obstacles, indicating that conceptual difficulties in geometry are multidimensional and cannot be explained solely by incorrect procedural performance. Participant S1 exhibited both epistemological and ontogenic obstacles, reflecting fragmented conceptual understanding accompanied by limited cognitive readiness in organizing mathematical reasoning. In contrast, Participant S2 predominantly demonstrated ontogenic obstacles characterized by low learning readiness, limited self-confidence, and difficulty initiating problem-solving processes despite having partial procedural knowledge. Meanwhile, Participant S3 primarily experienced epistemological obstacles, as evidenced by the ability to apply memorized formulas without sufficient conceptual understanding or mathematical justification. These findings indicate that successful procedural performance does not necessarily represent meaningful conceptual understanding, particularly in geometry topics requiring students to relate multiple mathematical concepts and visual representations. The triangulation of diagnostic test results, questionnaire responses, and interview data proved effective in revealing the underlying characteristics of students’ learning obstacles that could not be identified through written assessments alone. Furthermore, the absence of dominant didactical obstacles suggests that the major learning challenges identified in this study were more strongly associated with students’ conceptual understanding and cognitive readiness than with instructional implementation. The study therefore extends the practical application of Brousseau’s Learning Obstacle Theory by demonstrating how diagnostic assessment can be used to distinguish different categories of learning obstacles in circle and chord geometry. These findings emphasize the importance of implementing concept-oriented mathematics instruction that encourages students to explain mathematical relationships, justify solution strategies, and construct conceptual understanding rather than relying exclusively on procedural memorization. In addition, the integration of diagnostic assessment into routine classroom practice provides teachers with valuable evidence for designing targeted instructional interventions that address students' specific learning needs before misconceptions become persistent learning barriers. Overall, this study contributes to mathematics education by providing empirical evidence that understanding the nature and characteristics of students' learning obstacles is fundamental for developing more adaptive, evidence-based, and student-centered instructional practices that strengthen conceptual understanding and improve geometry learning outcomes.

AUTHOR CONTRIBUTIONS STATEMENT

Anggraini Puspitasari Conceptualization, methodology, investigation, data curation, formal analysis, validation, visualization, writing original draft preparation, and project administration. Sri Rahmawati Fitriatien Conceptualization, methodology, supervision, validation, formal analysis, writing review and editing, funding acquisition, and corresponding author. Both authors contributed substantially to the conception and design of the study, critically reviewed the manuscript for important intellectual content, approved the final version of the manuscript, and agreed to be accountable for all aspects of the work.

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