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Mapping students' probabilistic reasoning and literacy through solo taxonomy in discrete random variable problem solving among preservice mathematics teachers

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Abstract

Background: Probability learning requires not only procedural computation but also probabilistic reasoning and probabilistic literacy to interpret uncertainty meaningfully. However, many mathematics education students still experience difficulties in understanding discrete random variable concepts and communicating probabilistic meaning contextually. Previous studies have generally examined probabilistic reasoning and probabilistic literacy separately, with limited integration using SOLO Taxonomy as a cognitive framework.
Aims: This study aimed to investigate students' probabilistic reasoning and probabilistic literacy in solving discrete random variable problems through SOLO Taxonomy.
Method: A mixed-methods design was employed involving 20 fifth-semester mathematics education students enrolled in an Introduction to Probability Theory course. Data were collected through probabilistic reasoning tests, probabilistic literacy questionnaires, and semi-structured interviews. Students' responses were classified into five SOLO Taxonomy levels and analyzed using descriptive and thematic approaches.
Results: The findings revealed that most students were categorized at the Unistructural (45%) and Prestructural (30%) levels, indicating fragmented conceptual understanding and limited probabilistic interpretation. Only a small proportion achieved Relational (5%) and Extended Abstract (5%) levels. Higher probabilistic literacy was associated with more integrated reasoning, coherent interpretation, and stronger logical justification.
Conclusion: Probabilistic literacy and probabilistic reasoning are closely interconnected competencies in solving discrete random variable problems. SOLO Taxonomy provides an effective framework for identifying students' probabilistic thinking structures and supporting higher-order probabilistic learning development.

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INTRODUCTION

Mathematics is a fundamental discipline that contributes significantly to the development of analytical thinking, logical reasoning, and problem-solving abilities across various educational levels. Within mathematics education, probability serves as an essential branch that enables students to understand uncertainty, evaluate information critically, and make decisions based on quantitative evidence (Develaki, 2025; Ingram, 2024). Probability concepts are widely applied in scientific

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research, economics, engineering, healthcare, technology, and social analysis, making probabilistic understanding increasingly important in contemporary society (Coccia, 2022; Maertens et al., 2022). At the university level, probability learning is expected to support students in developing not only procedural competence but also conceptual understanding and interpretive reasoning. One of the central topics in probability education is discrete random variable distribution, which requires students to integrate symbolic representation, probabilistic interpretation, and mathematical reasoning simultaneously (Kazak & Pratt, 2021; Zorzos & Avgerinos, 2023). Students are expected to understand concepts such as probability mass functions, expected value, variance, and cumulative probability while interpreting the contextual meaning of probabilistic information logically (Gok & Goldstone, 2024). However, probability learning remains challenging for many students because probabilistic concepts are abstract and often disconnected from real-life experiences. Many students are able to apply formulas procedurally but still experience difficulties in explaining probabilistic meaning and interpreting probability results contextually. Such conditions indicate that meaningful probability learning requires deeper cognitive engagement than procedural computation alone. Therefore, probability instruction should emphasize conceptual understanding, contextual interpretation, and higher-order probabilistic thinking to support meaningful learning experiences.

Difficulties in probability learning are frequently associated with misconceptions, heuristic reasoning, and fragmented conceptual understanding. Many students rely on intuitive assumptions and personal beliefs rather than formal probabilistic logic when solving probability problems involving uncertainty (Elbehary, n.d.; Kaplar et al., 2021). This condition often leads to inconsistencies in reasoning, misinterpretation of probabilistic language, and difficulties in constructing coherent mathematical arguments. In discrete random variable problems, students commonly struggle to connect mathematical representations with contextual situations because they focus primarily on computational procedures. As a result, probability learning often becomes procedural and mechanistic rather than conceptual and reflective. These difficulties are particularly concerning in mathematics teacher education because preservice mathematics teachers are expected to teach probability concepts meaningfully in future classroom practice. Consequently, prospective teachers need strong probabilistic reasoning and probabilistic literacy abilities to support effective mathematical communication and instructional decision-making (Elbehary, 2022; Muñiz-Rodríguez et al., 2020). Probabilistic reasoning refers to the cognitive process of constructing logical conclusions and making decisions under uncertainty, whereas probabilistic literacy involves the ability to interpret, evaluate, and communicate probabilistic information critically in contextual situations (Develaki, 2025; Elbehary, 2022; Muñiz-Rodríguez et al., 2020). In solving discrete random variable problems, students require both competencies simultaneously because they must interpret probabilistic information while also justifying mathematical conclusions logically. Therefore, probabilistic reasoning and probabilistic literacy should be viewed as complementary competencies that collectively support students' understanding of probability concepts comprehensively.

To analyze the complexity of students' probabilistic thinking structures, SOLO Taxonomy provides a hierarchical framework that classifies cognitive development into five levels: Prestructural, Unistructural, Multistructural, Relational, and Extended Abstract. This framework explains how students' understanding develops from fragmented and isolated knowledge toward integrated reasoning and abstract generalization. Students at lower SOLO levels typically demonstrate incomplete understanding and procedural responses, whereas students at higher levels are able to connect concepts, justify reasoning, and transfer knowledge across contexts (Davies & Mansour, 2022; Schmude, 2022). In probability education, SOLO Taxonomy offers an effective framework for identifying the structure and quality of students' probabilistic thinking (Davies & Mansour, 2022; Karapanos et al., 2025; Serrano et al., 2025). Several studies have investigated probabilistic reasoning, probability misconceptions, probabilistic literacy, and SOLO Taxonomy in different educational contexts. Research on probabilistic reasoning mainly focused on heuristic thinking, conceptual difficulties, and probabilistic decision-making, while studies on probabilistic literacy emphasized interpretation, communication, and contextual understanding of probabilistic information (Ashari, 2025; Zhang, 2026). Other studies involving SOLO Taxonomy predominantly analyzed mathematical reasoning, inferential reasoning, computational thinking, and conceptual understanding in mathematics learning (Schmude, 2022; Serrano et al., 2025). However, previous studies generally examined probabilistic reasoning, probabilistic literacy, and SOLO Taxonomy separately rather than integrating them into a comprehensive analytical framework. Furthermore, most studies focused on general probability concepts and relied primarily on either quantitative or qualitative approaches independently. Therefore, there remains a significant research gap regarding the integration of probabilistic reasoning, probabilistic literacy, and SOLO Taxonomy within a mixed-methods framework to analyze students' thinking structures in solving discrete random variable problems among preservice mathematics teachers.

Table 1. Distinction between probabilistic reasoning and probabilistic literacy

Aspect	Probabilistic Reasoning	Probabilistic Literacy
Main focus	Logical reasoning under uncertainty	Interpretation and communication of probabilistic information
Cognitive activity	Drawing conclusions and justification	Reading, interpreting, and evaluating probability information
Role in learning	Conjecturing, argumentation, and justification in solving probabilistic problems	Interpretation, representation, communication, and understanding probabilistic meaning in context

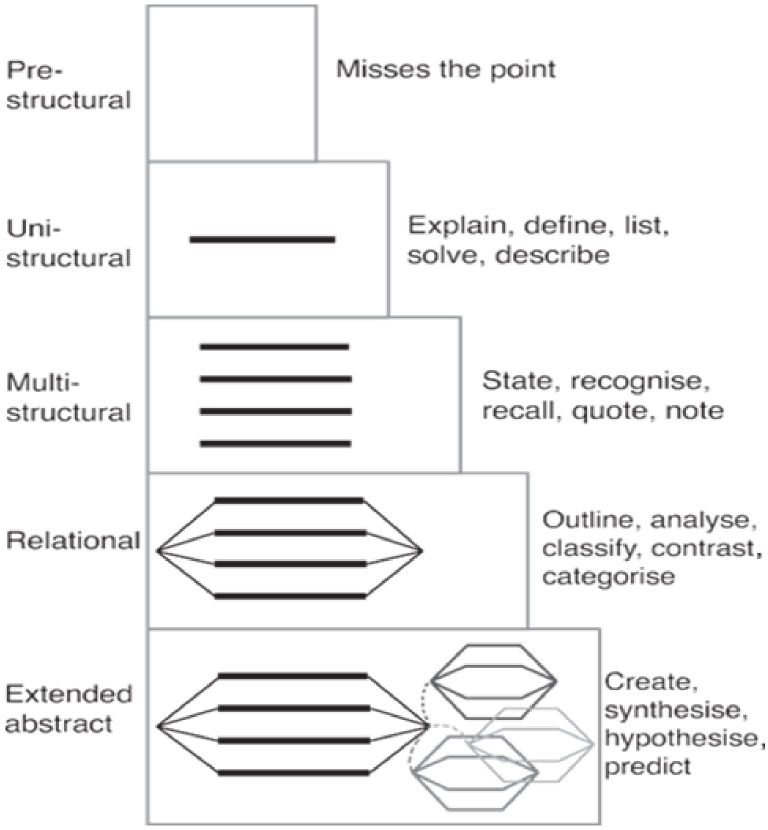

Figure 1. SOLO Taxonomy Levels

Tabel 2. Taxonomy levels and characteristics

Level	Characteristics
Pre-structural	Incomplete or irrelevant understanding
Uni-structural	Focus on one relevant aspect
Multi-structural	Several aspects are understood separately
Relational	Integrated understanding between concepts
Extended Abstract	Generalization and abstraction beyond the immediate context

Table 3. Summary pf previous studies and research gaps

Author	Focus	Method	Limitation
Primi et al.	Probabilistic reasoning	Quantitative	Did not analyze SOLO levels
Elbehary	Preservice teachers' reasoning	Case study	No literacy analysis
Appiah et al.	SOLO taxonomy in combinatorics	Qualitative	Not focused on probability
Álvarez-Arroyo et al.	Probabilistic literacy	Quantitative	No reasoning structure analysis

Previous studies have extensively investigated probabilistic reasoning, probabilistic literacy, SOLO Taxonomy, discrete random variables, and preservice mathematics teachers in various educational and mathematical contexts; however, these constructs have largely been examined separately rather than integratively. Research on probabilistic reasoning mainly focused on reasoning development, heuristic thinking, conceptual interpretation, and probabilistic thinking processes in general probability learning contexts (Kazak & Pratt, 2021; Placì et al., 2026), while studies on probabilistic literacy emphasized communication, contextual interpretation, language reception, and risk literacy related to probabilistic information (Büchter et al., 2026; Muñiz-Rodríguez et al., 2020). At the same time, SOLO Taxonomy has been widely utilized to analyze cognitive structures in mathematics learning, computational thinking, scientific learning, and educational assessment, but rarely within probability learning contexts specifically related to probabilistic reasoning and literacy development (Huang et al., 2024; Serrano et al., 2025; Svensäter & Rohlin, 2023). Furthermore, studies involving discrete random variables predominantly concentrated on stochastic processes, probability modeling, asymptotic analysis, reliability theory, and statistical computation rather than students' conceptual understanding and probabilistic thinking structures in mathematics education (Akbash et al., 2024; Geenens, 2020; Lileika & Mackevičius, 2020). In mathematics teacher education, previous studies involving preservice mathematics teachers mainly explored TPACK, AI literacy, pedagogical beliefs, instructional practices, professional competencies, and contextual problem-solving abilities without specifically examining probabilistic reasoning and probabilistic literacy in probability learning (Kartal & Çınar, 2024; Wijaya et al., 2025; Yang et al., 2022). Moreover, most existing studies employed either quantitative or qualitative approaches independently, resulting in limited comprehensive understanding of students' probabilistic cognitive structures and higher-order probabilistic thinking development. Therefore, there remains a significant research gap regarding the integration of probabilistic reasoning, probabilistic literacy, and SOLO Taxonomy within a mixed-methods framework to analyze preservice mathematics teachers' thinking structures in solving discrete random variable problems.

Based on the identified research gaps, this study aimed to investigate and characterize preservice mathematics teachers' probabilistic reasoning and probabilistic literacy in solving discrete random variable problems using SOLO Taxonomy as an analytical framework. This study specifically sought to identify students' probabilistic reasoning levels based on SOLO Taxonomy classifications, including Prestructural, Unistructural, Multistructural, Relational, and Extended Abstract levels. The study also aimed to analyze students' probabilistic literacy profiles in interpreting, understanding, and communicating probabilistic information contextually. In addition, this research intended to describe the characteristics of students' thinking processes and reasoning structures at each SOLO level through qualitative analysis. The study further explored the relationship between probabilistic literacy and probabilistic reasoning in solving discrete random variable problems among preservice mathematics teachers. By integrating probabilistic reasoning, probabilistic literacy, and SOLO Taxonomy within a mixed-methods framework, this study attempted to provide a more comprehensive understanding of students' probabilistic cognitive development. The findings of this study are expected to contribute theoretically to probability education literature by extending the application of SOLO Taxonomy in probabilistic thinking research. Practically, the study is expected to provide implications for mathematics education lecturers in designing probability instruction that emphasizes conceptual understanding, contextual interpretation, communication, and reasoning rather than procedural computation alone. Furthermore, the findings may support the development of differentiated instructional strategies based on students' probabilistic thinking structures and cognitive characteristics. Ultimately, this study is expected to contribute to the improvement of higher-order probabilistic thinking development among preservice mathematics teachers in higher education contexts.

LITERATURE REVIEW

Probability learning is closely associated with the development of students' reasoning, interpretation, and decision-making abilities under uncertain conditions. In mathematics education, probability is not merely viewed as a computational topic but also as a conceptual domain that requires students to understand uncertainty meaningfully and contextually (Cotič et al., 2024; Elbehary, 2022; Ingram, 2024; Kazak & Pratt, 2021). The study of probability involves interpreting random phenomena, evaluating probabilistic information, and constructing logical conclusions based on quantitative evidence (Ancker et al., 2025; Khodabakhshian et al., 2023). Consequently, probability learning requires students to integrate conceptual understanding, symbolic representation, and mathematical communication simultaneously. One of the important topics in probability learning is discrete random variable distribution, which includes concepts such as probability mass functions, expected value, variance, and cumulative probability. These concepts require students not only to calculate probabilities correctly but also to interpret probabilistic meaning within contextual situations. However, probability concepts are often considered abstract because students must reason about uncertain events that cannot always be observed directly. This condition frequently causes students to rely on procedural memorization rather than conceptual understanding during problem solving. As a result, many students experience difficulties in connecting mathematical formulas with probabilistic interpretation and contextual meaning. Therefore, effective probability learning should emphasize conceptual reasoning, interpretation, and communication rather than procedural computation alone.

Probabilistic reasoning is considered one of the fundamental competencies in mathematics education because it supports students in constructing logical conclusions and making decisions under uncertainty. Probabilistic reasoning refers to the cognitive process of interpreting probabilistic information, identifying relationships among variables, evaluating evidence, and constructing justified mathematical conclusions (Ancker et al., 2025; Braithwaite, 2022; Gok & Goldstone, 2024; Lee et al., 2023). In probability learning, students are expected to use reasoning processes to analyze random situations systematically rather than relying solely on intuition or personal belief (Erbas & Ocal, 2024; Lejarraga & Hertwig, 2021). Higher-order probabilistic reasoning is characterized by abstraction, flexibility of thought, and the ability to generalize probabilistic relationships across different contexts (Loyens et al., 2023). Students with strong probabilistic reasoning abilities are generally capable of interpreting probability concepts conceptually and communicating mathematical arguments coherently. However, previous studies reported that many students still demonstrate heuristic reasoning and intuitive misconceptions when solving probability problems. Students frequently misinterpret probabilistic language, confuse independent and dependent events, and experience difficulties in interpreting cumulative probability distributions (Lee et al., 2023; Zhang, 2026). Such misconceptions indicate that probabilistic reasoning development requires structured instructional experiences that emphasize conceptual understanding and reflective thinking. In mathematics teacher education, probabilistic reasoning becomes particularly important because preservice teachers are expected to facilitate meaningful probability learning in future classroom practice. Therefore, understanding students' probabilistic reasoning structures is essential for improving probability instruction in higher education contexts.

In addition to probabilistic reasoning, probabilistic literacy also plays an important role in supporting meaningful probability learning. Probabilistic literacy refers to the ability to interpret, evaluate, represent, and communicate probabilistic information critically in various contextual situations (Kazak & Pratt, 2021; Locoro et al., 2021; Muñiz-Rodríguez et al., 2020). This competency enables individuals to understand uncertainty, evaluate probabilistic claims, and make informed decisions based on quantitative evidence. Probabilistic literacy is not limited to computational ability because it also involves interpretation, communication, contextual understanding, and reflective evaluation of probabilistic information. Students with strong probabilistic literacy are generally more capable of understanding probability representations, interpreting graphs and numerical information, and explaining probabilistic meaning coherently (Ingram, 2024; Muñiz-Rodríguez et al., 2020). In solving discrete random variable problems, probabilistic literacy enables students to connect mathematical symbols with contextual interpretation and practical implications. However, many students still experience difficulties in interpreting probabilistic representations and communicating probabilistic ideas effectively. Students often focus primarily on formulas and numerical results without understanding the contextual meaning underlying probabilistic calculations. Consequently, probability learning may become procedural and mechanistic rather than conceptual and interpretive. Therefore, probabilistic literacy should be integrated with probabilistic reasoning to support comprehensive probabilistic thinking development in mathematics education.

To analyze students' cognitive development and probabilistic thinking structures, SOLO Taxonomy provides a hierarchical framework for classifying learning outcomes and reasoning complexity. SOLO Taxonomy categorizes students' understanding into five levels, namely Prestructural, Unistructural, Multistructural, Relational, and Extended Abstract. At the Prestructural level, students demonstrate fragmented understanding and frequently provide irrelevant or incomplete responses to mathematical problems. At the Unistructural level, students focus only on one relevant aspect of a problem without connecting ideas conceptually. Students at the Multistructural level are able to identify several relevant aspects separately but still experience difficulties integrating concepts coherently. At the Relational level, students demonstrate integrated conceptual understanding and logical connections among mathematical ideas. Finally, students at the Extended Abstract level are able to generalize concepts, transfer knowledge across contexts, and construct abstract reasoning beyond the immediate problem situation. SOLO Taxonomy has been widely applied in mathematics education research to analyze students' reasoning structures, conceptual understanding, and higher-order thinking development (Schmude, 2022; Serrano et al., 2025). In probability education, SOLO Taxonomy provides an effective framework for identifying the progression of students' probabilistic thinking from fragmented procedural understanding toward integrated and abstract probabilistic reasoning. Therefore, SOLO Taxonomy is considered suitable for investigating students' probabilistic reasoning and probabilistic literacy in solving discrete random variable problems.

Several previous studies have investigated probabilistic reasoning, probabilistic literacy, and SOLO Taxonomy in different educational contexts; however, these constructs have generally been examined separately rather than integratively. Research on probabilistic reasoning mainly focused on misconceptions, heuristic thinking, conceptual understanding, and probabilistic decision-making processes. Other studies emphasized probabilistic literacy from the perspectives of interpretation, communication, contextual understanding, and probabilistic information evaluation. Meanwhile, SOLO Taxonomy has frequently been used to analyze mathematical reasoning, inferential reasoning, computational thinking, and conceptual understanding in broader mathematics learning contexts. Although these studies contributed significantly to mathematics education literature, limited research has integrated probabilistic reasoning, probabilistic literacy, and SOLO Taxonomy simultaneously in probability learning contexts. Furthermore, most previous studies focused on general probability concepts rather than discrete random variable problems, which require higher-order abstraction and interpretive reasoning. Existing studies also predominantly relied on either quantitative or qualitative approaches independently, resulting in limited understanding of students' probabilistic cognitive structures comprehensively. In mathematics teacher education, previous research involving preservice mathematics teachers mainly explored pedagogical competence, technological integration, professional beliefs, and instructional strategies rather than probabilistic thinking structures specifically. Consequently, there remains limited understanding regarding how preservice mathematics teachers develop probabilistic reasoning and probabilistic literacy across different SOLO Taxonomy levels. Therefore, the integration of probabilistic reasoning, probabilistic literacy, and SOLO Taxonomy within a mixed-methods framework is important for providing a more comprehensive understanding of students' probabilistic thinking structures in solving discrete random variable problems.

METHOD

Research Design

This study employed a mixed-methods research design to investigate students' probabilistic reasoning and probabilistic literacy in solving discrete random variable problems using SOLO Taxonomy as an analytical framework. The mixed-methods approach was selected because the study aimed to obtain both quantitative measurements of students' probabilistic literacy and qualitative insights into students' reasoning structures and thinking processes. The quantitative approach was used to identify students' probabilistic literacy levels through questionnaire responses and classify probabilistic reasoning levels based on written test results. Meanwhile, the qualitative approach was employed to explore students' conceptual understanding, interpretation patterns, and reasoning processes through semi-structured interviews. The integration of quantitative and qualitative approaches enabled the study to provide a more comprehensive understanding of students' probabilistic thinking structures. This study adopted a convergent mixed-methods design in which quantitative and qualitative data were collected within the same research phase and integrated during interpretation. The design was considered appropriate because probabilistic reasoning and probabilistic literacy involve measurable performance as well as deeper cognitive interpretation. Through this approach, the study was able to identify the distribution of students' reasoning levels while simultaneously exploring the cognitive characteristics underlying each SOLO Taxonomy level. The use of mixed-methods research also strengthened the validity of the findings through triangulation among quantitative and qualitative data sources. Therefore, the research design supported a holistic investigation of students' probabilistic reasoning and literacy development in discrete random variable learning contexts.

Participant

The participants of this study consisted of 20 fifth-semester students enrolled in the Introduction to Probability Theory course in the Mathematics Education Study Program during the 2025/2026 academic year. The participants were selected purposively because the study required students who had previously learned discrete random variable distributions and demonstrated probabilistic reasoning processes during problem solving. The population of this study included undergraduate mathematics education students who had completed introductory probability learning materials. Preservice mathematics teachers were selected because they are future educators who will later teach probability concepts in schools and are therefore expected to possess strong probabilistic reasoning and literacy competencies. The participants represented students with different levels of mathematical understanding and academic performance in probability learning. All participants voluntarily agreed to participate in the study and were informed about the research objectives before the data collection process was conducted. To obtain deeper qualitative insights, five students representing each SOLO Taxonomy level were selected for semi-structured interviews. The interview participants represented Prestructural Probabilistic Thinking (PPT), Unistructural Probabilistic Thinking (UPT), Multistructural Probabilistic Thinking (MPT), Relational Probabilistic Thinking (RPT), and Extended Abstract Probabilistic Thinking (EPT) categories. The participant selection for interviews was based on students' written test results and SOLO Taxonomy classifications. Therefore, the participant selection process enabled the study to explore diverse probabilistic thinking structures across different cognitive levels comprehensively.

Instrument

This study used three research instruments, namely a probabilistic reasoning test, a probabilistic literacy questionnaire, and a semi-structured interview guide. The probabilistic reasoning test consisted of an open-ended discrete random variable problem involving the Poisson distribution. The test was designed to assess students' abilities in constructing conjectures, performing mathematical manipulation, interpreting probabilistic information, justifying solutions, and drawing logical conclusions. The main problem required students to analyze the probability of receiving at most three emergency calls per day using the Poisson distribution formula. Students were instructed not only to calculate the probability values correctly but also to explain and justify their reasoning processes systematically. The probabilistic literacy questionnaire used a five-point Likert scale ranging from strongly disagree to strongly agree. The questionnaire measured students' abilities in interpreting probabilistic information, understanding mathematical representations, evaluating contextual meaning, and communicating probabilistic ideas coherently. The semi-structured interview guide was designed to explore students' reasoning structures, conceptual understanding, interpretation strategies, and cognitive processes in greater depth. All instruments were validated by experts in mathematics education and probability learning before implementation. The reliability of the probabilistic literacy questionnaire was tested using Cronbach's Alpha and demonstrated acceptable reliability with $\alpha > 0.70$. Data collection was conducted during a two-month period in the 2025/2026 academic year through the administration of tests, questionnaires, and interviews.

Procedure

The study was conducted systematically over two months during the 2025/2026 academic year through several research stages. The first stage involved identifying research problems and reviewing relevant literature related to probabilistic reasoning, probabilistic literacy, SOLO Taxonomy, and discrete random variable learning. The second stage consisted of developing research instruments, including the probabilistic reasoning test, probabilistic literacy questionnaire, and semi-structured interview guide. After the instruments were developed, expert validation was conducted to evaluate the relevance, clarity, and appropriateness of the instrument contents. The validated instruments were revised according to suggestions and feedback from the validators before implementation in the actual study. During the data collection stage, participants completed the probabilistic reasoning test and probabilistic literacy questionnaire within scheduled probability learning sessions. Students' written responses were subsequently analyzed and classified according to SOLO Taxonomy levels. Based on the classification results, five representative students from different SOLO levels were selected for semi-structured interviews. The interview process explored students' reasoning strategies, conceptual understanding, probabilistic interpretation, and justification processes in solving discrete random variable problems. After all quantitative and qualitative data had been collected, descriptive and thematic analyses were conducted before integrating the findings comprehensively. Finally, the integrated findings were interpreted to identify students' probabilistic reasoning structures, literacy profiles, and cognitive characteristics within the SOLO Taxonomy framework.

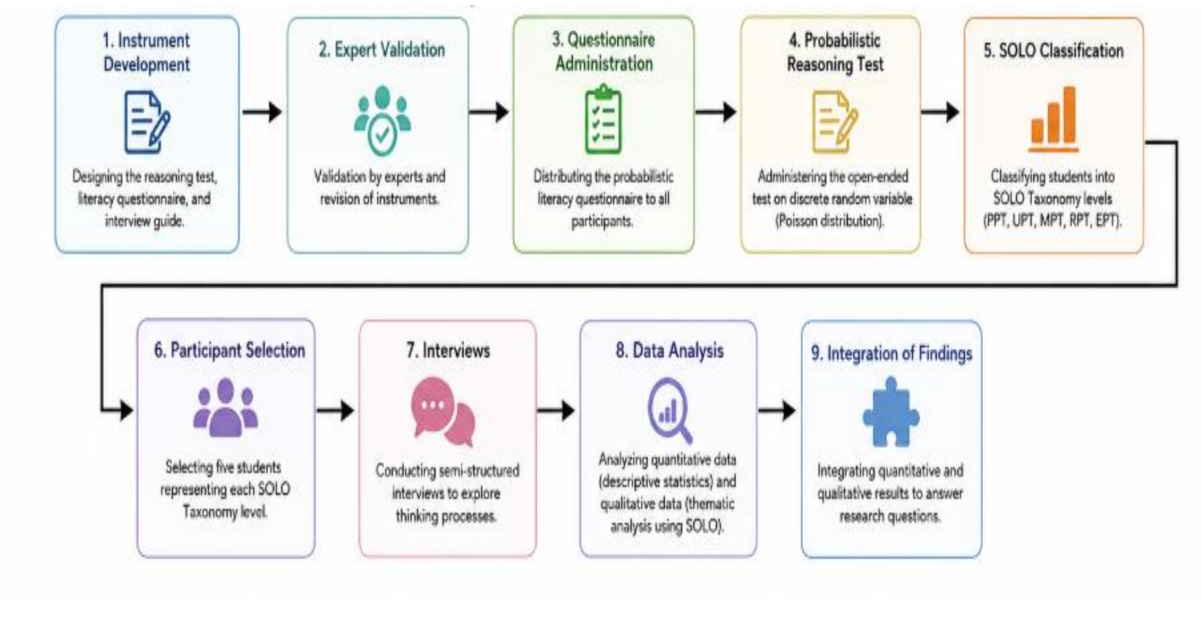

Figure 2. Research Procedure

Analysis plan

Quantitative and qualitative data were analyzed using complementary analytical procedures to support comprehensive interpretation of students' probabilistic thinking structures. Quantitative data obtained from the probabilistic literacy questionnaire were analyzed descriptively using percentages, mean scores, and literacy categorization levels. Students' probabilistic literacy levels were classified into low, moderate, and high categories based on predetermined scoring criteria. Students' probabilistic reasoning levels were classified according to SOLO Taxonomy categories, including Prestructural, Unistructural, Multistructural, Relational, and Extended Abstract Probabilistic Thinking levels. The classification process was conducted by analyzing students' written responses based on reasoning structures, conceptual understanding, integration of ideas, and justification quality. Cross-tabulation analysis was used to examine the relationship between probabilistic literacy and probabilistic reasoning levels descriptively. In addition, Spearman rank correlation analysis was conducted when necessary to support the interpretation of relationships between literacy and reasoning variables. Qualitative data obtained from written responses and interviews were analyzed thematically using SOLO Taxonomy characteristics as analytical indicators. Students' responses were categorized into PPT, UPT, MPT, RPT, and EPT levels based on their reasoning complexity, conceptual integration, abstraction, and probabilistic interpretation patterns.

Thematic analysis focused on how students interpreted probabilistic information, constructed mathematical arguments, and generalized probabilistic meaning during problem solving. Triangulation among test results, questionnaire data, and interview findings was conducted to ensure the credibility and trustworthiness of the analysis.

RESULTS AND DISCUSSION

Results

The results of this study describe preservice mathematics teachers' probabilistic reasoning and probabilistic literacy in solving discrete random variable problems based on SOLO Taxonomy levels. The findings were obtained from the probabilistic reasoning test, probabilistic literacy questionnaire, and semi-structured interviews. Quantitative findings were used to identify the distribution of students' probabilistic reasoning and literacy levels, whereas qualitative findings were employed to explore students' cognitive structures, conceptual understanding, interpretation patterns, and reasoning processes during probability problem solving. The integration of both quantitative and qualitative findings enabled the study to provide a comprehensive understanding of students' probabilistic thinking structures across different SOLO Taxonomy levels. The findings are presented systematically according to probabilistic reasoning levels, probabilistic literacy profiles, relationships between literacy and reasoning, characteristics of probabilistic literacy at each SOLO level, and qualitative exploration of students' higher-order probabilistic thinking.

Distributor of probabilistic reasoning levels based on SOLO taxonomy

The probabilistic reasoning test revealed that students' reasoning structures were distributed unevenly across the SOLO Taxonomy levels. Most participants were categorized at lower and middle cognitive levels, particularly within the Unistructural and Prestructural levels. Only a small proportion of students demonstrated higher-order probabilistic reasoning characterized by conceptual integration, abstraction, and contextual generalization. The complete distribution of probabilistic reasoning levels is presented in Table 4.

Table 4. Distribution of probabilistic reasoning levels based on SOLO taxonomy

SOLO Level	Frequency	Percentage	Representative Subject
Prestructural Probabilistic Thinking	6	30%	M21
Unistructural Probabilistic Thinking	9	45%	M06
Multistructural Probabilistic Thinking	3	15%	M03
Relational Probabilistic Thinking	1	5%	M10
Extended Abstract Probabilistic Thinking	1	5%	M17
Total	**20**	**100%**	**-**

Table 4 shows that the largest proportion of students was classified at the Unistructural Probabilistic Thinking (UPT) level, accounting for 45% of the participants. Students at this level were generally able to identify one relevant aspect of the probability problem, such as recognizing the Poisson distribution formula or identifying the parameter λ. However, these students still experienced substantial difficulties integrating probabilistic concepts and constructing complete reasoning processes. Their responses frequently focused on formula substitution without deeper interpretation of probabilistic meaning. Several students demonstrated procedural understanding but failed to explain why the applied procedures were conceptually appropriate. In many cases, students recognized mathematical symbols correctly but could not connect them to contextual situations involving probability interpretation. This condition indicates that students at the UPT level possessed partial conceptual understanding without coherent conceptual integration.

Furthermore, 30% of students were categorized at the Prestructural Probabilistic Thinking (PPT) level. Students within this category demonstrated fragmented understanding and frequently relied on intuition, guessing, or irrelevant reasoning during probability problem solving. Many students misunderstood the probabilistic meaning of "at most three calls" and interpreted the problem incorrectly. Some participants directly substituted values into formulas without identifying the appropriate probabilistic interpretation first. Their responses often lacked mathematical justification and coherent explanation. These findings indicate that students at the PPT level still experienced substantial conceptual difficulties in understanding discrete random variable distributions.

Only a limited number of students reached higher SOLO Taxonomy levels. Three students (15%) were classified at the Multistructural Probabilistic Thinking (MPT) level. These students were able to identify multiple relevant aspects of probability problems and perform procedural calculations appropriately. However, they still experienced difficulties integrating probabilistic concepts into coherent conceptual explanations. Meanwhile, only one student (5%) reached the Relational Probabilistic Thinking (RPT) level and one student (5%) achieved the Extended Abstract Probabilistic Thinking (EPT) level. Students at these higher levels demonstrated integrated understanding, coherent reasoning, and conceptual abstraction during probability problem solving. Overall, the findings indicate a substantial gap between lower-order and higher-order probabilistic reasoning structures among preservice mathematics teachers.

Students' probabilistic literacy profiles

Students' probabilistic literacy profiles were analyzed using questionnaire data focusing on probabilistic interpretation, representation, contextual understanding, and probabilistic communication abilities. The literacy results were categorized into low, moderate, and high literacy levels based on predetermined scoring criteria. The results are presented in Table 5.

Table 5. Students' probabilistic literacy categories

Literacy Category	Score Range	Frequency	Percentage
Low Literacy	0–59	5	25%
Moderate Literacy	60–79	11	55%
High Literacy	80–100	4	20%
Total	**-**	**20**	**100%**

Table 5 indicates that most students demonstrated moderate levels of probabilistic literacy, representing 55% of the participants. Students in this category were generally able to interpret basic probabilistic information and understand elementary probability representations. They demonstrated adequate procedural comprehension and could identify relevant probabilistic concepts within structured problems. Nevertheless, they still experienced difficulties in communicating probabilistic meaning comprehensively and connecting probability concepts to broader contextual situations. Several students correctly interpreted probability formulas but failed to explain their practical implications logically.

Meanwhile, 25% of students were categorized as having low probabilistic literacy. Students within this category demonstrated difficulties in interpreting probabilistic information, understanding mathematical representations, and communicating contextual probabilistic meaning. Their responses often emphasized numerical calculation without conceptual interpretation. Some students misunderstood probabilistic terminology and failed to explain cumulative probability situations coherently. These findings indicate that low probabilistic literacy is associated with limited conceptual understanding and weak probabilistic communication abilities.

Only 20% of students demonstrated high probabilistic literacy. These students showed stronger abilities in interpreting probabilistic information critically, understanding probability representations comprehensively, and communicating probabilistic meaning coherently. Students with high literacy levels were generally more capable of connecting mathematical symbols, contextual situations, and probabilistic reasoning systematically. Their responses reflected conceptual understanding rather than procedural memorization alone.

Relationship between probabilistic literacy and probabilistic reasoning

To examine the relationship between probabilistic literacy and probabilistic reasoning, cross-tabulation analysis was conducted based on literacy categories and SOLO Taxonomy levels. The results are presented in Table 6.

Table 6. Cross-tabulation of probabilistic literacy and probabilistic reasoning levels

SOLO Level	Low Literacy	Moderate Literacy	High Literacy	Total
Prestructural Probabilistic Thinking (PPT)	5	1	0	6
Unistructural Probabilistic Thinking (UPT)	0	8	1	9
Multistructural Probabilistic Thinking (MPT)	0	2	1	3
Relational Probabilistic Thinking (RPT)	0	0	1	1
Extended Abstract Probabilistic Thinking (EPT)	0	0	1	1
Total	**5**	**11**	**4**	**20**

Table 6 demonstrates a clear tendency that students with higher probabilistic literacy also tended to demonstrate higher probabilistic reasoning levels. Students categorized at the Prestructural level were predominantly classified within the low literacy category. These students experienced difficulties not only in performing probabilistic calculations but also in interpreting probabilistic information and contextual meaning. Their responses frequently relied on fragmented understanding, intuition, and incomplete reasoning structures. Students at the Unistructural level were predominantly categorized within the moderate literacy category. Although they recognized one important aspect of the problem, they still experienced difficulties integrating probabilistic representations and contextual interpretation coherently. Several students interpreted "at most three calls" as exactly three calls only, indicating limited understanding of cumulative probability interpretation. These findings suggest that moderate literacy abilities do not necessarily guarantee integrated probabilistic reasoning structures. Conversely, students classified at higher SOLO levels demonstrated high probabilistic literacy. Students at the Relational and Extended Abstract levels were able to interpret probabilistic information critically, communicate probabilistic meaning coherently, and justify mathematical conclusions logically. They connected probabilistic symbols, contextual interpretation, and conceptual reasoning simultaneously rather than relying solely on procedural calculation. These findings indicate that probabilistic literacy functions as an important cognitive foundation supporting higher-order probabilistic reasoning development.

Characteristics of probabilistic literay across SOLO taxonomy levels

Qualitative findings revealed distinctive probabilistic literacy characteristics across SOLO Taxonomy levels. Students at the Prestructural level demonstrated limited abilities in interpreting probabilistic information and contextual meaning. They frequently misunderstood probabilistic terminology, failed to identify relevant information, and provided incomplete mathematical explanations. Their responses were fragmented and lacked coherent probabilistic communication. Students at the Unistructural level began recognizing important probabilistic representations such as formulas and probability parameters. However, their literacy abilities remained partial because they interpreted information separately without integrating mathematical representations with contextual meaning. Their responses focused mainly on formulas rather than conceptual interpretation.

At the Multistructural level, students demonstrated improved literacy abilities by interpreting multiple probabilistic representations simultaneously. They were able to calculate probability values correctly and interpret numerical results appropriately. Nevertheless, they still experienced difficulties synthesizing probabilistic meaning into coherent conceptual explanations.

Students at the Relational level demonstrated integrated probabilistic literacy characterized by coherent interpretation, conceptual integration, and logical explanation. They connected symbolic representation, contextual interpretation, and probabilistic meaning consistently while constructing justified conclusions systematically. Finally, students at the Extended Abstract level exhibited advanced probabilistic literacy characterized by abstraction, generalization, reflective interpretation, and conceptual transfer across contexts. These students not only interpreted probabilistic information correctly but also generalized probabilistic concepts into broader decision-making and contextual situations.

Characteristics of M17 at the Extended Abstract Level

To further explore higher-order probabilistic reasoning and literacy, subject M17 was selected as the representative participant at the Extended Abstract Probabilistic Thinking (EPT) level. The written response of M17 is presented in Figure 6.

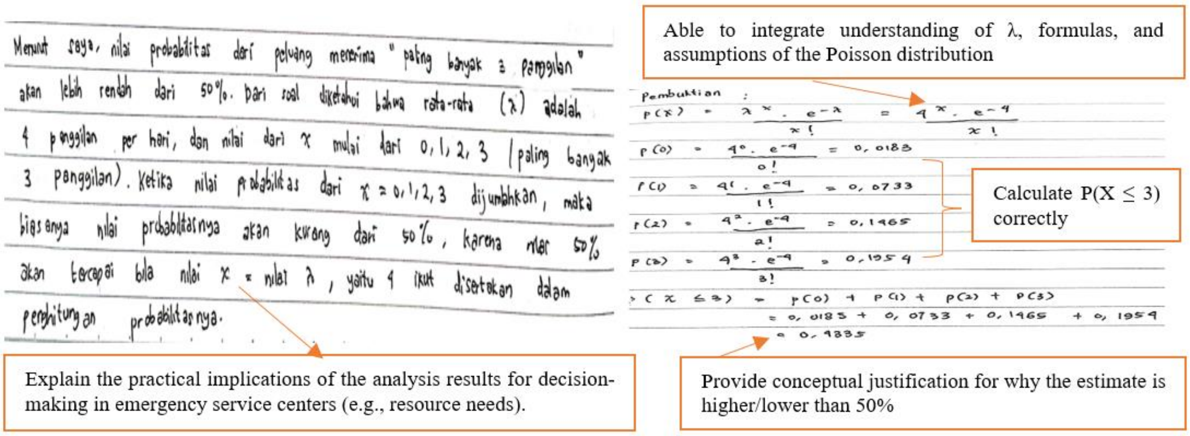

Figure 3. Written response of m17 demonstrating extended abstract probabilistic thinking and probabilistic literacy

Figure 2 illustrates that M17 demonstrated integrated probabilistic reasoning and probabilistic literacy simultaneously. The student correctly identified the parameter $\lambda = 4$, interpreted the phrase "at most three calls" as cumulative probability $P(X \leq 3)$, and calculated the cumulative probability accurately. More importantly, M17 constructed an initial conjecture before performing formal calculations by predicting that the resulting probability would be lower than 50% because the average number of calls exceeded the observed threshold value. Unlike students at lower SOLO levels who focused mainly on procedural substitution, M17 integrated conceptual understanding, contextual interpretation, and mathematical justification coherently. The student explained the practical implication of the probability result within the context of emergency service decision-making. Interview findings further confirmed that M17 developed probabilistic understanding through recognizing probabilistic patterns and conceptual relationships rather than relying solely on memorization. These responses indicate abstraction, metacognitive awareness, and conceptual generalization characteristic of the Extended Abstract level.

Integration of Quantitative and Qualitative Findings

The integration of quantitative and qualitative findings demonstrates that probabilistic reasoning and probabilistic literacy are strongly interconnected competencies in solving discrete random variable problems. Quantitative findings revealed that most students remained concentrated at moderate literacy and lower-order reasoning levels, whereas qualitative findings explained how these limitations emerged in students' interpretation patterns and reasoning structures.

Many students were able to apply probability formulas mechanically but still failed to interpret probabilistic meaning contextually and coherently. Students at lower SOLO levels frequently demonstrated fragmented understanding, limited probabilistic interpretation, and weak conceptual integration. In contrast, students at higher SOLO levels demonstrated balanced development between probabilistic literacy and probabilistic reasoning. They were capable of interpreting probabilistic language, constructing logical arguments, communicating probabilistic meaning coherently, and generalizing probabilistic concepts across contexts.

The findings indicate that probabilistic literacy functions as a cognitive foundation supporting higher-order probabilistic reasoning structures. Furthermore, the progression across SOLO Taxonomy levels suggests that probabilistic thinking develops hierarchically from fragmented procedural understanding toward integrated conceptual abstraction. Therefore, probability instruction should emphasize interpretation, communication, contextual reasoning, and conceptual understanding in addition to procedural computation to support meaningful probabilistic thinking development among preservice mathematics teachers.

Discussion

The findings of this study demonstrate that most preservice mathematics teachers were concentrated at lower and intermediate SOLO Taxonomy levels, particularly within the Prestructural and Unistructural categories, indicating that students' probabilistic reasoning structures remain largely fragmented and procedural. Many participants were able to identify formulas and perform symbolic substitutions correctly, yet they still experienced substantial difficulties interpreting probabilistic meaning contextually and constructing coherent mathematical arguments. This condition suggests that probability learning among preservice teachers is still dominated by procedural orientation rather than conceptual understanding and reflective reasoning. Similar findings have been reported in previous studies indicating that students frequently rely on intuitive judgments and heuristic reasoning rather than formal probabilistic logic during probability problem solving (Erbas & Ocal, 2024; Ghasemi et al., 2023; Suwarto et al., 2023). Earlier studies also showed that probability concepts are often perceived as abstract because students struggle to connect mathematical representations with contextual uncertainty situations. The dominance of lower SOLO levels in this study reinforces the argument that probabilistic reasoning development requires more than computational practice alone (An et al., 2023; Begolli et al., 2021). In particular, discrete random variable problems demand conceptual integration between symbolic representation, probabilistic interpretation, and contextual reasoning simultaneously. Students at lower SOLO levels tended to interpret probability formulas mechanically without understanding cumulative probability concepts or contextual probabilistic meaning comprehensively. These findings are consistent with studies showing that misconceptions in probability frequently emerge when students focus exclusively on formula manipulation without conceptual reflection. Therefore, the results suggest that probability instruction should shift from procedural emphasis toward conceptual interpretation, probabilistic communication, and reasoning-based learning approaches.

The findings further revealed that probabilistic literacy was strongly associated with students' probabilistic reasoning structures across SOLO Taxonomy levels. Students categorized within low probabilistic literacy levels predominantly demonstrated fragmented understanding, incomplete interpretation, and weak mathematical justification. In contrast, students with higher probabilistic literacy showed stronger abilities in interpreting probabilistic representations, communicating probabilistic meaning coherently, and constructing logically justified conclusions (English, 2023; Kappassova et al., 2025). These findings indicate that probabilistic literacy functions as an important cognitive foundation supporting higher-order probabilistic reasoning development. Previous studies similarly emphasized that probabilistic literacy extends beyond computational competence because it involves interpretation, contextual understanding, communication, and reflective evaluation of probabilistic information (Braun & Huwer, 2022). Students with stronger literacy abilities are generally more capable of connecting mathematical symbols with contextual situations and probabilistic meaning systematically (Douglas et al., 2020; Kusmaryono et al., 2024). The present findings also align with previous research reporting that students often misinterpret probabilistic language despite being able to calculate probability values correctly. For example, several participants in this study interpreted the phrase "at most three calls" as exactly three calls, demonstrating limited understanding of cumulative probability interpretation. Such findings highlight the importance of probabilistic language comprehension within probability learning contexts. Consequently, probability education should integrate literacy-oriented instructional approaches that encourage students to interpret probabilistic information critically and communicate probabilistic meaning coherently rather than relying solely on computational accuracy.

The progression of students' reasoning structures across SOLO Taxonomy levels demonstrates that probabilistic thinking develops hierarchically from fragmented understanding toward integrated abstraction and conceptual generalization. Students at the Prestructural level frequently provided irrelevant or incomplete responses, indicating limited conceptual understanding and weak probabilistic interpretation. At the Unistructural and Multistructural levels, students demonstrated procedural competence and partial conceptual recognition but still experienced difficulties integrating probabilistic ideas coherently. In contrast, students at the Relational and Extended Abstract levels were able to connect probabilistic representations, contextual meaning, and mathematical justification simultaneously. These findings support the theoretical assumptions of SOLO Taxonomy, which explain that cognitive development progresses hierarchically from isolated understanding toward relational integration and abstract generalization. Previous studies involving SOLO Taxonomy in mathematics education also reported similar patterns in students' conceptual reasoning and higher-order thinking development. The findings of this study extend previous SOLO Taxonomy research by demonstrating that probabilistic literacy and probabilistic reasoning develop simultaneously across SOLO levels within probability learning contexts. Furthermore, the findings suggest that students' probabilistic interpretation abilities become increasingly sophisticated as reasoning complexity increases. Students at higher SOLO levels not only solved probability problems correctly but also generalized probabilistic meaning into broader conceptual and contextual situations. Therefore, the results reinforce the suitability of SOLO Taxonomy as a framework for analyzing probabilistic reasoning and literacy development in higher education mathematics learning.

The qualitative findings obtained from subject M17 further illustrate the characteristics of higher-order probabilistic reasoning and literacy at the Extended Abstract level. Unlike students at lower SOLO levels who primarily relied on procedural substitution, M17 demonstrated conceptual integration, abstraction, and reflective probabilistic interpretation simultaneously (Schmude, 2022). The student not only calculated cumulative probability accurately but also constructed initial conjectures and justified probabilistic conclusions contextually before formal computation. Such reasoning indicates advanced metacognitive awareness because the student evaluated probabilistic relationships conceptually rather than relying solely on memorized procedures. Previous studies similarly reported that students at higher SOLO levels demonstrate abstraction, conceptual transfer, and flexible reasoning across different mathematical contexts. The findings also support earlier arguments that meaningful probability learning requires students to recognize probabilistic patterns, interpret uncertainty critically, and generalize probabilistic ideas beyond immediate problem situations. M17's ability to explain practical implications of probability results within emergency service contexts reflects integrated probabilistic literacy involving interpretation, communication, and contextual reasoning (Muñiz-Rodríguez et al., 2020). This characteristic contrasts sharply with lower-level students who frequently failed to explain probabilistic meaning despite producing numerical answers. The findings therefore suggest that higher-order probabilistic reasoning is strongly associated with conceptual understanding, abstraction, and probabilistic literacy simultaneously. Consequently, mathematics instruction should encourage reflective reasoning, contextual interpretation, and conceptual generalization to facilitate students' transition toward higher SOLO Taxonomy levels.

The integration of quantitative and qualitative findings provides important pedagogical implications for mathematics teacher education, particularly in probability instruction. The findings indicate that many preservice mathematics teachers still demonstrate limited conceptual understanding and fragmented probabilistic reasoning structures despite being able to perform procedural calculations. Such conditions are concerning because prospective teachers are expected to facilitate meaningful probability learning and support students' conceptual understanding in future classroom practice. Previous studies have similarly emphasized that procedural competence alone is insufficient for developing meaningful probabilistic reasoning and decision-making abilities. Therefore, probability instruction should emphasize conceptual understanding, interpretation, communication, and contextual reasoning in addition to procedural computation. Learning activities involving real-life probabilistic situations, reflective explanation tasks, and reasoning-oriented discussions may help students develop deeper probabilistic understanding. Furthermore, SOLO Taxonomy can function as an effective diagnostic framework for identifying students' probabilistic thinking structures and cognitive development levels systematically. By recognizing students' characteristics at different SOLO levels, lecturers may design differentiated instructional strategies and scaffolding approaches appropriate to students' conceptual needs. The findings also indicate that integrating probabilistic literacy into probability instruction may strengthen students' reasoning structures and conceptual understanding simultaneously. Ultimately, this study highlights the importance of integrating probabilistic reasoning, probabilistic literacy, and hierarchical cognitive frameworks within mathematics education to support the development of higher-order probabilistic thinking among preservice mathematics teachers.

Implications

The findings of this study provide important theoretical and pedagogical implications for probability education, particularly in mathematics teacher education programs. The dominance of students at the Prestructural and Unistructural SOLO levels indicates that probability instruction in higher education still tends to emphasize procedural computation rather than conceptual understanding and probabilistic interpretation. Consequently, mathematics lecturers need to design instructional approaches that integrate probabilistic reasoning and probabilistic literacy simultaneously within probability learning activities. Learning experiences involving contextual probability problems, reflective explanation tasks, and probabilistic communication activities may support students in developing deeper conceptual understanding and higher-order reasoning structures. The findings also demonstrate that probabilistic literacy functions as an important cognitive foundation supporting students' probabilistic reasoning development. Therefore, probability instruction should not focus exclusively on formula application and numerical calculation but should also emphasize interpretation, representation, communication, and contextual evaluation of probabilistic information. The application of SOLO Taxonomy in this study further suggests that hierarchical cognitive frameworks can function effectively as diagnostic tools for identifying students' probabilistic thinking structures and conceptual difficulties systematically. By recognizing students' cognitive characteristics at different SOLO levels, lecturers may develop differentiated instructional strategies and scaffolding techniques appropriate to students' conceptual needs. In addition, the findings imply that probabilistic literacy-oriented learning environments may facilitate students' transition from fragmented procedural understanding toward integrated and abstract probabilistic reasoning. The study also contributes theoretically by extending the application of SOLO Taxonomy within probability education research, particularly in analyzing probabilistic literacy and reasoning simultaneously. Furthermore, the integration of quantitative and qualitative findings demonstrates that mixed-methods approaches provide richer insights into students' cognitive development compared with single-method investigations. Ultimately, these findings highlight the importance of integrating conceptual interpretation, probabilistic communication, and contextual reasoning into probability instruction to support the development of higher-order probabilistic thinking among preservice mathematics teachers.

Limitations and Suggestions for Future Research

This study has several limitations that should be considered when interpreting the findings and developing future research. First, the study involved a relatively small number of participants consisting of only 20 preservice mathematics teachers from a single university, which may limit the generalizability of the findings to broader educational contexts. Second, the investigation focused specifically on discrete random variable problems involving the Poisson distribution, meaning that the findings may not fully represent students' probabilistic reasoning and literacy across other probability topics such as continuous distributions, inferential statistics, or conditional probability. Third, although the mixed-methods design provided comprehensive insights into students' probabilistic thinking structures, the qualitative analysis relied primarily on selected representative participants from each SOLO Taxonomy level. Consequently, some variations in students' reasoning characteristics and interpretation patterns may not have been fully captured. In addition, the study mainly emphasized cognitive aspects of probabilistic reasoning and literacy without examining affective variables such as mathematical anxiety, self-efficacy, motivation, or attitudes toward probability learning that may also influence students' reasoning development. The use of SOLO Taxonomy as the primary analytical framework also focused predominantly on hierarchical cognitive complexity rather than social interaction or collaborative learning dimensions. Furthermore, the study did not investigate the long-term development of probabilistic reasoning and literacy through longitudinal instructional interventions. Therefore, future research is recommended to involve larger and more diverse participant groups from multiple educational institutions to enhance the generalizability and external validity of the findings. Future studies may also explore probabilistic reasoning and literacy across broader probability and statistics topics to obtain more comprehensive understanding of students' probabilistic thinking development. In addition, longitudinal and experimental studies investigating the effectiveness of literacy-oriented probability instruction and reasoning-based learning approaches are strongly recommended. Future research may further integrate other theoretical frameworks, such as metacognitive theory, mathematical communication, or reflective thinking models, to enrich the analysis of students' probabilistic cognitive structures. Ultimately, further investigations are needed to develop instructional models capable of facilitating students' transition from fragmented procedural understanding toward higher-order probabilistic reasoning and abstract probabilistic literacy.

CONCLUSION

This study investigated preservice mathematics teachers' probabilistic reasoning and probabilistic literacy in solving discrete random variable problems using SOLO Taxonomy as an analytical framework through a mixed-methods approach. The findings revealed that most participants were concentrated at the Prestructural and Unistructural levels, indicating that many students still demonstrated fragmented probabilistic understanding, limited conceptual integration, and procedural-oriented reasoning structures. Students at lower SOLO levels frequently relied on formula substitution, intuitive interpretation, and incomplete probabilistic reasoning without fully understanding contextual probabilistic meaning. In contrast, students at higher SOLO levels demonstrated integrated probabilistic understanding characterized by conceptual interpretation, coherent justification, abstraction, and contextual generalization. The study also found that probabilistic literacy was strongly associated with probabilistic reasoning development across SOLO Taxonomy levels. Students with higher probabilistic literacy tended to demonstrate stronger abilities in interpreting probabilistic representations, communicating probabilistic meaning coherently, and constructing logically justified conclusions. Furthermore, the qualitative findings showed that probabilistic literacy developed hierarchically together with reasoning complexity, progressing from fragmented procedural understanding toward integrated abstract probabilistic thinking. The findings confirm that probabilistic reasoning and probabilistic literacy should not be viewed as separate competencies but rather as interconnected cognitive abilities supporting meaningful probability learning. The application of SOLO Taxonomy in this study successfully provided a systematic framework for identifying students' probabilistic thinking structures and cognitive development levels. The study also contributes theoretically by extending the use of SOLO Taxonomy within probability education research, particularly in analyzing probabilistic literacy and probabilistic reasoning simultaneously. Pedagogically, the findings suggest that probability instruction in higher education should emphasize conceptual understanding, contextual interpretation, communication, and reflective reasoning in addition to procedural computation. Ultimately, this study highlights the importance of integrating probabilistic literacy, probabilistic reasoning, and hierarchical cognitive frameworks to support the development of higher-order probabilistic thinking among preservice mathematics teachers.

AUTHOR CONTRIBUTIONS STATEMENT

Arfatin Nurrahmah was responsible for the research conceptualization, design of instruments, data collection, and manuscript drafting. Yulian Dinihari and Andri Surayana contributed to data analysis, interpretation of the research findings, and refinement of the manuscript. Nia Gardenia and Mohamed Aidil Subhan participated in the validation of the research instruments, assisted in qualitative and quantitative data analysis, and contributed to the substantive revision of the manuscript. Jarwani Afgani Dahlan supervised the research, provided methodological guidance, and reviewed the final manuscript for accuracy and coherence. All authors approved the final version and agreed to be accountable for all aspects of the work.

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