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Exploring students' ability to translate between mathematical representations in solving quadratic function problems

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Abstract

Background: Students often encounter difficulties in translating between mathematical representations when solving quadratic function problems, resulting in fragmented conceptual understanding. These difficulties indicate that the source representation may influence how students construct mathematical meaning across verbal, graphical, and symbolic forms.
Aim: This study explores students' ability to translate between mathematical representations in solving quadratic function problems by examining the influence of different source representations and identifying the characteristics of their translation processes.
Method: This qualitative study adopted an exploratory case study design. Participants were selected through purposive sampling based on variations in their representation translation ability. Data were collected through written problem-solving tasks and semi-structured interviews and analyzed using Braun and Clarke's Reflexive Thematic Analysis.
Results: Students demonstrated different translation patterns depending on the source representation. Verbal and graphical representations were more frequently translated into other forms, whereas symbolic representations were generally retained without further transformation. Three translation characteristics were identified: isolated, sequential, and fragmented, reflecting varying levels of representational coordination.
Conclusion: Students' ability to translate between mathematical representations is influenced by the source representation used during problem solving. Mathematics instruction should explicitly connect verbal, graphical, and symbolic representations to strengthen conceptual understanding and promote more meaningful representation translation.

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INTRODUCTION

Students continue to experience difficulties in translating between mathematical representations when solving mathematical problems, particularly in the topic of quadratic functions (Rahmawati et al., 2022). This condition indicates that the ability to translate between mathematical representations remains an important issue that requires greater attention in mathematics education. Mathematical representation is recognized as one of the essential process standards because it enables students to express, interpret, and communicate mathematical ideas through multiple forms while supporting problem solving, reasoning, communication, and mathematical connections. Representations also function as observable indicators of students' conceptual understanding and cognitive development because they reflect how mathematical ideas are organized and interpreted during learning. In mathematics learning, representations commonly include verbal, graphical, and symbolic forms, each providing unique perspectives for interpreting mathematical concepts (I. Putri et al., 2022). These different forms of representation complement one another and allow students to examine mathematical relationships from multiple viewpoints

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rather than relying on a single procedure. Consequently, students are expected to develop the ability to translate between representations while maintaining the underlying mathematical meaning throughout the translation process. Such translation requires students to coordinate information across different representational forms instead of merely converting symbols mechanically. The quality of this coordination reflects the depth of students' conceptual understanding because meaningful translation involves recognizing equivalent mathematical relationships across representations. Therefore, investigating students' ability to translate between mathematical representations is essential for understanding how conceptual knowledge is constructed during mathematical problem solving.

Translation between mathematical representations plays a central role in mathematical problem solving because students frequently encounter information presented in verbal, graphical, and symbolic forms within the same mathematical task. Each representation possesses distinct characteristics that require different cognitive processes for identifying relevant mathematical information and constructing appropriate solutions (García-García & Dolores-Flores, 2021; Munfaridah et al., 2021; Rach & Ufer, 2020). Consequently, the translation process from a source representation to a target representation may vary depending on the characteristics of the initial representation used by students. Different mathematical thinking processes are associated with different representational forms, indicating that representation selection influences students' reasoning during problem solving (Ferretti et al., 2024; Jäder & Johansson, 2025; Wahab, 2026). Source representations provide the initial mathematical information from which students identify relationships, interpret concepts, and determine subsequent representational transformations (García-García & Dolores-Flores, 2021; Gulkilik et al., 2020; Moyer-Packenham et al., 2022; Pedersen et al., 2021; Tytler et al., 2023). Successful translation therefore depends not only on procedural knowledge but also on students' ability to preserve mathematical meaning while coordinating multiple representations. This coordination enables students to understand the structure of mathematical problems more comprehensively and supports flexible reasoning across different representational forms. The ability to move effectively between representations has also been associated with deeper conceptual understanding, creative mathematical thinking, and improved problem-solving performance (Sari & Susanah, 2023). Nevertheless, previous studies consistently indicate that students' difficulties in mathematics arise primarily from translating between representations rather than from understanding individual representations in isolation (Ariningtyas et al., 2021; Kusumaningtyas et al., 2022). These findings suggest that examining the translation process itself is necessary to obtain a more comprehensive understanding of students' representational thinking during mathematical problem solving.

The challenges associated with translation between mathematical representations become particularly evident in learning quadratic functions because this topic requires students to coordinate verbal, graphical, and symbolic information simultaneously (Ermilia et al., 2024). Previous studies have shown that many students experience difficulties when constructing quadratic function equations from graphical information, interpreting graphical characteristics from symbolic expressions, or connecting contextual situations with mathematical models (Castro et al., 2022; Nurrahmawati et al., 2021; Safitri et al., 2025). These findings indicate that students often rely on procedural manipulation without establishing meaningful conceptual relationships among different representations of quadratic functions. Existing studies have predominantly focused on students' translation ability, translation outcomes, or common translation errors across different representational forms (Ariningtyas et al., 2021; Kusumaningtyas et al., 2022). However, relatively little attention has been given to how different source representations influence students' translation processes throughout problem solving. The role of source representations is particularly important because the initial representation may determine how mathematical information is identified, interpreted, and coordinated during representational transformation. Differences in source representations may therefore produce different reasoning pathways and distinct characteristics of representation translation among students. Understanding these characteristics is essential for explaining why students demonstrate different levels of representational coordination even when solving mathematically equivalent problems. Such knowledge may also provide valuable insights for designing instructional approaches that explicitly support translation between verbal, graphical, and symbolic representations. Accordingly, exploring students' translation processes based on different source representations is expected to provide a more comprehensive understanding of representational thinking and contribute to improving mathematics instruction on quadratic functions.

Previous studies have consistently highlighted the importance of multiple mathematical representations in supporting conceptual understanding, mathematical creativity, problem solving, and representational competence across various mathematical contexts (Bicer, 2021; Cullen et al., 2020; Pedersen et al., 2021). Other studies have further examined mathematical representations from semiotic and meaning-making perspectives, emphasizing representational transformations, mathematical connections, and the role of semiotic systems in students' mathematical reasoning (Capone et al., 2021; Erbilgin & Gningue, 2023; Ferretti et al., 2024; Godino et al., 2022). In the context of quadratic functions, existing research has primarily investigated students' conceptual understanding, visualization, representational fluency, self-efficacy, and the integration of technology to support learning (Canonigo, 2024; Pedersen et al., 2021). Although these studies have substantially advanced knowledge regarding the use and function of mathematical representations, they predominantly focus on students' representational performance, learning outcomes, or theoretical interpretations rather than examining how students dynamically translate between representations during problem solving. Moreover, limited attention has been given to how different source representations (verbal, graphical, and symbolic) shape students' translation processes and influence the construction of mathematical meaning throughout the solution process. Addressing this gap, the present study explores students' ability to translate between mathematical representations in solving quadratic function problems by examining the influence of different source representations and identifying the characteristics of the resulting translation processes, thereby providing a more comprehensive understanding of representational coordination in mathematics learning.

Based on the identified research gap, this study aims to explore students' ability to translate between mathematical representations when solving quadratic function problems. Specifically, the study investigates how different source representations influence students' translation processes across verbal, graphical, and symbolic representations. It also seeks to examine how students construct mathematical meaning while moving from one representation to another during problem solving. Rather than focusing solely on the correctness of translation outcomes, this study emphasizes the underlying cognitive processes that characterize students' representational coordination. Particular attention is given to identifying the patterns of translation that emerge from different source representations and the ways these patterns reflect students' conceptual understanding of quadratic functions. By adopting a qualitative exploratory case study approach, this research provides an in-depth description of students' representational thinking in authentic mathematical problem-solving situations. The findings are expected to extend the current understanding of translation between mathematical representations by highlighting the influence of source representations on students' reasoning processes. Furthermore, this study is intended to contribute to the theoretical development of representation translation by providing empirical evidence regarding the characteristics of students' translation processes. From a practical perspective, the findings are expected to assist mathematics teachers in designing instructional activities that explicitly integrate verbal, graphical, and symbolic representations to strengthen students' conceptual understanding and representational flexibility. Ultimately, this study contributes to mathematics education by offering new insights into how source representations shape students' ability to translate between mathematical representations and by informing more effective pedagogical practices for teaching quadratic functions.

LITERATURE REVIEW

Mathematical representation constitutes one of the fundamental components of mathematical thinking because it enables learners to express, interpret, and communicate mathematical ideas in various forms. Representations provide cognitive tools that support students in organizing information, constructing relationships among concepts, and developing conceptual understanding during mathematical learning. In mathematics education, representations are generally classified into verbal, graphical, symbolic, numerical, and visual forms, each highlighting different aspects of the same mathematical concept (Hatisaru, 2020; Liu et al., 2026). Rather than functioning independently, these representational forms complement one another by providing alternative perspectives for interpreting mathematical situations. Students who can coordinate multiple representations are generally more capable of explaining mathematical ideas, making logical inferences, and solving unfamiliar problems than students who rely on a single representational form (Abrahamson et al., 2020). The ability to work flexibly with multiple representations also promotes conceptual understanding by helping students recognize equivalent mathematical meanings across different representational systems. Consequently, representation has become one of the core process standards in mathematics education because it supports reasoning, communication, problem solving, and mathematical connections. Recent studies further emphasize that representational competence contributes significantly to students' mathematical achievement, creativity, and higher-order thinking skills (I. Putri et al., 2022). From a cognitive perspective, representations serve as external manifestations of internal mathematical thinking, allowing researchers and teachers to observe how students organize and process mathematical information. Therefore, understanding students' use of mathematical representations provides an important foundation for investigating their conceptual development and problem-solving processes.

Translation between mathematical representations refers to the process of expressing mathematical information from one representational form into another while preserving the underlying mathematical meaning. According to the Theory of Semiotic Representation, translation involves two complementary processes, namely treatment and conversion, which together support the construction of mathematical understanding (Komatsu & Jones, 2020; Ovodenko & Kouropatov, 2025; Svensson & Campos, 2022; Viseu et al., 2021). Treatment refers to transformations performed within the same representational register, whereas conversion involves transforming information across different representational registers while maintaining conceptual equivalence. Among these processes, conversion is generally considered more cognitively demanding because students must recognize the same mathematical object despite changes in representational form. Successful translation therefore requires students to coordinate mathematical meaning rather than merely performing procedural transformations. During this process, the source representation functions as the initial reference from which mathematical information is interpreted before being reconstructed into a target representation. The effectiveness of this translation depends on students' ability to identify essential mathematical relationships and preserve those relationships throughout representational transformation. Difficulties frequently arise when students focus on surface features of representations instead of recognizing their underlying conceptual equivalence. Consequently, failures in representation translation often reflect incomplete conceptual understanding rather than computational weaknesses. For this reason, translation between mathematical representations has become an important construct for explaining students' reasoning processes and conceptual development in mathematics learning.

Problem solving requires students to construct, interpret, and coordinate multiple mathematical representations throughout the reasoning process rather than relying exclusively on symbolic manipulation. During mathematical problem solving, students continuously identify relevant information, interpret mathematical relationships, and determine appropriate representational forms to support their reasoning (Al Farra et al., 2022; Altindis & Fonger, 2025; Torres-Peña et al., 2025). This process involves dynamic interactions between verbal descriptions, graphical interpretations, and symbolic expressions that together contribute to the development of meaningful mathematical solutions. The effectiveness of problem solving is therefore closely associated with students' representational flexibility, namely their ability to move appropriately between different representations according to problem demands. Students with high representational flexibility tend to construct stronger conceptual connections because they can examine mathematical situations from multiple perspectives. Conversely, students who rely predominantly on a single representation often experience difficulties when mathematical information must be interpreted or transformed into alternative representational forms. Previous studies have consistently reported that many students experience obstacles when translating verbal information into symbolic expressions or graphical representations and vice versa (Ariningtyas et al., 2021; Kusumaningtyas et al., 2022). However, existing research has predominantly evaluated students' translation accuracy or learning outcomes rather than examining the cognitive characteristics of the translation process itself. Furthermore, relatively limited attention has been given to the influence of different source representations on students' representational coordination during mathematical problem solving. Understanding these translation processes is essential because it provides deeper insight into how students construct mathematical meaning and develop conceptual understanding while solving mathematical problems.

Quadratic functions provide a particularly suitable context for investigating translation between mathematical representations because solving quadratic function problems requires students to coordinate verbal, graphical, and symbolic representations simultaneously. Conceptual understanding of quadratic functions extends beyond performing algebraic procedures, as students are expected to interpret the meaning of coefficients, identify graphical characteristics, explain mathematical relationships verbally, and connect these representations consistently (Ayeh, 2025; Wilkie, 2024). Effective coordination among these representations enables students to recognize that different representational forms describe the same mathematical object from complementary perspectives. However, empirical studies indicate that many students experience difficulties when translating graphs into symbolic equations, constructing quadratic graphs from algebraic expressions, or interpreting contextual situations using quadratic models (Castro et al., 2022; Nurrahmawati et al., 2021; Safitri et al., 2025). These difficulties suggest that students frequently rely on procedural manipulation without establishing meaningful conceptual connections among representations. As a result, successful symbolic computation does not necessarily indicate that students understand the mathematical relationships represented graphically or verbally. Such conditions demonstrate that representation translation in quadratic functions is not merely a procedural activity but also a cognitive process involving the construction and coordination of mathematical meaning. The complexity of quadratic functions therefore offers an appropriate context for examining how students utilize different representations throughout the problem-solving process. Investigating students' translation processes in this topic can reveal how representational coordination supports or constrains conceptual understanding during mathematical reasoning. Consequently, quadratic functions serve as an important domain for exploring the relationship between representation translation and students' conceptual development in mathematics.

Previous research has made substantial contributions to understanding mathematical representations by examining representational competence, translation accuracy, conceptual understanding, and common errors across different mathematical topics (Ariningtyas et al., 2021; Kusumaningtyas et al., 2022). Recent studies have also emphasized the importance of multiple representations and semiotic perspectives in supporting students' mathematical reasoning and conceptual learning (Ferretti et al., 2024). Nevertheless, existing studies have predominantly concentrated on evaluating students' representational performance or identifying translation outcomes rather than examining how translation processes develop during problem solving. Moreover, relatively limited attention has been given to the influence of different source representations on the ways students coordinate mathematical meaning throughout representational transformation. The source representation may shape students' selection of mathematical information, determine the sequence of representational transformations, and influence the construction of conceptual relationships during problem solving. Consequently, students may demonstrate different translation processes even when solving mathematically equivalent quadratic function problems that begin from different representations. Understanding these variations is essential because they provide richer evidence of students' cognitive processes than analyses based solely on final answers or translation accuracy. A process-oriented perspective also offers opportunities to identify distinctive characteristics of representation translation that cannot be captured through outcome-based assessment alone. Such knowledge is expected to strengthen theoretical understanding of representation translation while informing the design of instructional practices that explicitly connect verbal, graphical, and symbolic representations. Accordingly, investigating students' translation processes based on different source representations provides an important contribution to advancing research on mathematical representations and improving the teaching and learning of quadratic functions.

METHOD

Research Design

This study employs a qualitative approach using an exploratory case study design. This design was chosen because the study aims to describe the translation between students' mathematical representations based on source representations whether verbal, graphical, or symbolic. Furthermore, the selection of this design who states that an exploratory case study is used when a phenomenon needs to be understood in depth within a real-world context and the boundary between the phenomenon and its context cannot be clearly separated. In this study, the process of translation between mathematical representations is viewed as a phenomenon influenced by the source representations encountered by students; thus, this design is deemed appropriate for uncovering these dynamics. Furthermore, this design was chosen because the translation between students' mathematical representations based on verbal, graphical, and symbolic representations has not yet been fully and clearly mapped out in the literature. Based on these considerations, an exploratory case study design was chosen to provide a more comprehensive understanding of this phenomenon.

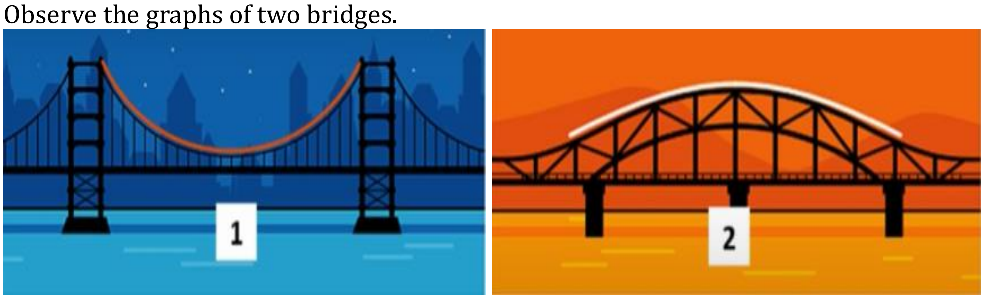
Participant

The subjects of this study were 36 tenth-grade students at a high school in Malang, East Java, Indonesia, who had studied quadratic functions. This school was selected because it had completed the quadratic function unit before the study began, ensuring that all students had learning experiences relevant to the study's focus. All students took a written test as the initial stage of data collection. The class was selected through purposive sampling not to represent the general population of high school students, but because it provided information-rich cases aligned with the study's objectives. Next, several participants were selected for in-depth analysis based on variations in their responses to the initial written test. These variations were identified from the initial representations used by the students, the problem-solving strategies demonstrated, the completeness of the solution steps, and the tendency to switch between representations evident in their written answers. These criteria were established before the in-depth analysis was conducted so that participant selection was not based on the final results of the translation process analysis but rather on the variation in students' initial responses to the given questions. Since this study involved only one class at a single school, the research results are intended to provide an analytical generalization of the phenomenon under study, rather than a statistical generalization to a broader population.

Instrument

The instruments used in this study consisted of a written test on quadratic functions and a semi-structured interview guide. Both instruments were developed to complement each other in order to obtain data on the process of translation between students' representations when solving quadratic function problems. The written test was designed to identify the source representations, target representations, and translation processes used by students. This written test included three open-ended questions developed based on three source representations: verbal, graphical, and symbolic. Each question was designed to elicit a different translation process corresponding to the given source representation, thereby allowing for a variety of translations between mathematical representations. Therefore, these three questions are not intended to measure the number of problems students solve, but rather to describe how students identify mathematical information, coordinate various representations, and translate source representations into other representations during the problem-solving process. Thus, each question represents one source representation that is the focus of the study. These problems are presented in Table 1 below.

Table 1. Test Item Instrument

No.	Question
	Verbal Source Representation
1	Angga is a soccer player who is about to kick a ball toward the goal. The horizontal distance of the ball is measured from the kicking point (in meters), and the height of the ball is measured from the ground surface (in meters). The height of the ball as a function of horizontal distance can be represented by a quadratic function. As the ball travels toward the goal, it reaches a maximum height of 6 meters at a position 6 meters from the kicking point and has a height of 5 meters at a position 2 meters from the maximum point. Determine the position of the ball (goal/no goal) when the goal is located 10 meters from the kicking point and the crossbar height is 2 meters.
2	Observe the graphs of two bridges.  Compare the arch curves of Bridge 1 and Bridge 2 based on their opening direction and steepness.
3	The profit of a production business depends on the number of goods produced in each period and is modeled by the quadratic function $f(x) = -2x^2 + 80x - 400$ where $x$represents the number of goods produced (in units) and $f(x)$represents the profit earned (in thousands of rupiah).

According to the financial records, the maximum profit ever achieved by the business in one production period was 400 thousand rupiah.. The owner wishes to evaluate the number of goods that should be produced so that the profit obtained remains at this maximum value. Based on the quadratic function that models the business profit, determine the most appropriate number of goods to produce.

Next, a semi-structured interview guide was developed based on the students' responses to the written test. The interviews focused on the students' processes in identifying information, selecting representations, translating between representations, and explaining the reasoning behind each step of their solution. During the interviews, the researcher used probing questions to elicit more in-depth explanations of the students' answers. The interview data were then integrated with the written test data to confirm and enrich the interpretation of the students' processes of translation between mathematical representations. Before being used in the study, both research instruments the written test on quadratic functions and the semi-structured interview guide were first validated through expert judgment by mathematics education faculty members to ensure content appropriateness, clarity of representations, and the potential to elicit translations between representations. Based on the validators' feedback, revisions were made to the wording of the instruments before they were used in the data collection process.

Data Analysis

In this study, the data obtained were analyzed using Reflexive Thematic Analysis is a qualitative data analysis technique used to identify and develop themes based on the researcher's reflective and in-depth interpretation of the data. Therefore, this technique is relevant to the research process conducted to identify, analyze, and interpret students' mathematical representation translation processes. In this study, the data obtained were analyzed According to Reflexive Thematic Analysis is a qualitative data analysis technique used to identify and develop themes based on the researcher's reflective and in-depth interpretation of the data. This technique was chosen because it allows the researcher to identify and interpret the characteristics of the translation process between students' representations based on the source representations used. Thus, the analysis focuses not only on the results of the translation but also on the mathematical meanings constructed during the process of solving quadratic function problems.

The analysis process in this study was conducted based on the framework the first stage included familiarization with the data and generating initial codes. At this stage, the researcher repeatedly read the written test results and interview transcripts to gain a comprehensive understanding of how students used verbal, graphical, and symbolic representations to solve quadratic function problems. Next, data units representing the use of source representations, translation processes, problem-solving strategies, and students' explanations were assigned codes inductively based on the meanings emerging from the data. The second stage involved searching for themes, reviewing themes, and defining and naming themes. Codes with related meanings were grouped into provisional themes. Subsequently, the themes were re-examined by comparing them against the entirety of the written test and interview data to ensure their consistency with the empirical data. At this stage, the conceptual boundaries of each theme were also refined so that each theme clearly and consistently interpreted the characteristics of the translation process between representations. The third stage involved producing the report. At this stage, the results of the analysis are interpreted by combining findings from the written test and interviews to explain the characteristics of the students' translation process between mathematical representations based on the source representations used in solving quadratic function problems.

RESULTS AND DISCUSSION

Results

This study involved six subjects who were purposively selected based on the variety of source representations they used to solve quadratic function problems. The analysis focused on the subjects' written responses, which included verbal, graphical, and symbolic source representations.

Table 2. Students' Mathematical Representation Translation Processes

Source Representation	Representation Translation Process
Question 1 (Verbal)	V → S
	V → G → S
	V → G
Question 2 (Graphical)	G → V
	G → S
Question 3 (Symbolic)	S → S

Note: V = Verbal; G = Graphical; S = Symbolic.

Table 2 shows that each type of source representation has a tendency toward a particular type of inter-representational translation process that appears most frequently, based on the results of the analysis of written responses and interviews. In Question 1, which involved a verbal source representation, most students showed a tendency to translate toward a symbolic representation, either directly or via a graph first. This finding suggests that verbal representations tend to be perceived as requiring a mathematical model. However, there were linear translations that directly converted to graphs without an intermediate symbolic step. In this study, linear translation refers to a translation process that proceeds sequentially from one representation to another without returning to a previous representation. Meanwhile, for Question 2 with a graphical source representation, the translation process between mathematical representations showed solutions using both verbal and symbolic methods. This indicates that graphical representations give rise to variations in interpretation strategies, where some students first use verbal representations before moving to symbolic ones, while others directly translate to symbolic form. As for Question 3, which uses a symbolic source representation, the majority of students remained within the same form of representation. These findings indicate that procedural operations dominate the resolution of symbolic problems. Overall, the results in Table 2 show that students' cross-representation translation appears to be related to the source representation used, suggesting that their ability to translate between mathematical representations is representation-sensitive. A more in-depth analysis of students' translation process between mathematical representations is provided below.

1. Representation-Induced Translation
   The results of the analysis show that the process of translation between students' mathematical representations appears to be influenced by the source representation provided. Source representations—whether verbal, graphical, or symbolic appear to guide the direction of the translation process and are also related to the complexity of the translation process carried out by students. Thus, the translation between mathematical representations that occurs is representation-induced, meaning that the direction and characteristics of the translation process are influenced by the source representation used as the starting point for problem-solving.
   a) Translation from Verbal Source Representations
      In Question 1, the source representation is presented in the form of a contextual verbal narrative. Therefore, students must first interpret the information provided in the context of the question before determining the mathematical representation to use in solving the problem. Based on Table 2, the identified translation processes include the translation from a verbal source representation to a symbolic target representation (V → S), the translation from a verbal source representation to a symbolic target representation via a graph (V → G → S), and

the translation from a verbal source representation to a graphical target representation (V → G). These findings indicate that verbal representations tend to guide students toward translation between symbolic representations, either directly or with the aid of graphical visualizations. One example of this translation process is demonstrated by subject S1-VGS, who used the V → G → S translation to solve the problem.

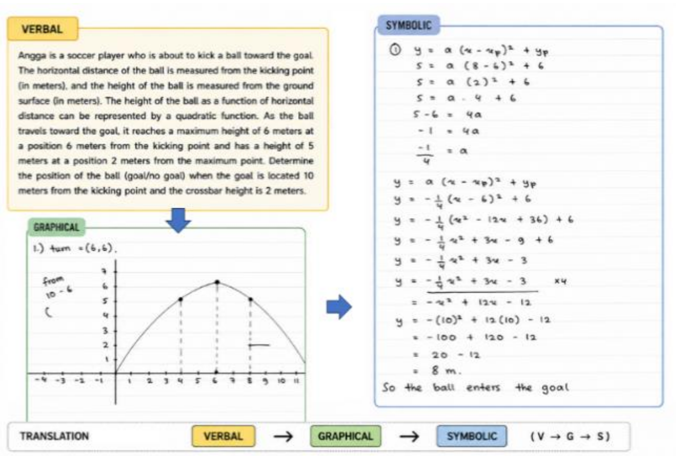


Figure 1. Answer S1-VGS

The translation between the S1 and VGS representations shown in Figure 1 indicates that, in the problem-solving process, the subject first converts verbal information into a visual form a sketch of a parabola with a vertex at (6,6) and a point of interest at (8,5). Then, from the graphical representation presented, the subject proceeds to a symbolic inter-representational translation using the quadratic function equation of the form y = a(x - x_p)^2 + y_p to determine the value of a. The value of a obtained from the quadratic function equation yields a quadratic function that determines the height of the ball when x is substituted with 10. In this calculation, the value y = 8 was obtained, leading the subject to conclude that the ball did not enter the goal. This translation process demonstrates that graphical representations serve not only as intermediaries but also as a means to coordinate information with mathematical models. Through graphical representation, the subject connects verbal information regarding the ball's trajectory with the form of the quadratic function before writing it in symbolic form. Thus, graphical representation acts as a bridge that facilitates coordination between representations during the problem-solving process. This is supported by the results of an interview with S1-VGS as follows.

Researcher : *"What is the first step you take when solving the problem?"*
S1-VGS : *"I draw an illustration first."*
Researcher : *"So, do you usually have to draw the graph first before using the formula?"*
S1-VGS : *"Yes."*
Researcher : *"Why do you need to draw the illustration first?"*
S1-VGS : *"So that I know the shape of the trajectory first."*
Researcher : *"What helps you determine the quadratic function equation?"*
S1-VGS : *"The graph, because from it I can determine the equation."*
Researcher : *"What if you use the formula directly without drawing the graph?"*
S1-VGS : *"I can't."*

Based on the presentation of the interview results, S1-VGS indicates that graphing serves as a strategy for understanding verbal information before formulating a mathematical model in symbolic form. For S1-VGS, without the use of graphs, a mathematical model of a quadratic function problem cannot be formulated.

In contrast, S2-VS demonstrated translation between representations with a verbal source representation to a symbolic target representation (V → S). S2-VS did not use graphical visualization and directly converted verbal information into symbolic form through the use of formulas that had been learned or memorized. The process of translation between representations was shorter and focused on symbolic manipulation without first going through a visualization process. Meanwhile, S3-VG was the only subject to demonstrate a translation from a verbal source representation to a graphical target representation (V → G). S3 used graphical illustrations to understand the situation in the problem, but the translation process did not lead to a symbolic representation. This finding indicates that graphical representations can serve as a tool for understanding verbal context and solving problems. Thus, the translation process from a verbal source representation generally occurs through a stepwise representational process. These characteristics form the basis for identifying the translation process, which is categorized in the following section as linear translation. The form of the verbal source representation may trigger a need for visualization in some students, while in others it directly activates either symbolic or visual procedures. Thus, the process of translation between verbal representations is influenced by how students construct initial meaning from the contextual information provided in the problem.

b) Translation of Graphical Source Representations

In Question 2, the source representation is presented in the form of a quadratic function graph. The results of the analysis, shown in Table 2, indicate that the graphical representation yields various types of inter-representational translations, including translations from the graphical source representation to the verbal target representation (G → V) and from the graphical source representation to the symbolic target representation (G → S). One such translation is demonstrated by S3-GV, who solved the problem using the G → V inter-representation translation.

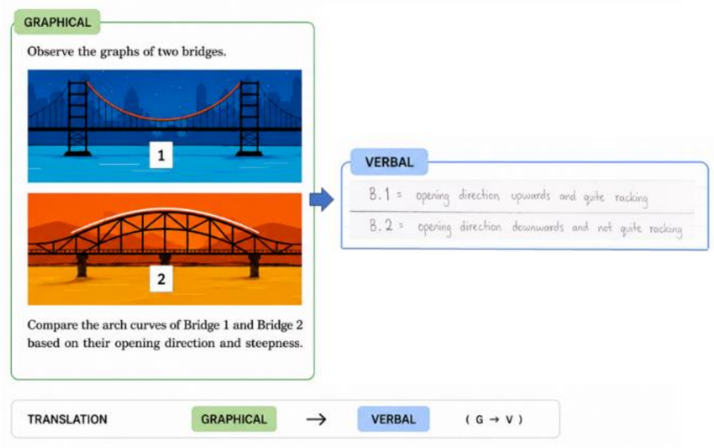


Figure 2. Answer S3-GV

The translation between S3-GV representations shown in Figure 2 indicates that, in the problem-solving process, the subject converts graphical information into verbal descriptions. S3-GV identifies basic visual characteristics of the graph, such as the direction of the parabola's opening and the steepness of the curve, and then interprets them. This is evident in the statements "it opens upward and is quite steep" and "it opens downward and is not very steep" in the responses. Furthermore, these findings are supported by the following interview with S2.

Researcher : *"What is the second problem about?"*
S3-GV : *"Comparing the shapes of the arches."*
Researcher : *"What is the comparison based on?"*
S3-GV : *"Based on the graph."*
Researcher : *"Is there any connection to a formula?"*
S3-GV : *"Yes, but I forgot it."*

Based on the presentation of the interview results, S3-GV indicated that graphs serve as the primary source of information for constructing verbal mathematical explanations. Although S3-GV understands that problems can be solved using symbolic representations, the subject first uses graphical representations to interpret the information in the problem before constructing symbolic representations.

In contrast, S4-GS demonstrated a different translation process in which the source representation was a graph and the target representation was symbolic (G → S). The student translated the graph into a symbolic representation by associating the direction in which the parabola opens with the sign of the coefficient (a), where ($a > 0$) indicates a parabola opening upward and ($a < 0$) indicates a parabola opening downward. Thus, the graph as the source representation served as a visual interpretation that could be translated into either verbal or symbolic representations, depending on the student's representation translation strategy.

c) Translation from Symbolic Source Representation

In Question 3, the source representation was presented in symbolic form as a quadratic function equation in the problem. The analysis results showed that all students tended to stick to the problem-solving process based on the symbolic representation (S). Students solved the problem through direct algebraic manipulation without involving graphical or verbal representations. This finding indicates that symbolic representations encourage students to use algebraic procedures mechanically and sequentially. S1-SS illustrates the transition from the symbolic source representation to the symbolic target representation (S → S), as shown in the following figure.

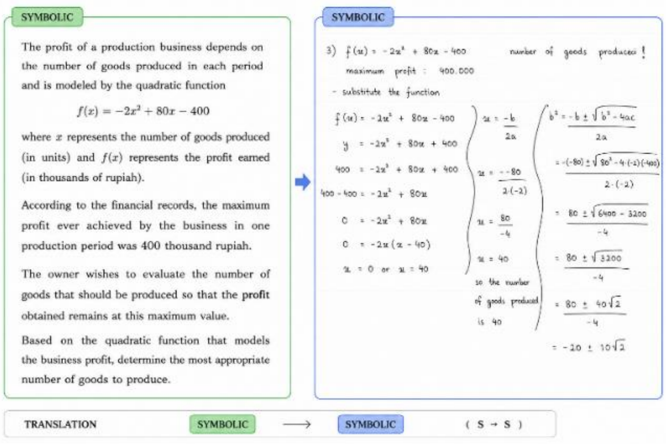


Figure 3. Answer S1-SS

The translation between S1's representations, as shown in Figure 3, indicates that in the problem-solving process, the subject directly applied the formula for the vertex $x = \frac{-b}{2a}$, to determine the production quantity that yields maximum profit. The entire process took place within a symbolic representation without involving graphical visualization or verbal explanation. This finding is supported by the results of the interview with S1-SS, as follows.

Researcher	:	*"What was the first thing you did when solving the problem?"*
S1-SS	:	*"I simplified the function first."*
Researcher	:	*"What method did you use after that?"*
S1-SS	:	*"The quadratic formula to find the value of (x)."*
Researcher	:	*"Why did you then use the vertex formula?"*
S1-SS	:	*"Because I could not find the value of (x), so I followed the method suggested by a friend."*

Based on the presentation of the interview results, S1-SS showed that the student initially attempted to solve the problem through symbolic manipulation by simplifying the function and using the ABC formula to determine the value of x. However, because they encountered difficulties in continuing this process, the students then switched to using the vertex formula $x = \frac{-b}{2a}$ until they obtained the value $x = 20$. Thus, symbolic representations tend to result in a shift between dominant registers compared to verbal or graphical representations. Students tend to maintain the problem-solving process within the symbolic register through algebraic manipulation and the direct application of formulas.

Research findings indicate that source representations are related to differences in the translation processes students use between mathematical representations. In this regard, students do not use the same translation process; rather, they adapt their translation process to the characteristics of the initial representation provided. Verbal source representations encourage students to construct graphical or symbolic representations; graphical representations facilitate coordination between visual and symbolic representations; whereas symbolic representations tend to maintain the problem-solving process within the symbolic representation. These findings indicate that source representations play a role in shaping how students coordinate various mathematical representations during problem-solving.

2. Fragmented Translation Meaning

The results of the analysis indicate that fragmented translation meaning occurred in this study. Fragmented translation meaning refers to the translation process in which students are able to convert information from one mathematical representation to another but are not yet able to maintain consistency in mathematical meaning across those representations. As a result, even though the translation procedure appears to be complete, there are still discrepancies in mathematical meaning during the problem-solving process. In this study, fragmentation is evidenced by inconsistencies in the relationships between representations, incomplete coordination of mathematical information, or inconsistencies between the constructed representations and the problem-solving results.

a) Fragmented Translation Meaning in Verbal Source Representations

In verbal source representations, fragmented translation occurs when students are able to construct a target representation from narrative information, but the mathematical meaning across representations has not yet been fully coordinated during the translation process. This is evident in S4-VS, as shown in Figure 4 below.

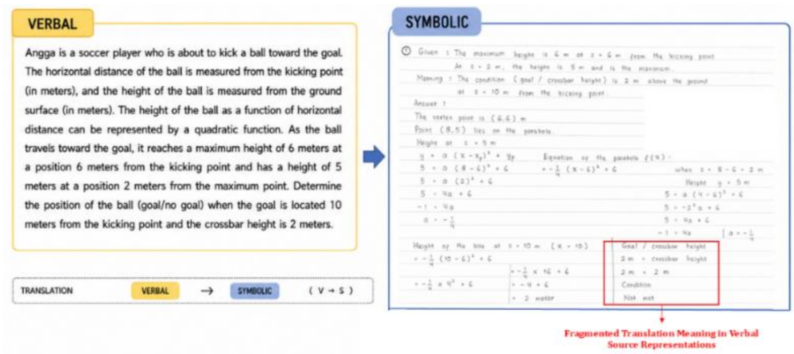


Figure 4. Answer S4-VS

Based on Figure 4, the student first visualized the trajectory of the ball by sketching a parabolic graph and positioning the vertex and the goalpost as the primary sources of information. The student then constructed a quadratic function model using the vertex form $y = a(x - x_p)^2 + y_p$ and proceeded with substitution to determine the height of the ball at a given horizontal distance. However, fragmentation became apparent when the student translated the visual representation into a symbolic representation. This was evidenced by the student's determination of $x = 8$ through the operation $6 + 2$, however, based on the interview, the student did not yet understand the conceptual rationale underlying this step. Furthermore, when determining the ball's height at $x = 10$, the student obtained $f(10) = 2$ and concluded that the "ball enters the goal" because the ball's height was equal to the height of the crossbar. These findings are supported by the interview excerpt with S4-VS presented below.

Researcher : *"What is your final conclusion?"*
S4-VS : *"The ball can go in, but it hits the crossbar."*

The results of the interviews indicate that the process of translation between representations is more oriented toward procedural outcomes than toward conceptual interpretations of the situations being represented. This is consistent with the S1-VGS condition shown in Figure 1, which indicates that students understand the trajectory of the ball as a parabolic curve but have difficulty determining the vertex and the relationship between coordinates on the graph.

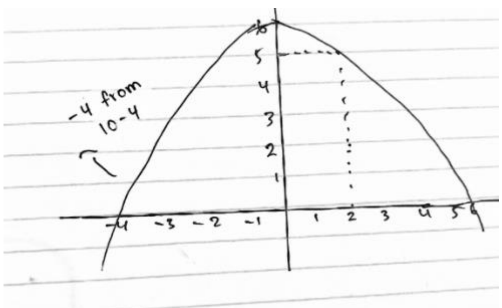


Figure 5. Graphical Sketch of Initial Understanding S1-VGS

Figure 5 shows that S1-VGS has understood the general shape of a lower-open parabola, but this figure was constructed intuitively. The positions of the data points on the graph are not yet relevant to the information in the problem, so S1-VGS is still uncertain about determining the correct position of the ball's trajectory. Therefore, the translation from the verbal source representation to the target graphical representation has not yet been fully conceptually coordinated. After constructing the graph, S1-VGS solved the problem by performing a symbolic translation to form the equation of a quadratic function of the vertex form $y = a(x - x_p)^2 + y_p$. However, fragmentation reemerged when S1 performed a substitution to determine the height of the ball at the 10-meter position. In the process shown in Figure 1, S1-VGS made a calculation error, resulting in a final answer of $y = 8$, and interpreted the conclusion incorrectly when answering the question.

Another form of fragmentation is also evident in S2-VS, which involves the translation between representations, specifically from a verbal source representation to a symbolic target representation (V → S).

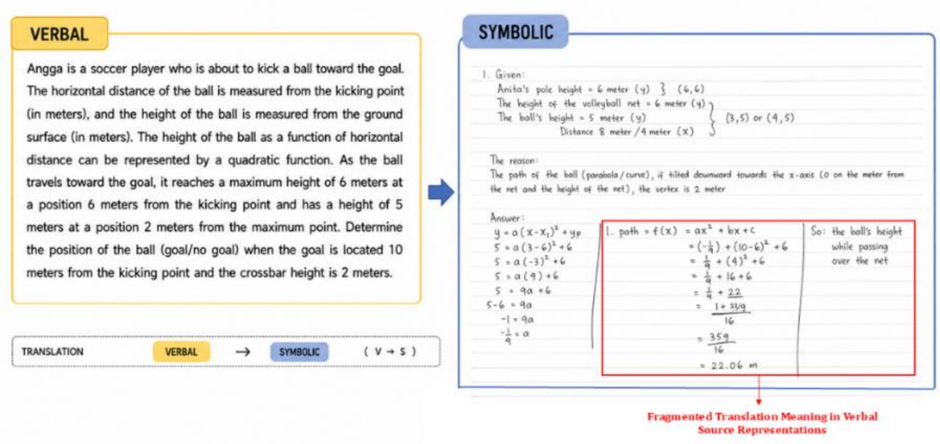


Figure 6. Answer to S2-VS

Figure 6 shows that the student first identified important information in the problem, such as the maximum point (6,6) and another point (8,5), and then directly constructed a quadratic function model using the vertex form $y = a(x - x_p)^2 + y_p$ obtaining the value of $a = -\frac{1}{4}$. However, fragmentation emerged when the student performed substitution to determine the height of the ball at a position of 10 meters, namely $f(10) = -\frac{1}{4}(10 - 6)^2 + 6$. Errors in symbolic manipulation caused the student to obtain a result of 22.06 meters. Although this result contradicted the context of the problem, the student was still able to use it to conclude that the ball did not enter the goal. This indicates that the symbolic procedure was carried out without a complete coordination of meaning with the verbal source representation in the problem.

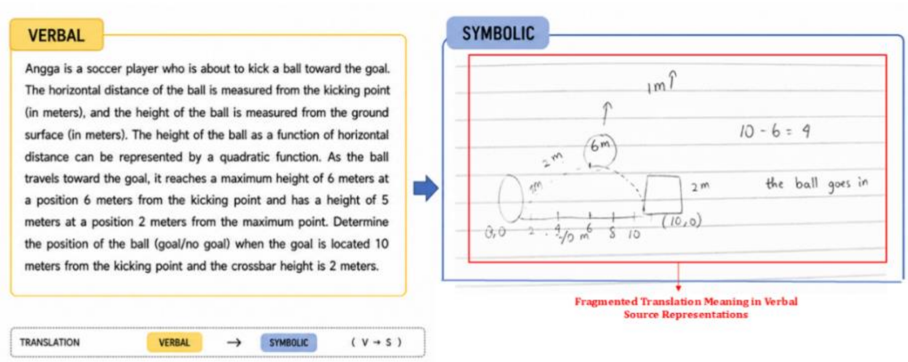


Figure 7. Answer for S3-VG

In addition, another form of fragmentation was observed in S3-VG, which involved a translation between representations from a verbal source representation to a graphical target representation (V → G). As shown in Figure 7, students drew the ball's position as a downward-opening parabola and immediately compared the ball's position to the height of the goalpost. Students understood that the ball moved upward and then downward until it approached the 2-meter height of the goalpost. This indicates that students' understanding still relies on intuitive interpretations of graphs without strong and appropriate conceptual coordination. Thus, these findings suggest that fragmentation in verbal representation occurs when students are able to construct target representations, whether visual, symbolic, or verbal, but the semantic relationships between representations have not yet been conceptually integrated during the translation process.

b) Fragmented Translation Meaning in Graphical Source Representations

In graphical source representations, fragmented translation occurs when students are able to use visual information about a parabola to construct another representation, but the semantic relationships between the representations have not yet been fully coordinated during the translation process. This is evident in S5-GV, as shown in Figure 8 below.

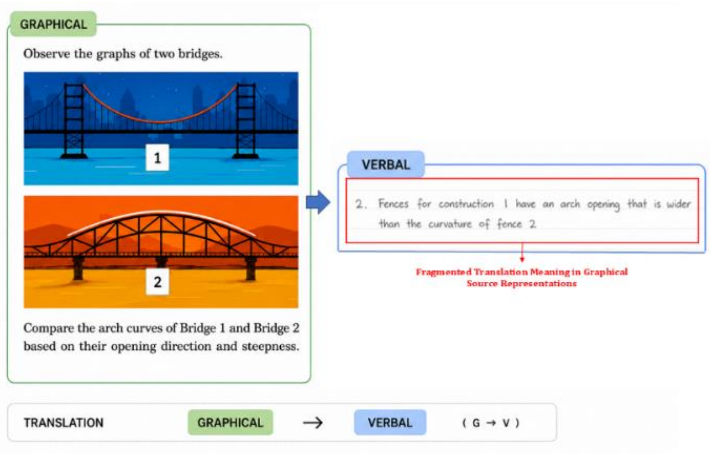


Figure 8. Answer S5-GV

Figure 8 shows that the students concluded that the curve of the first bridge railing was "more curved than the steepness of railing 2," and according to the interview results with S5, this conclusion was based on the shape of the figure. Fragmentation occurred with S5-GV, who was not yet able to differentiate between the concepts of the direction of the graph's slope and the steepness of a parabola. As a result, different visual characteristics were interpreted as having the same meaning during the translation process.

In line with this fragmentation, S4-GS uses symbolic representations to explain the shape of the bridge railing. In this regard, S4-GS is able to link the coefficient a to the direction of the parabola's opening through $a > 0$ (opening upward) and $a < 0$ (opening downward), but is not yet able to apply this to the steepness. In fact, the steepness of a parabola is related to the absolute value of $|a|$. This indicates that S5-GV successfully constructed a symbolic representation of the graph, but the conceptual meaning of the parabola's characteristics has not yet been fully integrated. Thus, these findings indicate that fragmentation in the source representation of the graph occurs when students are able to construct the target representation of the graph but are not yet able to conceptually integrate the meanings of the parabola's visual characteristics during the translation process.

c) Fragmented Translation Meaning in Symbolic Source Representations

In symbolic source representations, fragmented translation occurs when students are able to perform algebraic manipulations and construct target representations from symbolic forms, but the mathematical meaning of the procedures used has not yet been fully integrated. This is evident in S5-SS, as shown in Figure 9 below.

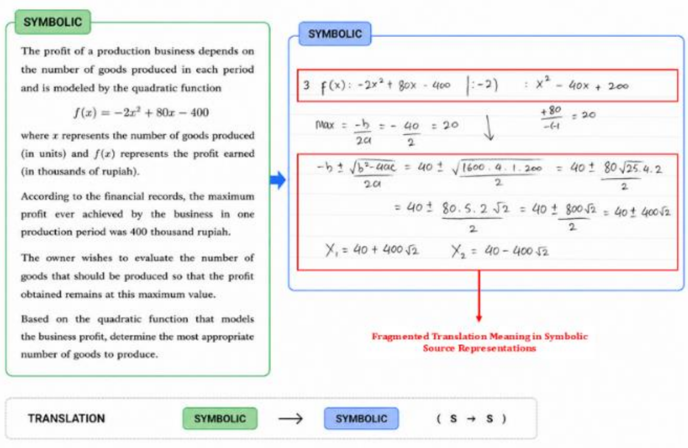


Figure 9. Answer to S5-SS

Figure 9 shows that S5-SS used the axis of symmetry formula to determine the maximum value of the quadratic function and obtained the value $x = 20$ However, fragmentation occurred after obtaining the value $x = 20$, a finding corroborated by the results of the interview with S5-SS as follows.

Researcher	*"What method did you use to solve the problem?"*
S5-SS	*"I used the formula* $x = \frac{-b}{2a}$ *to determine the maximum value."*
Researcher	*"What did you do after obtaining that result?"*
S5-SS	*"I substituted that value back into the equation and obtained* $x_1 = 40 + 400\sqrt{2}$ *and* $x_2 = 40 - 400\sqrt{2}$*."."*

In response to the interview, S5-SS indicated that during the problem-solving process after finding the maximum value via the axis of symmetry, the student continued to use the quadratic formula or the ABC method to find the value of x again. This indicates that S5-SS has not yet understood the conceptual relationship between the axis of symmetry, the maximum point of a quadratic function, and the roots of a quadratic equation. As a result, several symbolic procedures were used separately without a coherent coordination of meaning during the process of translation between representations. This is consistent with S1-SS, who used the same method, namely the quadratic formula or ABC, to determine the maximum value. However, based on interview results, students stated that they used the axis of symmetry because they could not find the value of x using the quadratic formula or the ABC method, following a classmate's suggestion. This process was carried out mechanically without a complete conceptual understanding of why the chosen formula was used. Thus, these findings indicate that fragmentation in symbolic representations can occur even when students successfully arrive at the correct answer. Fragmentation occurs because symbolic procedures are used mechanically without a comprehensive conceptual understanding of the relationships between mathematical representations during the problem-solving process.

The findings presented earlier indicate that translation between representations does not always preserve the mathematical meaning contained in the representations presented. Discrepancies in meaning manifest in various forms, such as the use of incomplete information, misinterpretation of graphs, procedural errors, and errors in drawing final conclusions. This demonstrates that successfully translation between representations does not guarantee the formation of a coherent mathematical meaning throughout the problem-solving process.

3. Linear Patterns in Representation Translation

The results of the analysis show that the process of translation between mathematical representations proceeds linearly in the cases analyzed. Students performed inter-representational translation step by step from one form of representation to another without returning to the previous representation during the problem-solving process. In Problem 1, linear translation was evident through V→G→S, V→S, and V→G. S1-VGS in Figure 1 performed inter-representational translation from the verbal source representation to the symbolic target representation via a graph without returning to the previously constructed form. Similarly, S2-VS and S3-VG perform a translation between the verbal source representation and the symbolic or graphical target representation without returning to the source representation.

Meanwhile, in Question 2, linear translation is evident through the paths G→V and G→S. S3-GV in Figure 2 shows the construction of a verbal representation based on visual observation of the graph, while S4-GS directly links the graph to the symbolic representations $a>0$ and $a<0$. Based on questions 1 and 2, it was found that students' transitions between representations tended to move toward a specific representation and continue to the next representation without revisiting the previous form of representation. In all these cases, students moved from one representation to the next without returning to the representation they had previously constructed. Therefore, the translation process observed in this study indicates a linear rather than a cyclical or back-and-forth coordination.

4. Dominance of a Single Representational Register in Representation Translation

The results of the analysis show that in the process of translation between mathematical representations and symbolic source representations, students tend to use a specific representational register throughout problem-solving. This dominance is evident when students maintain their thought processes within a single form of representation, such as the source representation. This is supported by their solutions to Problem 3, where the source representation provided was symbolic. Figure 3 illustrates that students tend to maintain their problem-solving process in the form of symbolic manipulation without coordinating with verbal or graphical representations, whether using quadratic formulas, the ABC method, or axes of symmetry. This finding indicates that, for the problems presented, the symbolic register tends to be the primary strategy for maintaining continuity in the problem-solving process.

5. The Dynamic Continuum of Mathematical Representation Translation

Based on the research findings, this study proposes a Dynamic Continuum of Translation Between Mathematical Representations. This framework was developed from the characteristics of translation between mathematical representations identified in the research data, namely linear translation, the dominance of a representation register, and fragmented translation meaning. These three characteristics do not appear as mutually exclusive categories but rather indicate variations in the process of translating mathematical representations depending on the source representation and the problem-solving context. Therefore, these characteristics are better understood as a continuum rather than as standalone categories. This continuum is empirically constructed based on the translation characteristics observed in the analyzed cases. Each characteristic in the continuum represents a form of translation that consistently appears across various subjects and types of problems; thus, these characteristics not only function as conceptual features but also have an empirical basis that can be traced in the research data. Furthermore, the characteristics in this continuum do not appear in a fixed manner but can change according to the source representation within the context of the problems students encounter. Thus, the Dynamic Continuum of Translation Between Mathematical Representations illustrates that the characteristics of mathematical representation translation are dynamic and can vary according to the source representation in the problem-solving context, rather To facilitate the visualization of the relationships among characteristics, the translation continuum among mathematical representations is presented as follows.

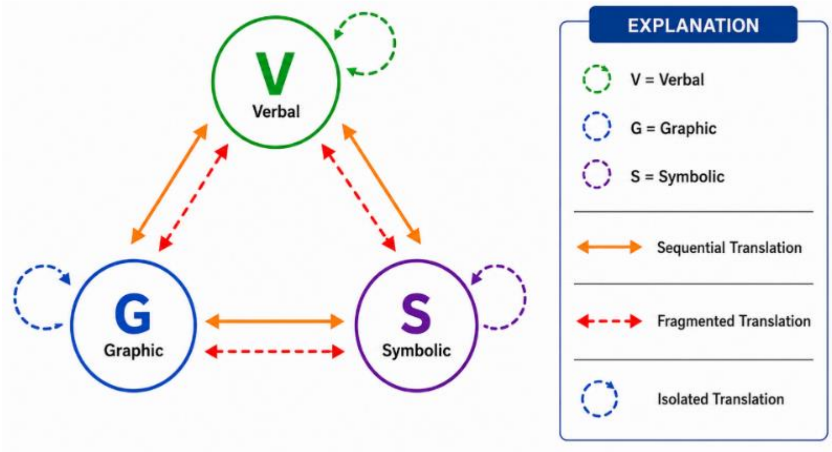


Figure 10. Dynamic Continuum of the Translation Between Mathematical Representations Identified in This Study

Table 3. Characteristics of the Dynamic Continuum of the Translation Between Mathematical Representations Identified in This Study

Process characteristics	Description	Concrete examples
Isolated	Students make representational translations while maintaining the same form of representation as the source representation throughout the problem-solving process, so coordination with other representations is not used.	S1-SS tends to retain the symbolic representation (S→S) throughout the problem solving process and does not conceptually reconnect the symbolic result with other representations.
Sequential	Students make representational translations from the source representation to the target representation in a linear and step-by-step manner, but do not return to the form of representation previously used during the problem solving process.	In Question 2, S3-GV demonstrates that G→V occurs gradually without reverting to the previously used representation.
Fragmented	Students make representational translations from the source representation to the target representation, but the coordination of mathematical meaning across representations has not yet been fully established, resulting in inconsistencies in both the process and the final outcome of problem solving.	In Question 1, S1-VGS and S4-VS show V→G→S and V→S, but the symbolic procedure was conducted without a complete alignment of meaning with the source representation; moreover, S1-VGS made a mistake in the calculation process. In Question 2, S4-GS shows that G→S can link the coefficient a with the direction of the parabola's opening, but is not yet able to apply this to the slope. Furthermore, S5-GV indicates G→V, but the interpretation of the parabola's slope is still mixed with the direction of the graph's opening. Meanwhile, in question 3, S5-SS maintains S→S, but the symbolic procedure used no longer aligns with the context of maximum profit in the question.

The continuum of translation between mathematical representations in this study, as presented in Table 3, shows that students' translation characteristics do not consistently manifest in any one particular characteristic. A student may demonstrate sequential translation on one problem but show fragmented translation on another. Thus, students' ability to translate mathematical representations is dynamic, contextual, and representation-sensitive, meaning it is influenced by the form of the source representation used as well as the mathematical interpretation of the problem the student is facing. Furthermore, this continuum is not intended to replace theory of semiotic representation registers, nor is it intended to expand upon that theory. Rather, these findings provide an empirical illustration of the characteristics of the translation process between mathematical representations that emerge in the context of solving quadratic function problems and demonstrate the variations in the translation processes carried out by students based on the source representations used.

Discussion

The findings of this study demonstrate that students' translation between mathematical representations in solving quadratic function problems forms a dynamic continuum of representational coordination consisting of isolated, sequential, and fragmented translation processes. These characteristics emerged inductively through cross-case analyses of students' written responses and interview data rather than being predetermined by an existing theoretical framework. Accordingly, the identified characteristics should not be interpreted as hierarchical levels of students' representational competence but rather as different manifestations of representational coordination that arise in response to the source representation, problem context, and students' mathematical interpretation during problem solving. This finding extends previous work suggesting that representation translation is highly dependent on the cognitive demands imposed by mathematical tasks and the representational forms embedded within those tasks (Bach et al., 2024). The present study further indicates that the source representation not only provides initial mathematical information but also shapes the pathway through which students coordinate mathematical meaning across different representations. This perspective complements previous studies that primarily emphasized representational fluency and translation accuracy by demonstrating that representational coordination is dynamic rather than fixed across different mathematical situations. The dynamic nature identified in this study also suggests that students may employ different translation processes even when solving mathematically equivalent problems because each source representation activates different cognitive resources. Consequently, representation translation should be viewed as a context-dependent cognitive process instead of a uniform procedural skill. These findings strengthen the argument that successful representation translation requires flexible coordination of mathematical meaning rather than simple procedural transformation between representational forms. Therefore, this study contributes to the growing body of research by providing empirical evidence that students' representation translation develops as a dynamic interaction between representational characteristics, contextual demands, and conceptual understanding during mathematical problem solving.

The first characteristic identified in this study is isolated translation, which refers to situations in which students remain within a single representational register throughout the entire problem-solving process. This characteristic was predominantly observed when students solved quadratic function problems presented in symbolic form, where they consistently maintained symbolic manipulation without coordinating verbal or graphical representations. Such findings correspond to the concept of treatment proposed in the Theory of Semiotic Representation, in which transformations occur within the same representational register without requiring representational conversion. Rather than constructing relationships across multiple representations, students concentrated on manipulating algebraic expressions through familiar procedures to obtain solutions. Similar tendencies have been reported in previous studies showing that symbolic tasks frequently encourage students to remain within symbolic registers because algebraic procedures are perceived as the most efficient strategy for solving formal mathematical problems (Erbilgin & Gningue, 2023). However, the present findings provide a more nuanced interpretation by showing that this symbolic dominance does not characterize all students but instead emerges only among those demonstrating isolated translation processes. This suggests that maintaining a single representational register should not automatically be interpreted as evidence of weak representational competence because students may intentionally select symbolic procedures when they perceive them as sufficient for solving a particular problem. Nevertheless, excessive reliance on symbolic manipulation may restrict opportunities to establish conceptual connections with graphical and verbal representations, thereby limiting deeper mathematical understanding. These findings reinforce the importance of designing instructional activities that encourage students to coordinate multiple representations rather than relying exclusively on symbolic procedures. Consequently, isolated translation should be understood as one possible form of representational coordination whose effectiveness depends on both the mathematical context and the representational demands of the task.

The second characteristic identified in this study is sequential translation, which describes students' ability to coordinate more than one representation through a gradual and linear translation process. This characteristic was reflected in translation pathways such as V→G→S, V→S, V→G, G→V, and G→S, demonstrating that students were capable of constructing mathematical solutions by moving systematically across different representational forms. These findings indicate that students selected translation pathways according to the characteristics of the source representation rather than applying a single translation strategy across all problems. Such behaviour supports the view that representation translation is representation-sensitive because different source representations activate different reasoning processes during mathematical problem solving. Similar conclusions have been reported who argued that representational transformation depends on the mathematical relationships emphasized within a problem, allowing students to select representations that best support their reasoning (Schifter & Russell, 2022). Likewise, students' familiarity with particular representations influences the sequence of representational transformations they construct during learning (Takaoğlu, 2024). From the perspective of Duval's Theory of Semiotic Representation, sequential translation reflects the process of conversion, in which mathematical meaning is reconstructed across different representational registers while preserving conceptual equivalence. Unlike isolated translation, sequential translation demonstrates that students actively coordinate multiple representations to facilitate mathematical reasoning rather than depending on a single representational system. Nevertheless, the findings also suggest that sequential translation does not necessarily guarantee complete conceptual understanding because students may successfully perform representational conversion while still experiencing difficulties in coordinating mathematical meaning across representations. Therefore, sequential translation should be interpreted as evidence of increasing representational flexibility, although the quality of conceptual coordination ultimately determines whether translation between mathematical representations becomes meaningful and mathematically coherent.

Although the findings demonstrate that many students were able to perform both isolated and sequential translation processes, the analysis also revealed that successful representational transformation did not always ensure successful coordination of mathematical meaning across representations. This condition is reflected in the characteristic of fragmented translation, in which students successfully translated information from one representation to another but failed to preserve conceptual consistency throughout the translation process. Fragmentation was identified across verbal, graphical, and symbolic source representations, indicating that this phenomenon is not restricted to a particular representational register but may emerge whenever students experience difficulties integrating mathematical meaning during problem solving. The presence of fragmented translation suggests that representation translation should not be evaluated solely based on the completion of representational conversion because students may produce mathematically appropriate representations while simultaneously misunderstanding the conceptual relationships underlying those representations. Similar observations have been reported in previous studies showing that many representational errors originate from students' inability to establish meaningful connections among different representations rather than from deficiencies in computational skills (O. R. U. Putri & Zukhrufurrohmah, 2022). The present findings extend this perspective by demonstrating that fragmentation can also occur when students successfully complete procedural transformations but fail to maintain semantic coherence throughout the entire translation process. In several cases, students correctly identified relevant information, selected appropriate mathematical procedures, and generated target representations, yet their final interpretations remained inconsistent with the mathematical meaning embedded in the original representation. This inconsistency indicates that conceptual integration constitutes a crucial component of successful representation translation and should be considered alongside procedural accuracy during assessment. From a pedagogical perspective, these findings imply that mathematics instruction should explicitly support students in coordinating mathematical meaning across verbal, graphical, and symbolic representations instead of emphasizing procedural conversion alone. Therefore, fragmented translation provides important evidence that representational competence involves both representational transformation and the preservation of conceptual meaning throughout mathematical problem solving.

Overall, the findings indicate that students' ability to translate between mathematical representations is dynamic, context-dependent, and strongly influenced by the characteristics of the source representation from which problem solving begins. The variation observed across the same participant, particularly in cases where isolated, sequential, and fragmented translation characteristics appeared in different tasks, demonstrates that representational coordination cannot be regarded as a stable personal attribute but rather as a flexible cognitive process that adapts to changing representational demands. This finding provides empirical support for the proposition that students activate different representational strategies depending on how mathematical information is initially presented and how meaning is subsequently constructed throughout problem solving. Consequently, the Dynamic Continuum of Translation Between Mathematical Representations proposed in this study should not be interpreted as a replacement for Duval's Theory of Semiotic Representation but rather as an empirical framework that illustrates how the processes of treatment and conversion are manifested in authentic classroom problem-solving situations. Unlike existing theoretical descriptions that primarily distinguish between representational transformation within and across registers, the proposed continuum explains how these transformations vary according to representational coordination during students' reasoning. The continuum also provides a more detailed explanation of why students may exhibit different translation characteristics despite working with mathematically equivalent concepts because the source representation influences the direction, sequence, and quality of representational coordination. This perspective contributes to the literature by emphasizing that translation between mathematical representations is inherently representation-sensitive and cannot be fully understood through analyses of translation outcomes alone. From a theoretical standpoint, the findings enrich current discussions on mathematical representation by introducing a process-oriented interpretation of representational coordination during quadratic function problem solving. Practically, the findings suggest that mathematics teachers should design learning experiences that intentionally engage students with multiple source representations to strengthen representational flexibility and conceptual understanding while reducing fragmented translation. Accordingly, the Dynamic Continuum of Translation Between Mathematical Representations offers both a theoretical contribution to representation research and a practical framework for supporting more meaningful mathematics learning through coordinated representational thinking.

Implications

The findings of this study provide both theoretical and practical implications for mathematics education by demonstrating that students' translation between mathematical representations is dynamic and influenced by the source representation used during problem solving. The proposed Dynamic Continuum of Translation Between Mathematical Representations enriches the understanding of representational coordination by illustrating how isolated, sequential, and fragmented translation processes emerge in different mathematical contexts. These findings complement the Theory of Semiotic Representation by providing empirical evidence of how treatment and conversion are manifested in students' reasoning when solving quadratic function problems. From a practical perspective, the results suggest that mathematics instruction should intentionally integrate verbal, graphical, and symbolic representations to strengthen students' conceptual understanding and representational flexibility. Teachers are also encouraged to evaluate not only students' final answers but also their translation processes to identify conceptual difficulties that may not be apparent from procedural performance alone. Furthermore, the continuum proposed in this study may serve as an analytical framework for diagnosing students' representational thinking and designing instructional strategies that promote meaningful coordination among mathematical representations. Ultimately, these findings contribute to improving the teaching and learning of quadratic functions by emphasizing the importance of representation-sensitive instruction that supports deeper conceptual understanding.

Limitations and Suggestions for Future Research

This study has several limitations that should be considered when interpreting its findings. First, the study involved a limited number of participants within a qualitative exploratory case study, restricting the generalizability of the results. Second, the analysis focused exclusively on students' translation processes in solving quadratic function problems, so the findings may not represent representational translation in other mathematical topics. Third, the proposed Dynamic Continuum of Translation Between Mathematical Representations was developed from a specific research context and therefore requires further empirical validation. Future research is encouraged to involve larger and more diverse samples using qualitative, quantitative, or mixed-methods approaches. Studies across different mathematical topics and educational levels are also needed to examine the consistency of the identified translation characteristics. In addition, future research should investigate factors such as representational fluency, conceptual understanding, and instructional strategies that may influence students' translation processes. Experimental studies are recommended to evaluate the effectiveness of learning interventions designed to strengthen representational coordination. Longitudinal research may also provide insights into how students' translation processes develop over time. Overall, further studies are expected to refine and validate the proposed continuum while contributing to more effective mathematics instruction that supports meaningful translation between mathematical representations.

CONCLUSION

This study demonstrates that students' translation between mathematical representations in solving quadratic function problems is a dynamic and representation-sensitive process that is influenced by both the source representation and students' ability to coordinate mathematical meaning throughout problem solving. The findings reveal that successful representation translation depends not only on transforming information across verbal, graphical, and symbolic representations but also on maintaining conceptual consistency among those representations. Based on the analysis, this study proposes the Dynamic Continuum of Translation Between Mathematical Representations, consisting of isolated, sequential, and fragmented translation characteristics, as an empirical framework for describing variations in students' representational coordination. These characteristics should not be interpreted as fixed levels of representational ability but rather as contextual patterns that emerge in response to different representational demands. The proposed continuum complements existing theories of mathematical representation by providing a process-oriented perspective on how students construct and coordinate mathematical meaning during translation. Therefore, mathematics instruction and assessment should emphasize not only students' ability to shift between representations but also their ability to establish meaningful conceptual connections among verbal, graphical, and symbolic representations to support deeper understanding and more effective problem solving.

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AUTHOR CONTRIBUTIONS STATEMENT

Sinta Agustin Ningseh contributed to the conceptualization of the study, research design, instrument development, data collection, data analysis, interpretation of findings, visualization of the results, and manuscript drafting. I Nengah Parta contributed to the supervision of the research, methodological guidance, validation of the research instruments, interpretation of the findings, and critical review of the manuscript. Anita Dewi Utami contributed to the research supervision, theoretical guidance, refinement of the data interpretation, manuscript revision, and overall evaluation of the final manuscript. All authors discussed the results, contributed to the improvement of the manuscript, and approved the final version for publication.

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