Journal of Advanced Sciences and Mathematics Education

Volume 6, Issue 2, 392-412.
e_ISSN: 2798-1606
DOI: 10.58524/jasme.v6i2.1258

Epistemological, ontogenic, and didactical learning obstacles in the Pythagorean Theorem: A systematic literature review

Yuce Sandra
Universitas Pendidikan Indonesia, INDONESIA

Didi Suryadi*
Universitas Pendidikan Indonesia, INDONESIA

Dadan Dasari
Universitas Pendidikan Indonesia, INDONESIA

Article Info

Article history:

Received: March 27, 2026
Revised: April 25, 2026
Accepted: Mei 31, 2026

Keywords:

Epistemological Obstacles; Learning Obstacles; Mathematics Education; Pythagorean Theorem; Systematic Literature Review.

Abstract

Background: Geometry is a fundamental branch of mathematics that develops students' spatial reasoning, logical thinking, and problem-solving abilities. One important topic in geometry is the Pythagorean Theorem, which remains challenging for many students due to persistent conceptual and procedural difficulties in solving related problems.
Aims: This study aimed to identify and analyze students' learning obstacles in the Pythagorean Theorem from epistemological, ontogenic, and didactical perspectives through a systematic literature review.
Method: This study employed a qualitative approach using a Systematic Literature Review (SLR) following the PRISMA protocol. Twenty articles published between 2019 and 2025 from 14 academic journals were selected based on predefined inclusion and exclusion criteria. The collected studies were analyzed qualitatively to identify patterns of learning obstacles and instructional challenges in learning the Pythagorean Theorem.
Results: The findings revealed that students' difficulties were predominantly associated with epistemological obstacles, including fragmented understanding of powers and roots, misconceptions regarding side relationships in right triangles, reliance on trial-and-error strategies when using Pythagorean triples, and inability to interpret the theorem as a meaningful geometric relationship. These difficulties were further reinforced by ontogenic obstacles such as limited prerequisite knowledge, weak visual–spatial reasoning, and poor problem-modeling ability. In addition, didactical obstacles emerged from instructional practices emphasizing formula memorization and procedural computation rather than conceptual reasoning, visualization, and proof.
Conclusion: This study concludes that learning obstacles in the Pythagorean Theorem are multidimensional and interconnected across epistemological, ontogenic, and didactical domains. Therefore, mathematics instruction should integrate conceptual understanding, visual representation, and contextual problem-solving activities to support meaningful learning of the Pythagorean Theorem.

To cite this article: Sandra, Y., Suryadi, D. & Dasari, D. (2026). Epistemological, ontogenic, and didactical learning obstacles in the Pythagorean Theorem: a systematic literature review. *Journal of Advanced Sciences and Mathematics Education, 6*(2), 392-412.

INTRODUCTION

Geometry is one of the fundamental branches of mathematics that plays an important role in developing students' logical thinking, visualization ability, spatial reasoning, and problem-solving skills. Through geometry learning, students are expected to understand the relationships among points, lines, planes, measurements, and spatial objects within both abstract and contextual situations (Bernabeu et al., 2024; Dorel, 2023; Hwang et al., 2020; Jablonski & Ludwig, 2023). Geometry also serves as an essential foundation for understanding many mathematical concepts used in science, engineering, architecture, and technology (Venetis, 2020). One of the important topics derived from geometry is the Pythagorean Theorem, which becomes a foundational principle

***Corresponding author:**
Didi Suryadi, Universitas Pendidikan Indonesia, Indonesia
didisuryadi@upi.edu

for understanding right triangles and geometric relationships (Blasjo, 2022; Hahn, 2022; Silva, 2022). The Pythagorean Theorem occupies a central position in mathematics learning because it supports the understanding of distance, shape, trigonometry, and spatial measurement (Keyik et al., 2026). In the context of geometry, the theorem establishes a mathematical relationship among the sides of a right triangle, where the square of the hypotenuse is equal to the sum of the squares of the other two sides. This relationship allows students to solve various geometric and measurement problems presented visually or contextually. Furthermore, the Pythagorean Theorem contributes significantly to the development of mathematical reasoning and conceptual understanding in geometry learning. However, despite its importance in mathematics education, many students still experience difficulties in understanding and applying the theorem meaningfully. Therefore, understanding students' learning difficulties in the Pythagorean Theorem becomes important for improving geometry instruction and supporting meaningful mathematical learning experiences.

In reality, the Pythagorean Theorem remains a difficult topic for many students because numerous conceptual and procedural errors continue to emerge during problem solving. Students frequently experience difficulties in understanding the concept of the theorem, selecting appropriate solution procedures, and performing arithmetic operations involving powers and roots (Alam & Mohanty, 2024; Baiduri et al., 2020; Barbieri & Rodrigues, 2025). These difficulties subsequently affect students' abilities to solve advanced and contextual problems involving the application of the Pythagorean Theorem (Deshpande et al., 2021; Dewi et al., 2025; Taamneh et al., 2024). Many students are also unable to transfer ideas from geometrical images into mathematical representations, resulting in weak interpretation of right-triangle relationships and measurement problems (Ulusoy, 2021). In addition, students often experience confusion when determining the hypotenuse or selecting the appropriate side lengths to substitute into the formula. Such conditions indicate that students' understanding of the theorem tends to remain procedural rather than conceptual. Students may memorize the formula successfully but still fail to understand when, why, and how the theorem should be applied meaningfully in problem-solving situations. As a consequence, students frequently encounter difficulties when solving story problems and contextual tasks requiring interpretation, visualization, and mathematical reasoning (Basid et al., 2024; Hughes et al., 2020; Kohen et al., 2022; Purcar et al., 2024; Verschaffel et al., 2020). These findings demonstrate that students' learning difficulties are not merely related to computational weakness but also involve conceptual misunderstanding and fragmented mathematical thinking. Therefore, investigating the nature and characteristics of students' learning obstacles in the Pythagorean Theorem is essential for supporting meaningful geometry learning.

Learning obstacles in the Pythagorean Theorem can be analyzed through epistemological, ontogenic, and didactical perspectives. Epistemological obstacles emerge when students possess limited conceptual understanding and are unable to transfer mathematical knowledge into different contexts (Fielding & Makar, 2022; Maknun et al., 2022; Rodriguez et al., 2020; Sukarma et al., n.d.). Students may understand formulas procedurally yet fail to interpret geometric relationships conceptually or apply the theorem in unfamiliar situations. Ontogenic obstacles are associated with students' cognitive readiness, prerequisite knowledge, visual–spatial reasoning ability, and mathematical maturity during learning processes (Silver & Libertus, 2022). Students with weak understanding of exponents, roots, and geometric representation often experience substantial difficulties in understanding the conceptual structure of the Pythagorean Theorem. Meanwhile, didactical obstacles originate from instructional practices that emphasize formula memorization, repetitive exercises, and procedural computation rather than conceptual reasoning and geometric visualization. Existing studies generally focused on empirical classroom investigations examining students' misconceptions, procedural errors, or specific instructional interventions separately. Although several studies discussed learning difficulties in geometry learning, limited attention has been given to systematically integrating epistemological, ontogenic, and didactical obstacles within a unified analytical framework. Furthermore, previous systematic literature review studies discussing learning obstacles predominantly focused on algebra or general mathematics concepts rather than specifically investigating the Pythagorean Theorem. Consequently, there remains insufficient comprehensive understanding regarding the interconnected nature of students' learning obstacles in the Pythagorean Theorem across conceptual, cognitive, and instructional dimensions.

Although previous studies have extensively investigated geometry learning, conceptual understanding, misconceptions, and instructional approaches in mathematics education, most research has focused on isolated aspects of students' difficulties rather than comprehensively synthesizing multidimensional learning obstacles within a unified framework. Several studies examined students' geometrical misconceptions, conceptual understanding, and visualization difficulties in geometry learning (Gurmu et al., 2024; Uygun et al., 2024), while other studies explored epistemological obstacles in mathematics topics such as trigonometry and transformation geometry (Kandaga et al., 2022; Maknun et al., 2022). In addition, technological interventions such as GeoGebra, virtual reality, and augmented reality have been widely investigated to improve students' geometry learning outcomes and spatial reasoning abilities. Research specifically related to the Pythagorean Theorem has predominantly emphasized instructional practices, proof construction, and mathematical argumentation rather than students' multidimensional learning obstacles (Due, 2024; Jackson & Johnson, 2024; Sol et al., 2025). Furthermore, studies on learning obstacles in mathematics have generally focused on probability learning or broader mathematics contexts rather than geometry learning specifically (Castellví, 2024; Sari et al., 2024). Although these studies contributed significantly to mathematics education literature, limited attention has been given to systematically integrating epistemological, ontogenic, and didactical obstacles simultaneously within the context of the Pythagorean Theorem. Existing research also tends to investigate procedural errors, misconceptions, or instructional interventions separately without examining how conceptual understanding, prerequisite knowledge, visual–spatial reasoning, and instructional practices interact in shaping students' learning difficulties. Moreover, systematic literature review studies specifically synthesizing multidimensional learning obstacles in the Pythagorean Theorem remain limited in recent mathematics education research. Consequently, there is still insufficient comprehensive understanding regarding the interconnected nature of epistemological, ontogenic, and didactical obstacles experienced by students in learning the Pythagorean Theorem. Therefore, this study addresses the existing research gap by systematically synthesizing previous studies to analyze learning obstacles in the Pythagorean Theorem through epistemological, ontogenic, and didactical perspectives within a unified systematic literature review framework.

Based on the identified research gaps, this study aimed to systematically identify and analyze students' learning obstacles in the Pythagorean Theorem from epistemological, ontogenic, and didactical perspectives through a systematic literature review approach. Specifically, this study sought to describe the dominant forms of conceptual, cognitive, and instructional obstacles experienced by students in learning the Pythagorean Theorem. The study also aimed to synthesize patterns of misconceptions, reasoning difficulties, visual spatial challenges, and instructional limitations reported in previous studies related to geometry learning. Furthermore, this study intended to examine how epistemological, ontogenic, and didactical obstacles interact and contribute to students' fragmented understanding of the Pythagorean Theorem. Through this systematic synthesis, the study is expected to provide a more comprehensive understanding of students' mathematical learning difficulties in geometry education. The findings are also expected to contribute theoretically by positioning learning obstacles as interconnected cognitive and instructional phenomena rather than isolated procedural errors. Practically, this study is expected to provide implications for mathematics educators in designing instructional strategies that emphasize conceptual understanding, visual representation, contextual reasoning, and reflective problem-solving activities. In addition, the findings may support the development of curriculum designs and geometry learning approaches that facilitate meaningful mathematical understanding among students. The study is further expected to serve as a reference for future research investigating conceptual and didactical obstacles in mathematics learning contexts. Ultimately, this systematic literature review aims to support the improvement of geometry instruction quality and students' conceptual understanding of the Pythagorean Theorem in mathematics education.

LITERATURE REVIEW

Geometry is an essential domain in mathematics education because it supports the development of visualization ability, logical reasoning, spatial understanding, and problem-solving skills. Through geometry learning, students are expected to construct relationships among shapes, measurements, positions, and mathematical representations meaningfully. One of the most important concepts in geometry learning is the Pythagorean Theorem, which becomes a foundational principle for understanding right triangles, distance relationships, and geometric measurement (Jablonski & Ludwig, 2023; Silva, 2022). The theorem is also closely connected with trigonometry, coordinate systems, algebraic reasoning, and mathematical proof. Consequently, understanding the Pythagorean Theorem is considered important for supporting students' mathematical competence at higher educational levels (Dewi et al., 2025; Taamneh et al., 2024). In mathematics learning, conceptual understanding of the Pythagorean Theorem requires students not only to memorize formulas but also to interpret geometric relationships among triangle sides conceptually and contextually (Keyik et al., 2026). Students are expected to understand why the relationship between the square of the hypotenuse and the squares of the other sides occurs mathematically. However, many students still perceive the theorem as merely a computational procedure involving formula substitution rather than as a meaningful geometric relationship. Such conditions frequently result in fragmented understanding and weak mathematical reasoning during problem solving. Therefore, understanding students' conceptual difficulties in learning the Pythagorean Theorem becomes important within mathematics education research.

Learning difficulties in the Pythagorean Theorem are frequently associated with misconceptions, procedural dependency, and limited conceptual understanding. Many students experience difficulties in distinguishing the hypotenuse from the perpendicular sides of a triangle, especially when the orientation of the figure changes. Students also frequently misunderstand the concepts of powers and roots, which subsequently affects their ability to apply the theorem correctly (Akolekar et al., 2025; Brown et al., 2020; Taamneh et al., 2024). In addition, students often rely on trial-and-error strategies when using Pythagorean triples without understanding the proportional relationship among side lengths conceptually. These conditions indicate that students tend to focus on procedural calculation rather than geometric reasoning and conceptual interpretation. Several studies reported that students are able to use the formula $a^2+b^2=c^2$ correctly in routine exercises but fail to interpret the geometric meaning underlying the formula. Students also experience difficulties transferring mathematical ideas from visual representation into symbolic representation during problem solving (Kaitera & Harmoinen, 2022; Ünal et al., 2023; Žakelj & Klančar, 2022). Consequently, many students encounter obstacles when solving contextual or story problems involving the application of the Pythagorean Theorem (Deshpande et al., 2021; Taamneh et al., 2024). Such findings demonstrate that conceptual understanding plays a crucial role in supporting meaningful geometry learning. Therefore, identifying the characteristics of students' conceptual difficulties is essential for improving instructional quality in geometry education.

In mathematics education, students' learning difficulties are commonly analyzed through the concept of learning obstacles. Learning obstacles refer to barriers experienced by students during learning processes that prevent them from achieving meaningful conceptual understanding (Bringula et al., 2021; Chew & Cerbin, 2021; Mailizar et al., 2020; Nouraey & Al-Badi, 2023). One important category is epistemological obstacles, which emerge when students possess limited conceptual schemas and are unable to transfer mathematical understanding into broader contexts (Alam & Mohanty, 2024; Maknun et al., 2022). Students experiencing epistemological obstacles may understand mathematical procedures mechanically yet fail to interpret concepts meaningfully in unfamiliar situations. In the context of the Pythagorean Theorem, epistemological obstacles often appear when students memorize formulas without understanding geometric relationships among the sides of right triangles (Maknun et al., 2022; Mukuka et al., 2023; Taamneh et al., 2024). Another category is ontogenic obstacles, which are related to students' cognitive readiness, prerequisite knowledge, visual–spatial reasoning, and developmental characteristics during learning. Students with weak understanding of exponents, roots, and geometric representation frequently experience substantial difficulties in understanding the conceptual structure of the Pythagorean Theorem (Bila et al., 2024; Mutambara & Tsakeni, 2022). Furthermore, didactical obstacles emerge from instructional practices emphasizing formula memorization, repetitive exercises, and teacher-centered learning rather than conceptual exploration and reflective reasoning. These three categories of learning obstacles are interconnected because students' conceptual difficulties are influenced simultaneously by cognitive, instructional, and contextual factors. Therefore, analyzing learning obstacles through epistemological, ontogenic, and didactical perspectives provides a more comprehensive understanding of students' mathematical learning difficulties.

Previous studies investigating the Pythagorean Theorem predominantly focused on students' misconceptions, procedural errors, instructional interventions, and technological integration in geometry learning. Several studies explored the use of visual media, GeoGebra, augmented reality, and contextual learning approaches to improve students' understanding of geometric concepts (Chonchaiya & Srithammee, n.d.; Nadzeri et al., 2024). Other studies examined students' difficulties in proof construction, mathematical argumentation, and representation during geometry problem solving (Campbell & Zelkowski, 2020; Dello Iacono, 2021; Ramírez-Uclés & Ruiz-Hidalgo, 2022; Stylianides et al., 2024; Winer & Battista, 2022). Although these studies contributed significantly to mathematics education literature, most investigations analyzed students' difficulties separately rather than integrating them within a multidimensional framework of learning obstacles. Existing research generally emphasized procedural mistakes or classroom interventions without comprehensively examining how epistemological, ontogenic, and didactical obstacles interact simultaneously during learning processes (Ario et al., 2025; Hendriyanto et al., 2024; Narciss & Alemdag, 2025). In addition, many studies were conducted as empirical classroom investigations with limited contextual scope and participant coverage. Systematic literature review studies discussing learning obstacles also predominantly focused on algebra, probability, or general mathematics concepts rather than specifically investigating geometry learning in the Pythagorean Theorem. Consequently, there remains limited synthesis regarding the interconnected nature of students' conceptual understanding, visual–spatial reasoning, prerequisite knowledge, and instructional practices within geometry learning contexts. Furthermore, the literature still lacks comprehensive analysis explaining how different types of learning obstacles contribute to students' fragmented understanding of the Pythagorean Theorem. Therefore, a systematic synthesis of previous studies is necessary to provide deeper understanding regarding multidimensional learning obstacles in geometry education.

Systematic literature review approaches have become increasingly important in mathematics education research because they enable researchers to synthesize findings from multiple studies systematically and comprehensively. Through systematic synthesis, researchers may identify recurring conceptual patterns, instructional issues, and cognitive difficulties experienced by students across different learning contexts. In the context of geometry learning, systematic reviews allow researchers to examine how conceptual understanding, reasoning ability, visualization, and instructional design interact in shaping students' mathematical learning experiences. Furthermore, systematic literature reviews support the identification of research gaps and theoretical inconsistencies that may not be visible in isolated empirical investigations. The PRISMA framework is frequently used in systematic review studies because it provides transparent procedures for article selection, screening, and synthesis processes. By applying systematic review methods, researchers may construct broader theoretical perspectives regarding students' learning obstacles and conceptual development in mathematics learning. In the context of the Pythagorean Theorem, systematic synthesis is important because students' learning difficulties are multidimensional and involve conceptual, cognitive, and instructional aspects simultaneously. Integrating epistemological, ontogenic, and didactical perspectives within a systematic literature review framework may therefore provide more comprehensive understanding regarding students' geometry learning obstacles. Such synthesis may also support the development of adaptive instructional strategies emphasizing conceptual reasoning, visualization, and contextual understanding rather than procedural memorization alone. Therefore, systematic literature review studies focusing on learning obstacles in the Pythagorean Theorem contribute significantly to mathematics education research and instructional improvement in geometry learning.

METHOD

Research Design

This study employed a qualitative research approach using a Systematic Literature Review (SLR) method to investigate students' learning obstacles in the Pythagorean Theorem from epistemological, ontogenic, and didactical perspectives. The SLR approach was selected because the study aimed to systematically identify, evaluate, and synthesize relevant previous studies to obtain a comprehensive understanding of learning obstacles experienced by students in geometry learning. The systematic review was combined with a qualitative meta-synthesis approach focusing on describing, interpreting, and integrating findings from previous research related to the Pythagorean Theorem. This approach enabled the researcher to analyze conceptual, cognitive, and instructional difficulties comprehensively across multiple educational contexts. The systematic review process in this study referred to the five stages proposed by Richter et al., namely formulating the study question and conceptual framework, establishing selection criteria, developing a search strategy, selecting and screening literature, and synthesizing and reporting findings. The conceptual framework of this study was based on the theory of learning obstacles, which classifies learning barriers into epistemological, ontogenic, and didactical obstacles. Epistemological obstacles refer to students' limited conceptual understanding and inability to transfer mathematical knowledge into different contexts. Ontogenic obstacles are associated with cognitive readiness, prerequisite knowledge, and students' developmental characteristics, while didactical obstacles originate from instructional approaches and classroom learning practices. To ensure transparency and methodological rigor, the literature identification and selection stages were reported using the PRISMA (Preferred Reporting Items for Systematic Reviews and Meta-Analyses) framework. Therefore, the qualitative systematic review design was considered appropriate for synthesizing multidimensional learning obstacles in the Pythagorean Theorem comprehensively.

Participant

The participants in this study were not individual students directly involved in classroom activities but rather research articles selected systematically based on predefined inclusion and exclusion criteria. The selected literature consisted of 20 research articles obtained from 14 national and international mathematics education journals discussing learning obstacles in the Pythagorean Theorem. The articles were published between 2019 and 2025 to ensure that the review captured recent issues and developments in mathematics education research. The selected studies included both qualitative and quantitative research investigating students' conceptual difficulties, misconceptions, instructional barriers, and geometry learning challenges related to the Pythagorean Theorem. The literature represented various educational contexts, research methodologies, instructional settings, and participant characteristics within mathematics learning environments. Articles discussing unrelated mathematics topics or studies lacking sufficient relevance to learning obstacles in the Pythagorean Theorem were excluded from the analysis. The selected literature functioned as the primary source of qualitative data for the systematic review process. The diversity of selected articles enabled the study to capture broader perspectives regarding conceptual, cognitive, and instructional difficulties experienced by students in geometry learning. Through the selected literature, the study synthesized recurring patterns of students' misconceptions, procedural errors, visual–spatial difficulties, and instructional limitations associated with the Pythagorean Theorem. Therefore, the selected research articles served as the principal data source for constructing comprehensive understanding regarding students' learning obstacles in geometry education.

Instrument

The primary instrument used in this study was a systematic review protocol developed based on the PRISMA framework to guide the processes of article identification, screening, eligibility assessment, and qualitative synthesis systematically. The review protocol included search strategies, inclusion criteria, exclusion criteria, data extraction forms, and thematic categorization guidelines. The literature search process was conducted using several academic databases, including Google Scholar and Scopus. Through Google Scholar, articles from Sinta-indexed journals were identified, while the Scopus search included articles from journals indexed in Springer, ScienceDirect, and Taylor & Francis databases. The search strategy used Boolean operators such as AND and OR to combine keywords systematically and consistently. The search strings used in the databases were ("learning obstacle" OR "learning difficulties") AND ("Pythagorean Theorem" OR "Pythagorean theorem") with publication year limitations between 2019 and 2025. After the initial search process, titles and abstracts were screened to ensure alignment with the focus of the study, namely students' learning obstacles in the Pythagorean Theorem. Articles meeting the inclusion criteria were then reviewed in full text to evaluate their relevance, methodological quality, and contribution to the study objectives. The inclusion criteria consisted of studies discussing student learning obstacles, the Pythagorean Theorem, or learning difficulties related to the Pythagorean Theorem, while exclusion criteria included articles lacking full-text access, insufficient scientific quality, or unrelated research focus. All stages of article identification, screening, selection, and inclusion were documented systematically and visualized using the PRISMA flowchart to ensure transparency and repeatability of the research process.

Procedure

The research procedure in this study followed several systematic stages based on the PRISMA framework and systematic review methodology. The first stage involved formulating the research problem and determining the conceptual framework related to learning obstacles in the Pythagorean Theorem. The second stage consisted of establishing inclusion and exclusion criteria as indicators for selecting relevant research articles. The inclusion criteria included studies discussing learning obstacles, the Pythagorean Theorem, or learning difficulties related to geometry learning published between 2019 and 2025 using qualitative or quantitative research methods. The third stage involved developing search strategies and conducting literature searches using academic databases such as Google Scholar and Scopus with predetermined keywords and Boolean search combinations. After the search process was completed, the fourth stage consisted of screening article titles and abstracts to remove duplicate and irrelevant studies. Eligible articles were then reviewed in full text to ensure alignment with the objectives and conceptual framework of the study. During the fifth stage, important information related to conceptual difficulties, instructional barriers, cognitive challenges, and geometry learning characteristics was extracted systematically using data extraction forms. The extracted findings were subsequently synthesized and classified into epistemological, ontogenic, and didactical obstacle categories. The final stage involved interpreting and reporting the synthesis findings comprehensively to construct broader theoretical understanding regarding multidimensional learning obstacles in the Pythagorean Theorem. Through these systematic procedures, the study ensured methodological transparency, consistency, and rigor throughout the systematic literature review process.

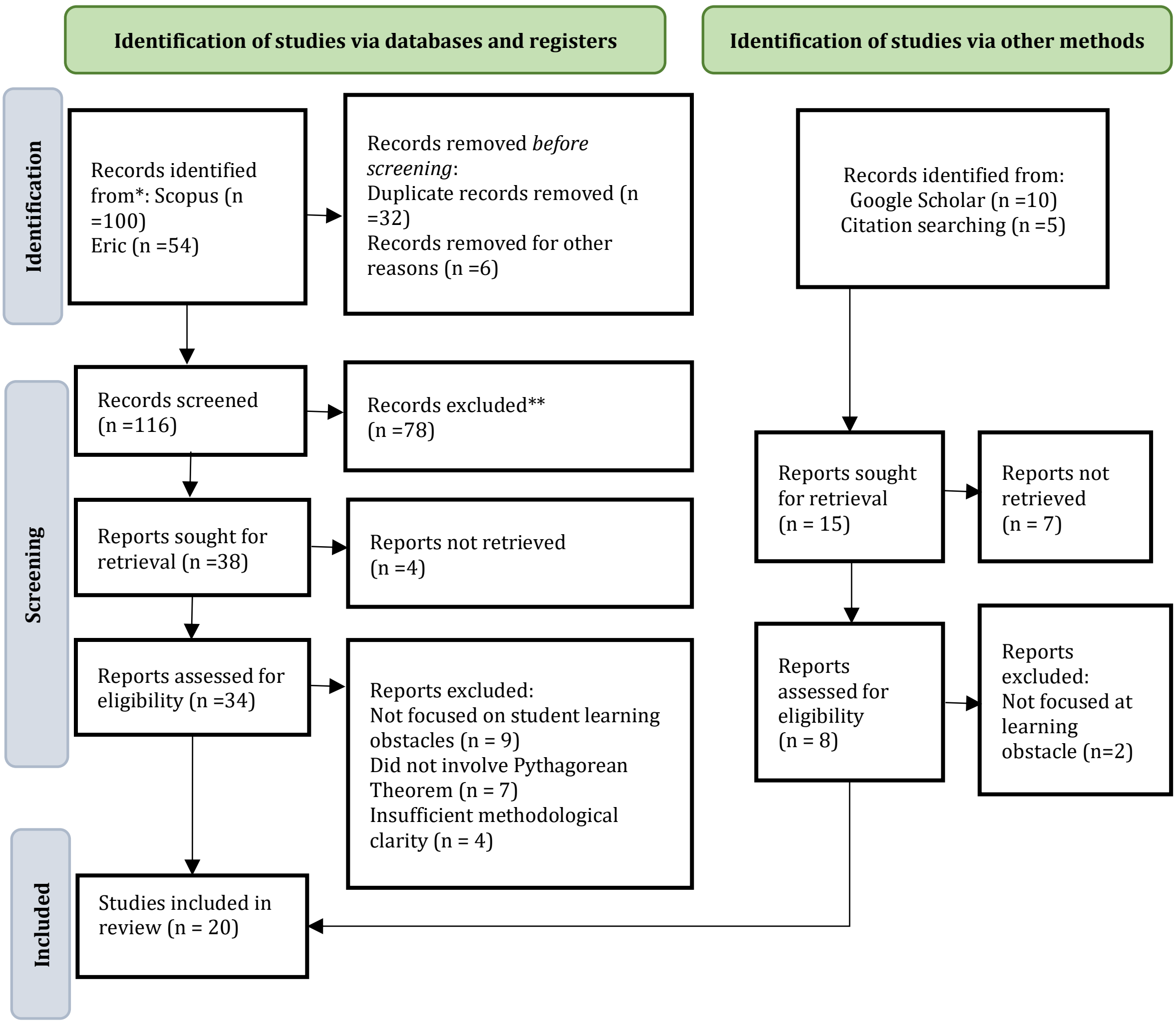

Figure 1. Result of flowchart of PRISMA on 100 articles

Analysis plan

Data analysis in this study was conducted through qualitative meta-synthesis techniques involving classification, comparison, and thematic synthesis of findings from the selected literature.

Each selected article was analyzed systematically based on key information including title, publication year, journal name, research objectives, research methods, participants or samples, and findings related to learning obstacles in the Pythagorean Theorem. The findings extracted from each study were subsequently grouped according to epistemological, ontogenic, and didactical obstacle categories. Epistemological obstacles were identified from studies discussing conceptual misunderstanding, fragmented knowledge, and students' inability to transfer mathematical understanding into broader contexts. Ontogenic obstacles were analyzed from findings related to prerequisite knowledge, cognitive readiness, visual–spatial reasoning ability, and developmental characteristics influencing students' learning processes. Meanwhile, didactical obstacles were identified from instructional practices emphasizing procedural computation, formula memorization, and teacher-centered learning rather than conceptual exploration and geometric reasoning. The thematic synthesis process aimed to identify recurring conceptual patterns, instructional issues, and cognitive difficulties across different studies systematically. Similarities and differences among previous findings were compared to construct broader theoretical understanding regarding multidimensional learning obstacles in geometry learning. The analysis also examined the relationships among conceptual understanding, instructional practices, and students' reasoning difficulties during problem solving. Through this synthesis process, the study obtained a comprehensive overview regarding the patterns and characteristics of students' learning obstacles in the Pythagorean Theorem. Therefore, qualitative meta-synthesis enabled deeper interpretation regarding the interconnected nature of epistemological, ontogenic, and didactical obstacles within mathematics education contexts.

RESULTS AND DISCUSSION

Results

Distribution of Learning Obstacles in the Pythagorean Theorem

The synthesis of the selected literature revealed that students experienced multidimensional learning obstacles in understanding and solving problems related to the Pythagorean Theorem. The reviewed studies consistently indicated that students' difficulties were not limited to procedural computation but also involved conceptual misunderstanding, weak prerequisite knowledge, visual–spatial difficulties, and instructional limitations. Analysis of the 20 selected articles demonstrated that learning obstacles could be categorized into three interconnected dimensions, namely epistemological obstacles, ontogenic obstacles, and didactical obstacles. Among these categories, epistemological obstacles emerged as the most dominant learning barrier experienced by students. These obstacles were primarily reflected in fragmented conceptual understanding of powers, roots, geometric relationships, and the application of the Pythagorean Theorem in contextual situations. Ontogenic obstacles were associated with students' cognitive readiness, prerequisite knowledge, visual–spatial reasoning ability, and affective conditions during problem solving. Meanwhile, didactical obstacles originated from instructional practices emphasizing procedural computation and formula memorization rather than conceptual reasoning, visualization, and reflective learning. The findings also revealed that these obstacles interacted dynamically rather than appearing independently during learning processes. Weak conceptual understanding frequently intensified students' anxiety and uncertainty during problem solving, while inappropriate instructional approaches reinforced procedural dependency and fragmented reasoning structures. Consequently, the synthesis indicates that learning obstacles in the Pythagorean Theorem are cumulative, multidimensional, and interconnected across conceptual, cognitive, and instructional dimensions.

Table 1. Distribution of learning obstacles identified in the reviewed studies

Types of Learning Obstacles	Percentage	Dominant Characteristics
Epistemological Obstacles	41%	Misconceptions, fragmented conceptual understanding, procedural dependency
Ontogenic Obstacles	32%	Weak prerequisite knowledge, visual–spatial difficulties, affective barriers
Didactical Obstacles	27%	Formula memorization, procedural teaching, inadequate instructional design

The findings presented in Table 1 indicate that epistemological obstacles were the most dominant category identified in the reviewed studies, accounting for 41% of the total findings. Students frequently experienced difficulties in understanding the conceptual meaning of the Pythagorean Theorem, interpreting geometric relationships, and transferring mathematical understanding into contextual situations. In many cases, students memorized formulas successfully but failed to understand why and when the theorem should be applied. Ontogenic obstacles accounted for 32% of the identified barriers and were associated with students' limited mastery of prerequisite concepts such as powers, roots, algebraic manipulation, and geometric representation. Furthermore, students' affective conditions, including anxiety, low confidence, and lack of motivation, were also identified as important ontogenic barriers influencing problem-solving performance. Didactical obstacles represented 27% of the findings and emerged primarily from instructional practices emphasizing repetitive procedural exercises rather than conceptual reasoning and geometric visualization. Several studies reported that students were rarely encouraged to construct proofs, interpret visual representations, or explain the conceptual meaning underlying the theorem. Consequently, students tended to develop procedural habits without reflective mathematical understanding. These findings demonstrate that learning obstacles in the Pythagorean Theorem originate from interconnected conceptual, cognitive, and instructional factors simultaneously. Therefore, effective geometry instruction requires comprehensive approaches capable of addressing all dimensions of learning obstacles systematically.

Learning Obstacles in Using the Concept of Powers and Roots

One of the most frequently reported learning obstacles involved students' difficulties in understanding and applying the concepts of powers and roots as prerequisite knowledge for the Pythagorean Theorem. The reviewed studies consistently revealed that students experienced recurring conceptual and procedural errors when performing exponentiation and radical operations. Many students demonstrated confusion between squaring operations and square roots, resulting in incorrect symbolic representations and computational procedures. For example, students frequently transformed expressions such as $c^2 = 17$ into $c^2 = \sqrt{17}$, indicating misunderstanding regarding the relationship between exponentiation and root operations. Similar misconceptions were also found in the misuse of radical notation, such as writing $AB^2 = 10$ instead of $AB = \sqrt{10}$. These errors were not isolated mistakes but rather recurring patterns appearing across multiple studies and educational contexts. Furthermore, students often experienced difficulties performing algebraic transposition during equation manipulation. Several studies reported that students incorrectly transformed equations such as $225 = a^2 + 144$ into $a^2 = 225 + 144$, demonstrating weak understanding of equality and inverse operations. Such findings indicate that students' difficulties in the Pythagorean Theorem are strongly influenced by incomplete understanding of prerequisite algebraic concepts. Consequently, conceptual misunderstanding of powers, radicals, and algebraic manipulation significantly hindered students' ability to apply the Pythagorean Theorem accurately during problem solving.

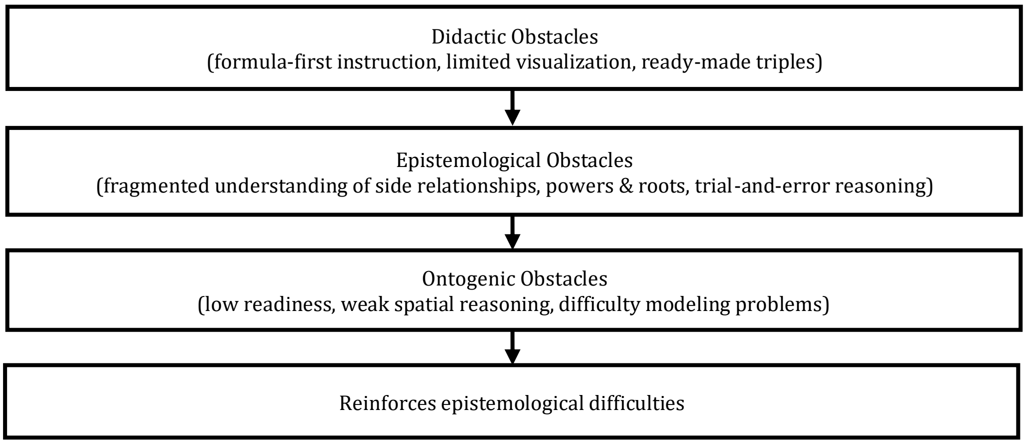

Figure 2. Interaction Among Learning Obstacles in the Pythagorean Theorem

The Sankey diagram illustrates the interaction among epistemological, ontogenic, and didactical obstacles identified in the reviewed studies. The visualization demonstrates that didactical obstacles frequently initiate chains of conceptual difficulties by emphasizing procedural teaching approaches over conceptual understanding and visual reasoning. These instructional limitations subsequently contribute to epistemological obstacles, which are further reinforced by ontogenic barriers such as weak prerequisite knowledge and low cognitive readiness. The diagram also indicates that students' learning obstacles are systemic and interconnected rather than isolated procedural errors. Consequently, ineffective instructional practices may intensify conceptual misunderstanding and affective difficulties simultaneously during geometry learning processes.

Learning Obstacles in Understanding Side Relationships in Right Triangles

The synthesis results revealed that students also experienced substantial difficulties in understanding the relationships among the sides of right triangles. These obstacles emerged through epistemological, ontogenic, and didactical dimensions simultaneously across the reviewed literature. Many students experienced difficulties sketching right triangles accurately and determining the values and positions of each side correctly. Students frequently misidentified the hypotenuse, adjacent side, and opposite side, especially when the orientation of the triangle changed. Misconceptions regarding altitude lines also emerged, where students incorrectly extended altitude lines beyond their proper perpendicular positions. Furthermore, students often failed to recognize all triangles formed within composite geometrical figures because they focused only on visible structures without considering hidden or overlapping components. Other obstacles included difficulties visualizing geometric figures, projecting points onto planes, identifying perpendicular relationships, and interpreting geometrical representation contextually. These conceptual limitations subsequently affected students' ability to apply the Pythagorean Theorem correctly in solving geometric problems. In several studies, students were unable to determine the correct side lengths to substitute into the formula because they lacked understanding regarding side relationships within right triangles. Such findings indicate that students' geometric reasoning and visual–spatial abilities remain underdeveloped during geometry learning. Therefore, strengthening students' visual representation skills and conceptual understanding of geometric relationships becomes essential for supporting meaningful learning of the Pythagorean Theorem.

Table 2. Dominant learning obstacles identified in the reviewed studies

Conceptual Area	Dominant Learning Obstacles
Powers and Roots	Misinterpretation of exponents and radicals
Triangle Relationships	Difficulty identifying hypotenuse and perpendicular sides

Pythagorean Triples	Reliance on trial-and-error strategies
Problem Modeling	Difficulty translating contextual situations into mathematical models
Conceptual Understanding	Formula memorization without conceptual interpretation

Table 2 demonstrates that students' learning obstacles were concentrated primarily in conceptual interpretation and geometric reasoning rather than in simple computational procedures. Difficulties in understanding powers, roots, and triangle relationships emerged consistently across the reviewed studies. Students also demonstrated substantial limitations in translating contextual situations into mathematical representations and geometric models. In addition, conceptual dependency on memorized formulas frequently prevented students from developing meaningful mathematical understanding. These findings reinforce the argument that geometry learning should emphasize conceptual reasoning, visualization, and contextual interpretation rather than procedural computation alone.

Learning Obstacles in Using Pythagorean Triples and Problem Solving

The reviewed studies further revealed that many students experienced conceptual difficulties when using Pythagorean triples to prove or identify right triangles. Several studies reported that students relied heavily on trial-and-error strategies by substituting numerical values repeatedly into formulas until suitable answers were obtained. Instead of understanding the proportional relationships among side lengths conceptually, students tended to depend on memorized numerical patterns mechanically. When asked to determine whether specific number combinations formed Pythagorean triples, many students were unable to provide mathematical justification and simply answered "yes" or "no" without proof. Furthermore, students frequently misunderstood the classification of triangles based on side relationships, particularly when comparing the square of the hypotenuse with the sum of the squares of the other sides. These misconceptions indicate weak understanding regarding the conceptual structure underlying the Pythagorean relationship. Students also encountered difficulties during the planning stage of problem solving, especially when converting contextual situations into mathematical representations and geometric diagrams. In some cases, students created nearly correct geometric sketches but omitted important angle markers or applied inappropriate Pythagorean triples to special triangles. Such conceptual errors subsequently resulted in incorrect final solutions. The findings also demonstrated that students frequently failed to interpret the purpose of contextual problems appropriately. Consequently, students often focused only on determining unknown side lengths without understanding broader problem objectives such as perimeter, area, or measurement relationships.

Obstacles in Understanding and Restating the Pythagorean Theorem

The reviewed literature also revealed that many students experienced difficulties in understanding and restating the Pythagorean Theorem meaningfully. Several studies reported that students incorrectly expressed the relationship among the sides of right triangles, indicating fragmented conceptual understanding of the theorem itself. In many cases, students could reproduce the formula symbolically but failed to explain the conceptual reasoning underlying the relationship mathematically. Furthermore, students often experienced difficulties connecting the general formula with contextual problem-solving situations. Some students stopped at writing quadratic forms without continuing the solution process or relating their answers to the conceptual meaning of the theorem. These conditions indicate that students' understanding remains procedural and symbolic rather than conceptual and relational. In addition to cognitive barriers, affective factors such as anxiety, fear of making mistakes, and low confidence also contributed significantly to students' difficulties during problem solving. Several studies indicated that students frequently recognized that their solutions were incorrect but hesitated to revise their answers due to uncertainty and lack of confidence. These affective obstacles interacted dynamically with epistemological and ontogenic barriers, reinforcing procedural dependency and fragmented reasoning structures. Consequently, students tended to terminate their problem-solving processes prematurely and avoid reflective evaluation of their solutions. Therefore, the findings suggest that effective geometry instruction should simultaneously strengthen conceptual understanding and create supportive learning environments encouraging reflection, confidence, and mathematical exploration.

Obstacles Affecting Students' Ability to Solve Contextual Problems

The synthesis findings further demonstrated that learning obstacles significantly affected students' ability to solve contextual and story problems related to the Pythagorean Theorem. Students frequently experienced difficulties translating contextual situations into mathematical models and geometric representations. Many students failed to identify important information from problem contexts and were unable to determine appropriate strategies for solving problems systematically. For example, in trapezoid problems requiring calculation of height using the Pythagorean Theorem, students frequently failed to recognize hidden right triangles embedded within the figure. Similarly, in contextual perimeter problems, students often focused solely on calculating unknown side lengths without understanding the broader objective of the problem. Such errors indicate weak problem-modeling ability and limited contextual interpretation during mathematical reasoning. Several studies also reported that students lacked experience in transforming verbal information into mathematical language and symbolic representation. Consequently, students often produced incomplete or irrelevant solutions despite possessing procedural knowledge of the formula. The findings demonstrate that conceptual understanding, visual representation, and contextual reasoning are strongly interconnected during geometry problem solving. Students who possessed fragmented conceptual understanding frequently encountered greater difficulties in contextual interpretation and mathematical modeling. Therefore, the reviewed studies emphasize the importance of instructional strategies encouraging problem solving, visualization, conceptual exploration, and contextual reasoning to support meaningful understanding of the Pythagorean Theorem.

Discussion

The findings of this study demonstrate that epistemological obstacles constitute the dominant source of students' learning difficulties in understanding and applying the Pythagorean Theorem. Most of the identified errors were related to fragmented conceptual understanding of exponentiation, radical operations, algebraic transposition, and geometric relationships rather than isolated procedural mistakes. Students frequently demonstrated incorrect simplification of powers and roots, misuse of radical notation, and misunderstanding of equality relationships during equation manipulation. These findings indicate that students' difficulties are rooted in incomplete conceptual schemas regarding inverse operations and symbolic representation. Similar findings were reported in previous mathematics education studies showing that students often memorize operational procedures without understanding the conceptual meaning underlying algebraic transformations (Alam & Mohanty, 2024; Borji et al., 2021; Harel, 2025; Pitta-Pantazi et al., 2020). Earlier research also found that misconceptions involving exponents and radicals tend to persist because instruction frequently emphasizes procedural fluency over conceptual reasoning and symbolic interpretation (Polydoros, 2026). In the context of geometry learning, such fragmented understanding subsequently affects students' ability to apply the Pythagorean Theorem accurately in problem-solving situations (Fauzi et al., 2024; Taamneh et al., 2024). The reviewed studies further revealed that ontogenic obstacles function mainly as reinforcing factors, particularly when students possess weak prerequisite knowledge and limited numerical reasoning ability. Meanwhile, didactical obstacles emerged through limited conceptual explanation and insufficient emphasis on the relationships among powers, roots, and geometric reasoning during instruction. Therefore, the findings suggest that conceptual understanding of exponents, radicals, and algebraic relationships must be strengthened systematically before students engage with higher-order geometry concepts such as the Pythagorean Theorem.

The synthesis findings also indicate that students' difficulties in understanding side relationships within right triangles arise from the interaction among epistemological, ontogenic, and didactical obstacles simultaneously. The core difficulty was primarily epistemological because students frequently failed to conceptualize the geometric meaning of perpendicularity, hypotenuse relationships, and triangle orientation accurately (Yao, 2020). Many students experienced confusion when identifying the hypotenuse or determining side relationships after geometric figures were rotated or presented in unfamiliar forms. Similar findings have been identified in previous studies reporting that students often rely on memorized visual prototypes of right triangles rather than conceptual geometric reasoning (Downton & Livy, 2022; Haj Yahya & Hershkowitz, 2026). Earlier research also emphasized that weak geometric visualization ability contributes significantly to students' misunderstanding of side relationships and perpendicular structures in geometry learning. Ontogenic obstacles intensified these conceptual difficulties because students demonstrated limited visual spatial reasoning ability and weak capacity to manipulate geometric representations mentally. Furthermore, students frequently failed to identify hidden triangles or construct accurate geometric sketches, indicating limitations in spatial cognition and mathematical representation skills (Dorel, 2023; Leung et al., 2024; Miragliotta & Baccaglini-Frank, 2021). Didactical obstacles further reinforced these conceptual barriers because classroom instruction often prioritized numerical calculation rather than geometric interpretation and visual exploration. The reviewed studies revealed that students rarely engaged in activities requiring them to construct, interpret, or analyze geometric diagrams meaningfully during learning processes. Consequently, students' understanding of the Pythagorean Theorem remained procedural and disconnected from the geometric relationships underlying the theorem itself.

The findings additionally demonstrate that students' difficulties in understanding Pythagorean triples and classifying triangle types are strongly associated with epistemological dominance and instructional limitations. Many students relied on trial-and-error substitution strategies when determining whether certain number combinations satisfied the Pythagorean relationship. Rather than understanding why particular numerical relationships produce right triangles, students tended to depend on memorized patterns and procedural experimentation. Similar findings have been reported in previous studies showing that students frequently approach geometry problems algorithmically without constructing conceptual justification or mathematical proof. Earlier research also indicated that students often fail to understand the relationship between squared side lengths and triangle classification because instruction focuses predominantly on procedural application of formulas. In this review, students frequently provided "yes" or "no" responses without presenting mathematical reasoning when asked to verify Pythagorean triples. Ontogenic obstacles emerged particularly during the planning and representation stages of problem solving because students struggled to construct meaningful geometric models and connect numerical values with geometric configurations. Several studies also reported that students' diagrams frequently lacked angle markers and geometric indicators necessary for accurate conceptual interpretation. Furthermore, didactical obstacles were evident because students were commonly introduced to Pythagorean triples as ready-made numerical sets rather than through exploratory activities emphasizing derivation, geometric meaning, and proof construction. Consequently, these findings suggest that geometry instruction should integrate reasoning-based proof, visual representation, and conceptual exploration to support meaningful understanding of Pythagorean triples and triangle classification.

Another important finding of this review is that students' learning obstacles in understanding and restating the Pythagorean Theorem involve interconnected conceptual, cognitive, and instructional dimensions. Students frequently demonstrated inability to express the Pythagorean relationship correctly or connect solution procedures with the conceptual meaning underlying the theorem. In several studies, students wrote incorrect quadratic relationships among triangle sides despite appearing familiar with the symbolic formula. Similar findings have been identified in previous mathematics education research indicating that students often reproduce formulas mechanically without understanding the rationale for their use in problem-solving situations (Barana, 2021; Verschaffel et al., 2020). Earlier studies also emphasized that students frequently fail to connect general formulas with contextual applications because their conceptual understanding remains fragmented and procedural. In addition to conceptual misunderstanding, ontogenic obstacles were reflected in students' limited ability to interpret problem statements, coordinate multi-step procedures, and determine relevant variables during problem solving. Several studies further revealed that students often stopped after writing quadratic forms because they lacked confidence and conceptual clarity regarding subsequent solution steps (Baybayon & Lapinid, 2024; Reid O'Connor & Norton, 2024). Didactical obstacles emerged when classroom instruction emphasized memorization and repetitive procedural exercises without providing opportunities for conceptual explanation, reflective reasoning, and mathematical discussion. The findings also demonstrate that affective factors such as anxiety, fear of making mistakes, and low confidence interacted dynamically with epistemological obstacles, further intensifying students' conceptual difficulties. Therefore, effective geometry instruction should simultaneously strengthen conceptual understanding, cognitive readiness, and supportive classroom environments encouraging reflection and error revision during mathematical learning processes.

The synthesis findings further reveal that students' difficulties in solving contextual and verbal problems involving the Pythagorean Theorem are strongly associated with weak mathematical modeling ability and fragmented contextual interpretation. Students frequently failed to convert verbal information into mathematical representations and geometric models accurately. In many reviewed studies, students focused only on determining unknown side lengths without understanding the broader contextual objectives of the problem. Similar findings have been reported in previous studies showing that students often experience difficulties connecting mathematical procedures with contextual meaning because they rely excessively on symbolic manipulation. Earlier research also emphasized that weak problem-modeling ability frequently results from limited opportunities to engage in contextual reasoning and mathematical communication during classroom instruction. In the reviewed studies, epistemological obstacles were evident in students' inability to interpret problem meaning, classify geometric objects correctly, and connect contextual information with mathematical concepts. Ontogenic obstacles appeared through weak visual–spatial reasoning ability and limited capacity to interpret geometric structures embedded within contextual situations. Meanwhile, didactical obstacles were associated with limited instructional emphasis on contextual learning, verbal interpretation, and mathematical modeling activities. The findings indicate that students require more opportunities to develop problem-solving strategies integrating conceptual understanding, visualization, contextual reasoning, and mathematical communication simultaneously. Therefore, geometry instruction should move beyond procedural calculation by incorporating contextual problem-solving activities, reflective discussion, and modeling-based learning approaches to support meaningful understanding of the Pythagorean Theorem.

Implications

The findings of this study provide important theoretical and practical implications for mathematics education, particularly in geometry instruction involving the Pythagorean Theorem.

The dominance of epistemological obstacles indicates that mathematics learning should prioritize conceptual understanding rather than procedural memorization and repetitive numerical exercises alone. Instructional practices need to emphasize the meaning of geometric relationships, the interpretation of mathematical symbols, and the reasoning underlying the Pythagorean relationship to help students construct deeper conceptual understanding. The results also imply that prerequisite concepts such as exponents, radicals, algebraic manipulation, and visual–spatial reasoning should be strengthened systematically before introducing formal applications of the Pythagorean Theorem. Furthermore, the interaction between epistemological, ontogenic, and didactical obstacles suggests that students' learning difficulties cannot be addressed through procedural remediation only, but require integrated pedagogical approaches that consider cognitive readiness, affective conditions, and instructional design simultaneously. These findings highlight the importance of using visual representations, geometric exploration, and contextual problem-solving activities to support students' conceptual reasoning and mathematical modeling ability. In addition, instructional strategies should provide opportunities for students to construct proofs, interpret diagrams, explain mathematical reasoning verbally, and evaluate solution processes reflectively. The findings also imply that teachers need to develop diagnostic sensitivity toward different categories of learning obstacles so that instructional interventions can be designed more adaptively according to students' conceptual needs. From a curriculum perspective, geometry learning materials should integrate conceptual explanation, visual reasoning, and contextual application rather than focusing predominantly on formula application and algorithmic procedures. The findings further suggest that supportive classroom environments are necessary to reduce students' anxiety, fear of making mistakes, and procedural dependency during mathematical problem solving. Therefore, mathematics teacher education programs should also emphasize the development of pedagogical content knowledge related to conceptual instruction, visual representation, and learning obstacle identification in geometry learning. Ultimately, this study contributes to the development of more meaningful, reflective, and conceptually oriented mathematics instruction capable of supporting students' long-term understanding of the Pythagorean Theorem and geometry learning in general.

Limitations and Suggestions for Future Research

This study has several limitations that should be considered when interpreting the findings and drawing broader conclusions regarding learning obstacles in the Pythagorean Theorem. First, the study relied exclusively on secondary data obtained from previously published articles, meaning that the analysis was limited by the quality, scope, and methodological characteristics of the selected studies. Second, the reviewed articles originated from different educational contexts, participant characteristics, and instructional settings, which may have influenced the consistency and comparability of the synthesized findings. Third, although the systematic review focused on studies published between 2019 and 2025 to capture recent developments in mathematics education, relevant studies published outside this period may not have been included in the synthesis. Fourth, the analysis primarily emphasized qualitative interpretation of learning obstacles and did not quantitatively measure the magnitude or statistical relationship among epistemological, ontogenic, and didactical obstacles. In addition, the reviewed studies varied considerably in their definitions, classifications, and operationalization of learning obstacles, potentially affecting the uniformity of interpretation during synthesis. Another limitation is that affective factors such as anxiety, confidence, and mathematical disposition were identified indirectly through previous studies rather than examined empirically within a single integrated research design. Furthermore, this study focused specifically on the Pythagorean Theorem and therefore may not fully represent learning obstacles occurring in other geometry topics or broader mathematics learning contexts. Future research is recommended to conduct empirical classroom investigations integrating qualitative and quantitative approaches to examine the interaction among conceptual understanding, affective factors, and instructional practices more comprehensively. Further studies are also encouraged to develop and evaluate instructional interventions specifically designed to reduce epistemological, ontogenic, and didactical obstacles simultaneously in geometry learning. In addition, future research should explore the role of visual representation, digital learning media, and inquiry-based instructional models in improving students' conceptual understanding of the Pythagorean Theorem. Comparative studies involving different educational levels, cultural contexts, and curriculum systems are also necessary to obtain broader perspectives regarding students' geometry learning obstacles. Ultimately, future investigations should focus not only on identifying students' errors but also on designing adaptive pedagogical frameworks capable of supporting meaningful, reflective, and conceptually oriented mathematics learning experiences.

CONCLUSION

This systematic literature review demonstrates that students' learning obstacles in the Pythagorean Theorem are multidimensional and interconnected across epistemological, ontogenic, and didactical domains. The findings indicate that epistemological obstacles are the most dominant barriers, particularly in students' fragmented understanding of exponents, radicals, geometric relationships, and the conceptual meaning of the Pythagorean Theorem. Students frequently relied on procedural memorization and trial-and-error strategies without fully understanding the mathematical relationships underlying right triangles and Pythagorean triples. In addition, ontogenic obstacles were identified through students' limited prerequisite knowledge, weak visual–spatial reasoning ability, low cognitive readiness, and affective conditions such as anxiety and lack of confidence during problem solving. Didactical obstacles also contributed significantly to students' difficulties because instructional practices often emphasized formula memorization and repetitive calculation rather than conceptual reasoning, geometric visualization, and reflective mathematical exploration. The synthesis findings further reveal that students experience substantial difficulties in interpreting contextual problems, constructing mathematical models, identifying geometric structures, and communicating mathematical ideas meaningfully. These obstacles interact dynamically and cumulatively, indicating that students' learning difficulties cannot be understood as isolated procedural errors alone. The study also demonstrates that students' conceptual understanding of the Pythagorean Theorem remains largely procedural rather than relational and reflective across many educational contexts. Consequently, meaningful geometry learning requires instructional approaches that integrate conceptual understanding, visual representation, contextual reasoning, and problem-solving activities simultaneously. The findings highlight the importance of strengthening prerequisite mathematical concepts and developing instructional designs that facilitate conceptual exploration and mathematical communication during geometry learning. Furthermore, the study contributes theoretically by positioning learning obstacles as interconnected cognitive and instructional phenomena that influence students' mathematical reasoning structures comprehensively. Ultimately, this systematic review provides important insights for mathematics educators and researchers in designing more adaptive, reflective, and conceptually oriented learning environments to improve students' understanding of the Pythagorean Theorem and geometry learning in general.

AUTHOR CONTRIBUTIONS STATEMENT

Yuce Sandra was responsible for the research idea, data collection from Scopus, and conducted the analysis using VOSviewer. Didi Suryadi and Dadan Dasari supervised the research, provided suggestions, and corrections. They both contributed to the final refinement of the manuscript and reviewed the research methodology for validation.

REFERENCES

Akolekar, H., Jhamnani, P., Kumar, V., Tailor, V., Pote, A., Meena, A., Kumar, K., Challa, J. S., & Kumar, D. (2025). The role of generative AI tools in shaping mechanical engineering education from an undergraduate perspective. *Scientific Reports, 15*(1), 9214. https://doi.org/10.1038/s41598-025-93871-z

Alam, A., & Mohanty, A. (2024). Unveiling the complexities of "Abstract Algebra" in university mathematics education (UME): Fostering "conceptualization and understanding" through advanced pedagogical approaches. *Cogent Education, 11*(1), 2355400. https://doi.org/10.1080/2331186X.2024.2355400

Ario, M., Suhendra, Jupri, A., & Nurlaelah, E. (2025). Students' errors and learning obstacles in solving algebraic word problems: Hermeneutic phenomenology. *Education Sciences, 15*(12). https://doi.org/10.3390/educsci15121674

Baiduri, B., Putri, O. R. U., & Alfani, I. (2020). Mathematical connection process of students with high mathematics ability in solving PISA problems. *European Journal of Educational Research, 9*(4), 1527–1537. https://doi.org/10.12973/eu-jer.9.4.1527

Barana, A. (2021). From formulas to functions through geometry: A path to understanding algebraic computations. *European Journal of Investigation in Health, Psychology and Education, 11*(4), 1485–1502. https://doi.org/10.3390/ejihpe11040106

Barbieri, C. A., & Rodrigues, J. (2025). Leveraging cognitive load theory to support students with mathematics difficulty. *Educational Psychologist, 60*(3), 208–232. https://doi.org/10.1080/00461520.2025.2486138

Basid, A., Sutrisno, E., & Aliyeva, L. R. (2024). Analysis of the effect of contextual problem solving on students' mathematical reasoning ability. *International Journal of Science and Mathematics Education, 1*(3), 24–33. https://doi.org/10.62951/ijsme.v1i3.258

Baybayon, G. G., & Lapinid, M. R. C. (2024). Students' common errors in quadratic equations: Towards improved mathematics performance. *Infinity Journal, 13*(1), 83–98. https://doi.org/10.22460/infinity.v13i1.p83-98

Bernabeu, M., Moreno, M., & Llinares, S. (2024). Polygon class learning opportunities: Interplay between teacher's moves, children's geometrical thinking, and geometrical task. *International Journal of Science and Mathematics Education, 22*(6), 1381–1403. https://doi.org/10.1007/s10763-023-10425-3

Bila, H., Maphutha, K., & Mutodi, P. (2024). Learners' algebraic and geometric connections when solving Euclidean geometry riders. *Pythagoras, 45*(1), 1–9. https://doi.org/10.4102/pythagoras.v45i1.810

Blasjo, V. (2022). Operationalism: An interpretation of the philosophy of ancient Greek geometry. *Foundations of Science, 27*(2), 587–708. https://doi.org/10.1007/s10699-021-09791-4

Borji, V., Radmehr, F., & Font, V. (2021). The impact of procedural and conceptual teaching on students' mathematical performance over time. *International Journal of Mathematical Education in Science and Technology, 52*(3), 404–426. https://doi.org/10.1080/0020739X.2019.1688404

Bringula, R., Reguyal, J. J., Tan, D. D., & Ulfa, S. (2021). Mathematics self-concept and challenges of learners in an online learning environment during COVID-19 pandemic. *Smart Learning Environments, 8*(1), 22. https://doi.org/10.1186/s40561-021-00168-5

Brown, S. A., Ronfard, S., & Kelemen, D. (2020). Teaching natural selection in early elementary classrooms: Can a storybook intervention reduce teleological misunderstandings? *Evolution: Education and Outreach, 13*(1), 12. https://doi.org/10.1186/s12052-020-00127-7

Campbell, T. G., & Zelkowski, J. (2020). Technology as a support for proof and argumentation: A systematic literature review. *International Journal for Technology in Mathematics Education, 27*(2), 113–123. https://doi.org/10.1564/tme_v27.2.04

Castellví, J. (2024). Epistemological obstacles to social studies education. *Trends in Higher Education, 3*(4), 1072–1080. https://doi.org/10.3390/higheredu3040062

Chew, S. L., & Cerbin, W. J. (2021). The cognitive challenges of effective teaching. *The Journal of Economic Education, 52*(1), 17–40. https://doi.org/10.1080/00220485.2020.1845266

Chonchaiya, R., & Srithammee, N. (n.d.). *Augmented reality as a tool for enhancing geometry learning and improving mathematical understanding*.

Dello Iacono, U. (2021). From argumentation to proof in geometry within a collaborative computer-based environment. *Digital Experiences in Mathematics Education, 7*(3), 395–426. https://doi.org/10.1007/s40751-021-00090-y

Deshpande, D. S., Riccomini, P. J., Hughes, E. M., & Raulston, T. J. (2021). Problem solving with the Pythagorean theorem: A think aloud analysis of secondary students with learning disabilities. *Learning Disabilities: A Contemporary Journal, 19*(1), 23–47.

Dewi, I., Hasratuddin, Andriani, A., & Siregar, N. (2025). Digital deep learning materials with culture: Improving students' Pythagorean theorem abstraction skills. *European Journal of STEM Education, 10*(1). https://doi.org/10.20897/ejsteme/17427

Dorel, L. (2023). The relationship between visual and abstract comprehension in spatial geometry, and its importance to developing spatial perception and vision. *International Journal for Technology in Mathematics Education, 30*(4), 219–226. https://doi.org/10.1564/tme_v30.4.3

Downton, A., & Livy, S. (2022). Insights into students' geometric reasoning relating to prisms. *International Journal of Science and Mathematics Education, 20*(7), 1543–1571. https://doi.org/10.1007/s10763-021-10219-5

Due, B. L. (2024). The matter of math: Guiding the blind to touch the Pythagorean theorem. *Learning, Culture and Social Interaction, 45*, 100792. https://doi.org/10.1016/j.lcsi.2023.100792

Fauzi, K. M. A., Hia, Y., Darari, M. B., & Siagian, M. D. (2024). Efforts to overcome students' learning difficulties in geometry: A didactic design of creative thinking skills through metacognitive approaches. *Beta: Jurnal Tadris Matematika, 17*(2), 159–182. https://doi.org/10.20414/betajtm.v17i2.648

Fielding, J., & Makar, K. (2022). Challenging conceptual understanding in a complex system: Supporting young students to address extended mathematical inquiry problems. *Instructional Science, 50*(1), 35–61. https://doi.org/10.1007/s11251-021-09564-3

Gurmu, F., Tuge, C., & Hunde, A. B. (2024). Effects of GeoGebra-assisted instructional methods on students' conceptual understanding of geometry. *Cogent Education, 11*(1), 2379745. https://doi.org/10.1080/2331186X.2024.2379745

Hahn, R. (2022). Thales, the "Pythagorean Theorem", and technological context. *Dialogues d'histoire ancienne, 482*(2), 25–51. https://doi.org/10.3917/dha.482.0025

Haj Yahya, A., & Hershkowitz, R. (2026). Interference of prototypical geometry representations in students' construction of concepts and development of proofs. *Mathematical Thinking and Learning, 28*(1), 53–74. https://doi.org/10.1080/10986065.2024.2386619

Harel, G. (2025). Promoting linear algebraic reasoning among students: Affordances and challenges. *PRIMUS, 35*(9–10), 930–950. https://doi.org/10.1080/10511970.2024.2327325

Hendriyanto, A., Suryadi, D., Juandi, D., Dahlan, J. A., Hidayat, R., Wardat, Y., Sahara, S., & Muhaimin, L. H. (2024). The didactic phenomenon: Deciphering students' learning obstacles in set theory. *Journal on Mathematics Education, 15*(2), 517–544. https://doi.org/10.22342/jme.v15i2.pp517-544

Hughes, E. M., Riccomini, P. J., & Lee, J.-Y. (2020). Investigating written expressions of mathematical reasoning for students with learning disabilities. *The Journal of Mathematical Behavior, 58*, 100775. https://doi.org/10.1016/j.jmathb.2020.100775

Hwang, W.-Y., Zhao, L., Shadiev, R., Lin, L.-K., Shih, T. K., & Chen, H.-R. (2020). Exploring the effects of ubiquitous geometry learning in real situations. *Educational Technology Research and Development, 68*(3), 1121–1147. https://doi.org/10.1007/s11423-019-09730-y

Jablonski, S., & Ludwig, M. (2023). Teaching and learning of geometry: A literature review on current developments in theory and practice. *Education Sciences, 13*(7). https://doi.org/10.3390/educsci13070682

Jackson, N., & Johnson, C. (2024). Five or ten new proofs of the Pythagorean theorem. *The American Mathematical Monthly, 131*(9), 739–752. https://doi.org/10.1080/00029890.2024.2370240

Kaitera, S., & Harmoinen, S. (2022). Developing mathematical problem-solving skills in primary school by using visual representations on heuristics. *LUMAT: International Journal on Math,* *Science and Technology Education, 10*(2), 111–146. https://doi.org/10.31129/LUMAT.10.2.1696

Kandaga, T., Rosjanuardi, R., & Juandi, D. (2022). Epistemological obstacle in transformation geometry based on van Hiele's level. *EURASIA Journal of Mathematics, Science and Technology Education, 18*(4). https://doi.org/10.29333/ejmste/11914

Keyik, N., Sengil Akar, S., & Saygı, E. (2026). Students' conceptualization of Pythagorean theorem (in RME): Examination with APOS. *Pedagogical Research, 11*(1), em0255. https://doi.org/10.29333/pr/18093

Kohen, Z., Amram, M., Dagan, M., & Miranda, T. (2022). Self-efficacy and problem-solving skills in mathematics: The effect of instruction-based dynamic versus static visualization. *Interactive Learning Environments, 30*(4), 759–778. https://doi.org/10.1080/10494820.2019.1683588

Leung, A., Baccaglini-Frank, A., Mariotti, M. A., & Miragliotta, E. (2024). Enhancing geometric skills with digital technology: The case of dynamic geometry. In *Handbook of digital resources in mathematics education* (pp. 409–437). Springer. https://doi.org/10.1007/978-3-031-45667-1_15

Mailizar, Almanthari, A., Maulina, S., & Bruce, S. (2020). Secondary school mathematics teachers' views on e-learning implementation barriers during the COVID-19 pandemic: The case of Indonesia. *EURASIA Journal of Mathematics, Science and Technology Education, 16*(7). https://doi.org/10.29333/ejmste/8240

Maknun, C. L., Rosjanuardi, R., & Jupri, A. (2022). Epistemological obstacle in learning trigonometry. *Mathematics Teaching Research Journal, 14*(2), 5–25.

Miragliotta, E., & Baccaglini-Frank, A. E. (2021). Enhancing the skill of geometric prediction using dynamic geometry. *Mathematics, 9*(8). https://doi.org/10.3390/math9080821

Mukuka, A., Balimuttajjo, S., & Mutarutinya, V. (2023). Teacher efforts towards the development of students' mathematical reasoning skills. *Heliyon, 9*(4). https://doi.org/10.1016/j.heliyon.2023.e14789

Mutambara, L. H. N., & Tsakeni, M. (2022). Cognitive obstacles in the learning of complex number concepts: A case study of in-service undergraduate physics student-teachers in Zimbabwe. *EURASIA Journal of Mathematics, Science and Technology Education, 18*(10). https://doi.org/10.29333/ejmste/12418

Nadzeri, M. B., Musa, M., Meng, C. C., & Ismail, I. M. (2024). The effects of augmented reality geometry learning applications on spatial visualization ability for lower primary school pupils. *International Journal of Interactive Mobile Technologies, 18*(16), 104. https://doi.org/10.3991/ijim.v18i16.47079

Narciss, S., & Alemdag, E. (2025). Learning from errors and failure in educational contexts: New insights and future directions for research and practice. *British Journal of Educational Psychology, 95*(1), 197–218. https://doi.org/10.1111/bjep.12716

Nouraey, P., & Al-Badi, A. (2023). Challenges and problems of e-learning: A conceptual framework. *Electronic Journal of E-Learning, 21*(3), 188–199. https://doi.org/10.34190/ejel.21.3.2677

Pitta-Pantazi, D., Chimoni, M., & Christou, C. (2020). Different types of algebraic thinking: An empirical study focusing on middle school students. *International Journal of Science and Mathematics Education, 18*(5), 965–984. https://doi.org/10.1007/s10763-019-10003-6

Polydoros, G. (2026). Exploring high school students' misconceptions of functions: A cross-grade comparative study. *Mathematics Education Research Journal*. https://doi.org/10.1007/s13394-026-00576-1

Purcar, A.-M., Bocoș, M., Pop, A.-L., Roman, A., Rad, D., Mara, D., Crișan, C., Răduț-Taciu, R., Mara, E.-L., Todor, I., Muntean-Trif, L., Neacșu, M.-G., Colareza, C. C., Maier, M., Tăușan-Crișan, L., Triff, Z., Baciu, C., Marin, D.-C., & Triff, D.-G. (2024). The effect of visual reasoning on arithmetic word problem solving. *Education Sciences, 14*(3). https://doi.org/10.3390/educsci14030278

Ramírez-Uclés, R., & Ruiz-Hidalgo, J. F. (2022). Reasoning, representing, and generalizing in geometric proof problems among 8th grade talented students. *Mathematics, 10*(5). https://doi.org/10.3390/math10050789

Reid O'Connor, B., & Norton, S. (2024). Exploring the challenges of learning quadratic equations and reflecting upon curriculum structure and implementation. *Mathematics Education Research Journal, 36*(1), 151–176. https://doi.org/10.1007/s13394-022-00434-w

Rodriguez, J.-M. G., Bain, K., & Towns, M. H. (2020). The role of epistemology and epistemic games in mediating the use of mathematics in chemistry: Implications for mathematics instruction and research on undergraduate mathematics education. *International Journal of Research in Undergraduate Mathematics Education, 6*(2), 279–301. https://doi.org/10.1007/s40753-019-00110-8

Sari, A. D., Suryadi, D., & Dasari, D. (2024). Learning obstacle of probability learning based on the probabilistic thinking level. *Journal on Mathematics Education, 15*(1), 207–226. https://doi.org/10.22342/jme.v15i1.pp207-226

Silva, C. M. S. (2022). Experimental geometry, Pythagorean theorem and Montessori. *Acta Scientiae, 24*(8), 376–404. https://doi.org/10.17648/acta.scientiae.7023

Silver, A. M., & Libertus, M. E. (2022). Environmental influences on mathematics performance in early childhood. *Nature Reviews Psychology, 1*(7), 407–418. https://doi.org/10.1038/s44159-022-00061-z

Sol, T., Ledezma, C., Sánchez, A., & Font, V. (2025). Teachers' practical argumentation on the teaching of the Pythagorean theorem. *International Journal of Science and Mathematics Education, 23*(8), 3707–3731. https://doi.org/10.1007/s10763-025-10591-6

Stylianides, G. J., Stylianides, A. J., & Moutsios-Rentzos, A. (2024). Proof and proving in school and university mathematics education research: A systematic review. *ZDM – Mathematics Education, 56*(1), 47–59. https://doi.org/10.1007/s11858-023-01518-y

Sukarma, I. K., Isnawan, M. G., & Alsulami, N. M. (2024). Research on nonroutine problems: A hybrid didactical design for overcoming student learning obstacles. *Education Research International, 2024*, 5552365. https://doi.org/10.1155/2024/5552365

Taamneh, M. A., Díez-Palomar, J., & Mallart-Solaz, A. (2024). Examining tenth-grade students' errors in applying Polya's problem-solving approach to Pythagorean theorem. *Eurasia Journal of Mathematics, Science and Technology Education, 20*(12), em2551. https://doi.org/10.29333/ejmste/15707

Ulusoy, F. (2021). Prospective early childhood and elementary school mathematics teachers' concept images and concept definitions of triangles. *International Journal of Science and Mathematics Education, 19*(5), 1057–1078. https://doi.org/10.1007/s10763-020-10105-6

Ünal, Z. E., Ala, A. M., Kartal, G., Özel, S., & Geary, D. C. (2023). Visual and symbolic representations as components of algebraic reasoning. *Journal of Numerical Cognition, 9*(2), 327–345. https://doi.org/10.5964/jnc.11151

Uygun, T., Guner, P., & Simsek, I. (2024). Examining students' geometrical misconceptions by eye tracking. *International Journal of Mathematical Education in Science and Technology, 55*(1), 68–93. https://doi.org/10.1080/0020739X.2022.2101954

Venetis, J. (2020). The role of mathematics in architecture and fine arts: A historical overview, problems and prospects. *Civil Engineering and Architecture, 8*(3), 258–267. https://doi.org/10.13189/cea.2020.080308

Verschaffel, L., Schukajlow, S., Star, J., & Van Dooren, W. (2020). Word problems in mathematics education: A survey. *ZDM, 52*(1), 1–16. https://doi.org/10.1007/s11858-020-01130-4

Winer, M. L., & Battista, M. T. (2022). Investigating students' proof reasoning: Analyzing students' oral proof explanations and their written proofs in high school geometry. *International Electronic Journal of Mathematics Education, 17*(2). https://doi.org/10.29333/iejme/11713

Yao, X. (2020). Unpacking learner's growth in geometric understanding when solving problems in a dynamic geometry environment: Coordinating two frames. *The Journal of Mathematical Behavior, 60*, 100803. https://doi.org/10.1016/j.jmathb.2020.100803

Žakelj, A., & Klančar, A. (2022). The role of visual representations in geometry learning. *European Journal of Educational Research, 11*(3), 1393–1411. https://doi.org/10.12973/eu-jer.11.3.1393