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Intuitive and reflective mathematical reasoning in solving quadratic function problems: A dual-process perspective

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Abstract

Background: Understanding students' mathematical reasoning in solving quadratic function problems is important because it reveals how they construct and regulate their thinking during problem-solving activities. According to Dual-Process Theory, reasoning involves intuitive processes (System 1) and reflective processes (System 2).
Aim: This study aimed to investigate how students engage System 1 and System 2 when solving quadratic function problems.
Method: A qualitative descriptive approach was employed involving two tenth-grade students representing lower and higher mathematical ability levels. Data were collected through written tasks, video recordings, and semi-structured interviews. Data credibility was established through method triangulation.
Results: The findings showed that both System 1 and System 2 were involved in solving quadratic function problems. Lower-ability students relied more frequently on System 1, resulting in automatic and less reflective reasoning. In contrast, higher-ability students demonstrated a more coordinated use of System 1 and System 2, leading to more systematic and accurate solutions.
Conclusion: Quadratic function problem solving requires the integration of intuitive and reflective reasoning. System 2 plays a critical role in evaluating and validating initial intuitive responses, thereby supporting more effective mathematical reasoning.

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INTRODUCTION

Quadratic functions play an important role in secondary mathematics learning because they involve conceptual understanding, graphical representation, and connections with other algebraic topics, making mastery of this concept important for students. Although students have been introduced to quadratic functions at the junior high school level, previous studies show that many senior high school students still experience difficulties in solving quadratic function problems. These difficulties are reflected in conceptual errors (40%), procedural errors (35%), and technical errors (25%) (Kurniasari et al., 2021). This finding is consistent with previous studies reporting that such difficulties may arise from students' limited understanding of quadratic function concepts and properties (Sigalingging & Siregar, 2025)confusion in applying systematic problem-solving procedures (Ruhma et al., 2023), difficulties in recalling relevant formulas (Wijaya & Khabibah, 2025). In addition, students also demonstrate limited reasoning habits in solving quadratic function problems (Faturohman & Afriansyah, 2020). According to dual process theory, reasoning consists of

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two systems, namely System 1 and System 2 (Darmawan, 2019; Kahneman, 2020). This theory, rooted in cognitive psychology, explains that human reasoning involves two distinct types of cognitive processing (Augusto, 2024; Darmawan, 2019; Kahneman, 2020; Kholifah & Darmawan, 2025; Manik et al., 2023; Ruhma et al., 2023). Given these issues, this research focuses on analyzing the activation of System 1 and System 2 in senior high school students based on dual process theory.

Furthermore, the classification of reasoning into System 1 and System 2 is based on the processes involved in generating solutions (Darmawan, 2019). Each system has distinct characteristics. System 1 is characterized by automatic and unconscious processing (Borodin, 2016; Darmawan, 2019; J. S. B. T. Evans, 2019; V. Evans, 2007; Kahneman, 2020). Automatic processing refers to the spontaneous generation of answers based on integrated prior learning experiences, whereas unconscious processing, on the other hand, occurs when individuals produce answers without analyzing the compatibility between prior knowledge and the given information (Gawronski et al., 2013). Meanwhile, subjective-empirical processing involves generating responses based on subjective interpretations derived from visual and auditory observations, causing System 1 reasoning to tend to occur less systematically (Kholifah & Darmawan, 2025; Manik et al., 2023); (Sukma, 2011). In other words, conclusions are often generated without explicit assumptions or structured reasoning processes. For example, when students are asked to determine the shape of a quadratic graph, they may immediately assume that a positive coefficient of x^2 indicates an upward-opening parabola based on prior learning experiences. Speed is the most observable characteristic due to the spontaneity of the response; however, automaticity also plays a role, as the answer is retrieved from previously memorized knowledge.

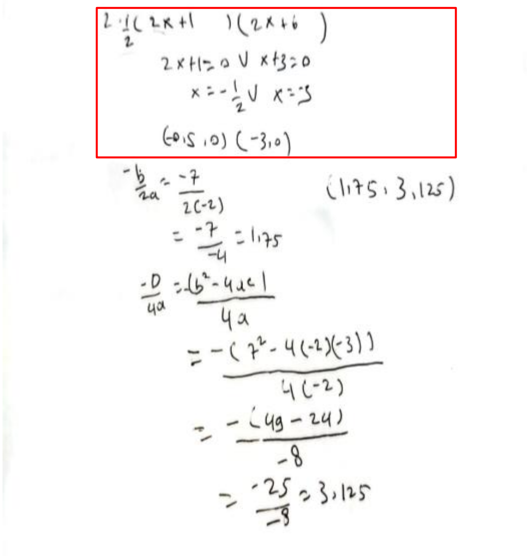
In contrast to System 1, System 2 refers to mental activities characterized by conscious processing and empirical accuracy (Borodin, 2016). Conscious processing involves comparing given information with prior knowledge to produce appropriate responses (Darmawan, 2019). Meanwhile, empirical accuracy refers to achieving accurate answers through systematic empirical steps (Darmawan, 2019; Leron & Hazzan, 2006). Examples of System 2 processes include performing arithmetic operations through procedural manipulations, such as long division, column multiplication, or the use of calculators, which require more deliberate and effortful thinking to evaluate and match relevant information (Manik et al., 2023). Because System 2 involves conscious evaluation and verification processes, the responses produced are generally more carefully considered.

Furthermore, (Wahyuni et al., 2023) argue that in the problem-solving process, System 1 often acts as an initial trigger in generating responses, due to its rapid and automatic nature. The responses produced by System 1 may provide both advantages and disadvantages for individuals (Kahneman, 2020). System 1 becomes advantageous when the problem context aligns with individuals' prior learning experiences (Darmawan, 2019). In such situations, responses can be generated quickly without extensive elaboration in working memory. However, when the problem context does not align with prior experiences and this mismatch is not recognized, the resulting responses may lead to difficulties or even failure in problem solving. This phenomenon is consistent with findings from the preliminary study. In this research, a problem was administered to a tenth-grade senior high school student in Malang City. The following problem was presented in Figure 1.

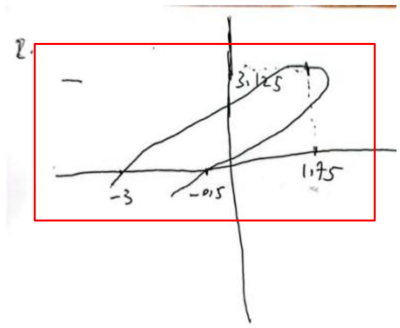
"Buatlah sketsa/ gambar fungsi kuadrat $y = -2x^2 + 7x - 3$**"**
Translate:
"Sketch the graph of quadratic function $y = -2x^2 + 7x - 3$**"**

Figure 1. Preliminary study problem

The response of one student is presented in Figure 2.



(A) Solution steps



(B) Graph/sketch of function

Figure 2. A student's response in the preliminary study

The response presented in Figure 2 indicates the activation of System 1, particularly unconscious processing during the problem-solving process, which leads to an incorrect answer. This interpretation is further supported by the student's statement in the following interview 1 excerpt.

Interview 1. Preliminary study

Researcher : What's the first thing that comes to your mind when you're working on this problem?

Subject : The first thing that comes to my mind is the roots of the given quadratic function and then find the vertex using such as $-b/2a$ and $-D/4a$ formula

Researcher : Are you sure that's what the graph of the quadratic function looks like?

Subject : yes, since it matches the calculations

Researcher : What kind of method did you use to find the roots of the quadratic function?

Subject : Factorization

Researcher : Why did you choose factorization?

Subject : Because it's easier and faster

Researcher : What does the quadratic function graph look like?

Subject : It's shaped like a hill

Based on the written response and interview 1, it is evident that the student directly applied factoring, as it was perceived to be easier and faster based on prior learning experiences. The student reported that the initial step considered was determining the roots, followed by identifying the vertex. This pattern reflects the activation of System 1, characterized by fast, automatic thinking that relies on prior experience. However, this matching process led to an incorrect result because the student failed to re-evaluate the compatibility of the algebraic form. As shown in Figure 2(A), the student wrote the expression $\frac{1}{2}(2x+6)(2x+6)$ which is incorrect since its expansion yields $y = 2x^2 + 7x + 3$ instead of $y = -2x^2 + 7x - 3$. Consequently, the graph constructed by the student in Figure 2(B) is also inaccurate. Furthermore, the student's statement that a quadratic graph "resembles a hill" indicates the use of subjective visual interpretation without further analytical verification, which reflects subjective-empirical processing in System 1. Therefore, the involvement of System 2 is crucial for evaluating the correctness of responses, thereby minimizing difficulties and potential failure in problem solving.

Furthermore, errors in solving mathematical problems are not always caused by misconceptions. In many cases, such errors arise from rapid and unverified thinking processes associated with System 1. Therefore, understanding the characteristics of mental processes involved in solving mathematical problems are essential. If these characteristics can be identified, explained, and explicitly taught to students, they are more likely to become cautious and reflective, thereby enabling the activation of System 2 in processing information. Such understanding also enables teachers to assess the extent of students' conceptual understanding. Furthermore, it can be utilized to identify and address students' difficulties in problem solving, determine appropriate instructional methods, learning models, or scaffolding, and design effective learning strategies to facilitate meaningful knowledge internalization.

Based on the preliminary study, research examining students' reasoning in solving quadratic function problems from a Dual Process Theory perspective remains limited, particularly in identifying the involvement of System 1 and System 2 during problem solving. Previous research that has been applying dual-process theory in mathematics has been conducted by several experts. For instance, (Kholifah & Darmawan, 2025) that examined junior high school students' thinking process involving the surface area and combined volume of three-dimensional shapes using Dual Process Theory. Similarly (Manik et al., 2023) focus on investigating the thinking characteristics of 9th-grade high school students in solving AKM problems also based on Dual Process Theory. Additionally, (Wahyuni et al., 2023) explored the thinking abilities of college students across visual, auditory, and kinesthetic learning styles in the context of solving simple closed-curve problems. Lastly, (Riset & Pendidikan, 2023) analyzed the interaction between elementary school students' default (automatic) and interventionist (analytical) thinking in solving geometry problems. Based on previous studies conducted by experts in mathematics education, the Dual Process Theory has generally been applied to geometric elements without any consideration of students' academic ability criteria.

Furthermore, there has been no such study on algebraic elements specifically in quadratic functions using the Dual Process Theory particularly the one that specifies a comprehensive approach to students' academic abilities. If students' thought processes remain unidentified, teachers cannot determine whether students truly understand the main concepts or just merely memorizing procedures, to the extent that the misconceptions may go undetected. This study aims to explore how System 1 and System 2 are involved in students' reasoning during quadratic function problem solving by considering differences in students' ability levels. This study contributes to the literature on Dual Process Theory in mathematics education and provides pedagogical insights for supporting students' reasoning during quadratic function problem solving.

LITERATURE REVIEW

Mathematical reasoning is widely recognized as a fundamental component of mathematics learning because it enables students to develop, justify, and evaluate mathematical ideas. Reasoning supports learners in constructing meaningful connections between concepts, procedures, and problem-solving strategies. In mathematics education, reasoning is not limited to obtaining correct answers but also involves explaining and validating the processes used to reach those answers (Alsina et al., 2021; Hjelte et al., 2020; Kollosche, 2021; Säfström et al., 2024). Previous studies have shown that students with stronger reasoning abilities tend to demonstrate greater flexibility when facing unfamiliar problems. Mathematical reasoning contributes to conceptual understanding by encouraging students to analyze relationships and identify patterns. It also facilitates the development of critical thinking and decision-making skills. Educational frameworks consistently emphasize reasoning as a core competency in mathematics instruction. As mathematical tasks become more complex, students are expected to employ increasingly sophisticated forms of reasoning. Consequently, investigating students' reasoning processes provides valuable insights into how mathematical understanding is developed. Such investigations are particularly relevant in topics that require both conceptual and procedural knowledge.

Dual-Process Theory offers a useful framework for understanding how individuals reason and make decisions in various contexts, including mathematics. According to this theory, cognitive activity is supported by two interacting systems known as System 1 and System 2. System 1 operates rapidly, automatically, and with minimal conscious effort. This system relies heavily on prior experiences, pattern recognition, and intuitive judgments. In contrast, System 2 is slower, deliberate, and reflective in nature. It becomes active when individuals encounter unfamiliar situations or need to evaluate the validity of an initial response. Researchers have argued that mathematical problem solving often involves continuous interaction between these two systems. Intuitive responses generated by System 1 may provide efficient starting points for solving problems. However, reflective processes associated with System 2 are necessary to assess and refine those initial judgments. Therefore, Dual-Process Theory provides a valuable perspective for examining students' mathematical reasoning.

The distinction between intuitive and reflective reasoning has received increasing attention in mathematics education research. Intuitive reasoning enables students to make rapid judgments based on familiar patterns and previously acquired knowledge. This form of reasoning often supports efficient problem solving when students encounter routine mathematical situations. Nevertheless, intuitive responses may sometimes lead to misconceptions or incorrect conclusions if they are not critically examined. Reflective reasoning allows learners to analyze assumptions, evaluate evidence, and verify the accuracy of their solutions (Milner & Wolfer, 2023; Mohamad & Tasir, 2023; Molerov et al., 2020; Richards et al., 2020). Through reflective thinking, students can identify inconsistencies and revise inappropriate strategies. Several studies have reported that successful mathematical problem solvers tend to balance intuitive and reflective reasoning. Rather than relying exclusively on one type of reasoning, effective learners integrate both processes according to task demands. This interaction enhances the quality and reliability of mathematical decisions. Consequently, understanding the relationship between intuitive and reflective reasoning remains an important area of investigation.

Quadratic functions constitute a significant topic in secondary mathematics because they integrate algebraic, graphical, and analytical representations. Students are expected to interpret quadratic relationships, determine key characteristics of functions, and solve related mathematical problems (Díaz et al., 2020; Hu et al., 2022; Odutayo & Fonseca, 2024; Reid O'Connor & Norton, 2024). However, many learners experience difficulties when transitioning between symbolic expressions and graphical representations. Such challenges often require students to engage in both intuitive and reflective reasoning processes. For example, students may initially estimate the shape of a parabola through intuition before applying formal procedures to verify their predictions. Solving quadratic function problems frequently involves identifying patterns, selecting appropriate methods, and evaluating solution accuracy (Ahmed et al., 2020; Faradiba et al., 2024; Mohammed et al., 2020). These activities activate different cognitive processes that correspond to System 1 and System 2. As a result, quadratic functions provide a suitable context for examining reasoning from a dual-process perspective. Investigating students' approaches to quadratic function tasks can reveal important characteristics of their mathematical thinking.

Although previous studies have explored mathematical reasoning and dual-process theory separately, limited research has specifically examined their interaction in the context of quadratic function problem solving. Existing literature has predominantly focused on general problem-solving performance, cognitive styles, or mathematical achievement. Comparatively fewer studies have investigated how students coordinate intuitive and reflective reasoning while solving specific mathematical topics. Understanding this coordination is important because reasoning patterns may differ according to students' mathematical abilities and task characteristics. Furthermore, examining cognitive processes can provide a deeper explanation of why students succeed or struggle in solving mathematical problems. Insights from such investigations may support the development of instructional strategies that encourage reflective thinking without suppressing intuitive insights. Teachers can use this knowledge to design learning environments that promote balanced reasoning processes. A dual-process perspective therefore offers a promising framework for understanding students' mathematical behavior. This study seeks to contribute to this area by exploring intuitive and reflective mathematical reasoning in solving quadratic function problems.

METHOD

Research Design

This study employed a qualitative descriptive design to investigate the involvement of intuitive and reflective reasoning during students' quadratic function problem-solving processes from the perspective of Dual-Process Theory. The qualitative approach was considered appropriate because it enables an in-depth exploration of students' cognitive processes, particularly the activation of System 1 and System 2 while solving mathematical problems. The study focused on understanding how students construct, regulate, and justify their reasoning rather than measuring performance quantitatively. The analytical framework was based on the characteristics of System 1 and System 2 proposed in Dual-Process Theory.

Participants

The participants consisted of two tenth-grade students from a senior high school who were selected using purposive sampling. The selection criteria included students who had completed learning on quadratic functions, demonstrated different levels of mathematical ability, and possessed adequate communication skills to explain their reasoning processes. One participant represented a high mathematical ability level, while the other represented a low mathematical ability level. The small number of participants was intentionally chosen to facilitate a detailed examination of individual reasoning processes and cognitive characteristics during problem solving.

Instruments

Data were collected through written problem-solving tasks, video recordings, researcher field notes, and semi-structured interviews. The mathematical task consisted of a quadratic function problem specifically designed to elicit indicators of both System 1 and System 2 activation. Students were asked to solve the task individually while their problem-solving activities were recorded using video equipment. The recordings captured students' behaviors, expressions, verbal comments, and solution strategies throughout the process. Following task completion, semi-structured interviews were conducted to explore students' reasoning, clarify solution steps, and identify cognitive processes that were not directly observable from written responses. To enhance data credibility, method triangulation was employed by comparing information obtained from written work, video recordings, interviews, and field notes.

1. Tentutakan nilai a, b, c serta sumbu simetri $f(x) = 3x^2 - 4x + 2$!
2. Buatlah sketsa/ gambar fungsi kuadrat $y = -2x^2 + 4x + 6$
3. Pak Dika memiliki sawah yang ditanami padi berbentuk persegi Panjang dengan luas membentuk $f(x) = x^2 - 4x - 45$. Apabila ukuran pajang kebun Pak Dika adalah $(x - 9)$, maka tentukan ukuran lebar kebun Pak Dika yang mungkin jika nilai x adalah 45!

Translate:

1. Determine the values of a, b, and c, as well as the axis of symmetry of $f(x) = 3x^2 - 4x + 2$!
2. Draw a sketch/graph of the quadratic function $y = -2x^2 + 4x + 6$
3. Mr. Dika has a rice field in the shape of a rectangle with an area represented $f(x) = x^2 - 4x - 45$.. If the length of the field is $(x - 9)$, then determine the possible width of Mr. Dika's field when $x = 45$.

Figure 3. Quadratic function problems given to the participants

Data Analysis

Data analysis followed the interactive model of qualitative analysis, including data collection, data reduction, categorization, data display, interpretation, and conclusion drawing. Students' written responses, interview transcripts, and video-recorded behaviors were examined to identify indicators of System 1 and System 2. The analysis was guided by a rubric developed from Dual-Process Theory and validated by two experts in mathematics education prior to implementation. Indicators of System 1 included automatic processing, unconscious processing, and subjective-empirical processing. Indicators of System 2 included conscious processing and empirical accuracy. Data from multiple sources were coded and categorized according to these indicators to determine the extent to which each cognitive system contributed to students' reasoning during quadratic function problem solving.

Table 1. Indicators of System 1 and System 2

Category	Mental Process	Indicator
System 1	Automatic	Answering spontaneously, memorizing formulas, and without effort.
	Without conscious awareness	Making errors in writing or answering without being aware of them.
	Subjective-empirical	Determining how to answer or writing answers based on visual impression.
System 2	With conscious awareness	Matching the answer with prior learning experiences to produce a response
	Empirical accuracy	Performing arithmetic operations using manual techniques such as long division, long multiplication, etc. Performing arithmetic operations using supporting tools, such as calculators, to ensure the accuracy of results.

Procedure

The study was conducted in four stages. First, participants were selected according to the predetermined criteria of mathematical ability and communication skills. Second, students were given a quadratic function problem-solving task and asked to complete it individually while being video-recorded. Third, semi-structured interviews were conducted immediately after task completion to investigate the reasoning underlying students' responses and solution strategies. Finally, the collected data, including written work, video recordings, interview transcripts, and field notes, were analyzed using the System 1 and System 2 indicators. The results from different data sources were then compared through triangulation to verify the consistency of findings and generate a comprehensive description of students' intuitive and reflective mathematical reasoning.

RESULTS AND DISCUSSION

This study investigates how System 1 and System 2 explain students' thinking in quadratic problem solving. Two senior high school students were purposively selected based on differences in academic ability. Although the small number of participants limits the generalizability of the findings, it allowed for a more in-depth exploration of students' reasoning processes. The study focuses on the involvement of System 1 and System 2 during problem solving rather than on comparing students' ability levels. Therefore, the participants were treated as sources of data to understand the dynamics of reasoning based on Dual Process Theory. System 1 and System 2 were analyzed using triangulated data consisting of students' written responses, video recordings during problem solving, and retrospective interviews to explain students' mathematical reasoning processes.

Results :

Activation of System 1 and System 2 in Subject 1

Student 1 was categorized as having low academic ability. Nevertheless, the student was able to correctly solve Problem 1 through the involvement of System 1 and System 2.

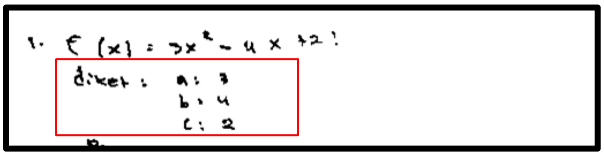
Automatic and Unconscious Processing (System 1)

For solving the first problem, which involved determining the axis of symmetry of a quadratic function:

$$fx = 3x^2 - 4x + 3$$

Figure 4. Research problem number 1

Student 1 was asked to identify the coefficients $a, b, c,$ and to determine the axis of symmetry using the appropriate formula. Based on the written response and video recording (see Figure 5), Student 1 immediately identified the coefficient values without thoroughly examining the structure of the quadratic function. The student wrote: $a = 3$; $b = 4$ ($incorrect;\ it\ should\ b = -4$); $c = 2$.



Translated:

1. $f(x) = 3x^2 - 4x + 21$

Know: $a = 3$

$b = 4$

$c = 2$

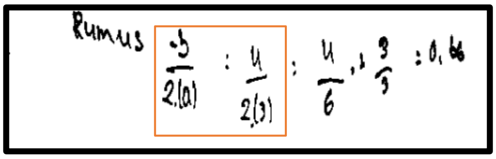
Figure5. Subject 1's answer to question 1

Such errors (as shown in Figure 5) occurred even though the function was explicitly written in the problem. In the retrospective interview, the student explained:

Interview 2. Activation of System 1 in Subject 1

Researcher : Now, what are the values of a, b, c?
Subject 1 : The values of a, b, c are $a = 3, b = -4, c = 2$
Researcher : Are you sure? Did you double-check your work?
Subject 1 : Yes, I'm sure, and I didn't double-check it.
Researcher : On your answer sheet, you wrote $b = 4$ for problem number 1. Are you sure?
Subject 1 : (She/He remained silent)
Subject 1 : It's wrong.
Researcher : What is the correct answer?
Subject 1 : Minus 4.

The interview data indicate that Student 1 possessed an understanding of the general form of a quadratic function; however, this knowledge was not accessed during the initial stage of the problem-solving process. Instead, the initial response was guided by a sense of confidence derived from prior exposure to similar problems, which unconsciously obscured the negative sign of the coefficient *b.* This finding reinforces the proposition that errors associated with System 1 are not necessarily caused by a lack of knowledge, but rather by rapid and unverified decision-making. Furthermore, Student 1 continued calculating the axis of symmetry using the formula $x = \frac{-b}{2a}$. However, the final result obtained was 0,66, representing the decimal form of $\frac{2}{3}$, but without the negative sign, as shown in Figure 6. This indicates that the initial error in identifying the coefficient b was carried through the subsequent calculations, leading to an incorrect result.



Translated:

Formula $\frac{-b}{2(a)} = \frac{4}{2(3)} = \frac{4}{6} : 2 = \frac{2}{3} = 0{,}66$

Figure 6. Subject 1’s Answer Related to the Axis of Symmetry

The following interview snippet below clarifies the process:

Interview 3. System 1 dominance in sign misinterpretation

Researcher : Since the correct value of b is -4. How should it be when you substitute it to the $\frac{-b}{2(a)}$formula?
Subject 1 : Minus four over two times three.
Researcher : Are you sure? Did you double-check your answer?
Subject 1 : Nope, it wasn’t necessary.
Researcher : The formula is $\frac{-b}{2(a)}$. Now, the value of b is minus 4, that means there’re two minuses.
Subject 1 : Ohh, yeah.
Researcher : So, you also just realised that? So, what happens now?
Subject 1 : Yes, since there’re two minuses, -4 is also a minus. Therefore, it is positive.

This indicates a discrepancy between the calculation process and the final result, reflecting a disruption in logical reasoning or verification processes, which is a common characteristic when System 1 dominates thinking. The error occurred because Student 1 made a rapid decision without engaging in thorough verification, as illustrated in the following interview excerpt:

Interview 4. System 1-based estimation without verification

Research : Inside the video, you wrote $\frac{4}{6} = 0{,}66$. Where did you get 0,66 from?
Subject 1 : I imagined it in my mind, and then tried to estimate its value. I thought it was already in its simplest form, but it turns out it can still be simplified.

This statement in interview 4 indicates the spontaneous use of a familiar numerical representation (0,66) without considering that the result should be negative. During member checking, the student stated:

Interview 5. System 2 activation during reflection

Researcher : So, what is the correct answer?
Subject 1 : If the value of b is 4, then the answer is -0,66. But since $b = 4$, the correct answer is 0,66.

The statement in interview 5 indicates that conceptual understanding is actually available in long-term memory, but is not accessed effectively when doing the problem at first. System 2 only activated when researcher re-evaluated the answer. This suggests that Subject 1 has the ability to reflect their answer, but the process is not spontaneously activated when solving the problems.

Activation of System 1 and System 2 in Subject 2

Subject 2 demonstrated correct solutions across all research problems, indicating relatively high academic ability. In the mathematics problem-solving process, the activation of System 1 and System 2 seems to complement each other.

Subject 2's Automatic (System 1) and Conscious (System 2) Processing in Solving Problem Number 1

In solving the first problem, that is determining the axis of symmetry of a quadratic function: $fx = 3x^2 - 4x + 3$. (See Figure 4). Subject 2 was asked to identify the value of coefficient a, b, c, and calculate the value of the axis of symmetry using the formula.

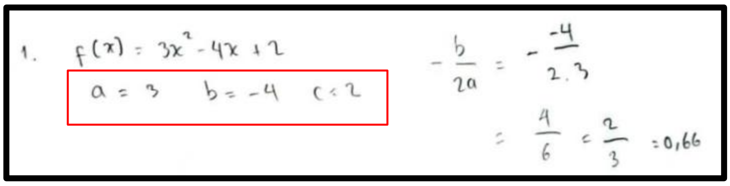


Figure 7. Subject 2's Answer number 1

According to the answer sheet (Figure 7), Subject 2 seems to clearly understand what is asked in the question and how to solve it. This argument is strengthened by the following excerpt of the researcher's interview with Subject 2, which clarifies their thinking:

Interview 6. System 1 Activation in Initial Problem Interpretation

Researcher : Based on the video, you seemed to start working on it right away. What information is given and what is being asked in problem number 1?

Subject 2 : We're asked to find the value of a, b, c, and axis of symmetry. I immediately started looking for the axis of symmetry. If I recall correctly, the formula is $-\frac{b}{2a}$, so I wrote it down first. Then I recalled the way to find the value of a, b, c. a is the one at the front, b is the constant with x, and c is the last one. So I just need to substitute the values into the formula. The b is -4 over 2 times a. The a is 3, then I calculated minus times minus, and finally the result is 4 over 6.

Furthermore, Subject 2 demonstrated coordinated involvement of System 1 and System 2 during problem solving. This coordination was reflected in the student's ability to make quick decisions while simultaneously recalling relevant formulas, identifying the values of a, b, and c, and verifying the substitution process based on prior learning experiences. This is clarified in the following interview excerpt:

Interview 7. Coordination of System 1 and System 2 in Problem Solving

Researcher : Where did you get the $-\frac{b}{2a}$ formula from?

Subject 2 : Automatically, because I memorized the axis of symmetry formula.

Researcher : So you were fully aware of it and just wrote it down right away?

Subject 2 : Yes

Researcher : What do you think this function $f(x) = 3x^2 - 4x + 3$ is?

Subject 2 : It's a quadratic function, since the x is squared and $a \neq 0$.

Researcher : Did you double-check everything after you've finished?
Subject 2 : Yes, I did.

Subject 2's Automatic and Conscious (System 1 and System 2) Process in Solving Problem Number 2

On the second problem, Subject 2 is asked to sketch the graph of a quadratic function.

$$y = -2x^2 + 4x + 6\,.$$

Figure 8. Research problem on graphing a quadratic function

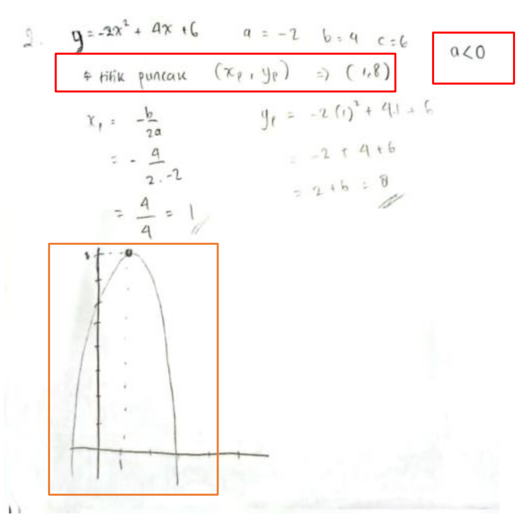
From the video recording, Subject 2 immediately wrote the equation and determined the turning point without first writing the "given" and "asked" information. Furthermore, Subject 2 explains in the interview 8.

Interview 8. Activation in problem understanding and planning

Researcher : Based on the video, you immediately start working on the question without writing what is given. At that time, what do you think the question is asking you to do and what information is given in the problem?

Subject 2 : Sketch the graph of quadratic function $y = -2x^2 + 4x + 6$. I wrote the equation, then immediately found the turning point. Also I look at the a, is it positive or negative, to determine the graph's shape.

The interview excerpt verifies the researcher's initial assumption that Subject 2 consciously and quickly understood what the problem was asking and immediately planned the solution steps by focusing on writing the turning point as the main focus. Furthermore, when Subject 2 quickly stated, "*since $a < 0$, the graph is shaped like an n*" (The graph can be seen in Figure 9), this indicated System 1 activation, namely an automatic response based on prior learning experiences. The answer sheet also shows that Subject 2 used the turning point and the direction in which the parabola opens to draw the graph without finding the x- and y-intercepts, indicating the dominance of intuitive processes in Subject 2.

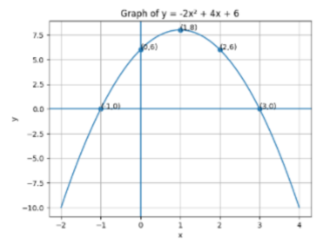


Translated:

2. $y = -2x^2 + 4x + 6 \qquad a < 0$

Turning point $(x_1, y_1) = (1,8)$

$$x_p = -\frac{b}{2a} = -\frac{4}{2.-2} = \frac{4}{4} = 1 \qquad y_p = -2(1)^2 + 4.1 + 6 = -2 + 4 + 6 = 2 + 6 = 8$$



A B

Figure 9. Subject 2's Graph Sketch

The following additional interview shows how Subject 2 automatically determines the shape of the parabola graph:

Interview 9. Subject 2's Automatic Recognition of Quadratic Graph Shape
Researcher : Inside the video, you stated that, "*since $a < 0$, the graph is shaped like an n*"?
Subject 2 : Yes, I've memorized it automatically.

The quick and confident response indicates the activation of System 1, where conceptual knowledge has been internalized and emerges automatically without the need for in-depth analysis. Meanwhile, when sketching the graph (as observed in Figure 9A and represented in Figure 9B), Subject 2 chooses to use the turning point (1,8) without finding the $x-$ or $y-$intercepts.

Activation of System 1 and System 2 in Solving Problem Number 3 by Subject 2
In the third problem, Subject 2 successfully solved the following contextual mathematics problem:

Pak Dika memiliki sawah yang ditanami padi berbentuk persegi panjang dengan luas membentuk $y = x^2 - 4x - 45$**. Apabila ukuran panjang kebun Pak Dika adalah** $(x - 9)$, **maka tentukanlah ukuran lebar kebun Pak Dika yang mungkin jika nilai** x **adalah** 45!

Translate: **Mr. Dika has a rice field in the shape of a rectangle with an area given by** $y = x^2 - 4x - 45$**. If the length of Mr. Dika's field is** $(x - 9)$, **determine the possible width of the field when the value of** x **is** 45!

Figure 10. Research problem number 3

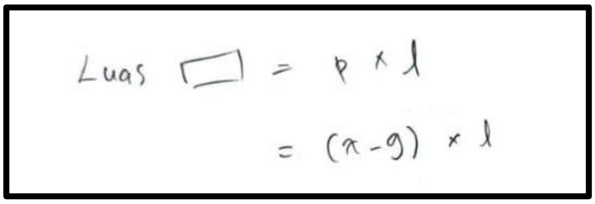


Figure 11. Subject 2's subjective-empirical representation

According to the video, Subject 2 immediately drew the shape of a rectangle (see Figure 11) after reading the "*rice field in the shape of a rectangle*" sentence from the question. This action is carried out without further calculation or consideration. The following interview illustrates this:

Interview 10. System 1 activation in visual representation
Researcher : At the beginning, you drew the rectangle first, right?
Subject 1 : Yes, because it's stated in the problem that the rice field is in the shape of a rectangle.
Researcher : So you drew it based on the visual impression?
Subject 1 : Yes, I did.

Based on Figure 11 and interview 10 excerpt above, Subject 2 immediately drew a rectangle upon reading the information in the problem, indicating the activation of System 1, which involves fast, automatic, and experience-based thinking.

Furthermore, in the counting process, Subject 2 appears to immediately substitute the value of $x = 45$ to the $(x - 9)$ expression. The following interview supports this observation:

Interview 11. Transition from System 1 to System 2 in Calculation
Researcher : Once you know the value of x. What did you do next?
Subject 2 : I automatically substituted the value of x.
Researcher : From the video, you used your fingers to calculate $45 - 9$, am I correct?

Subject 2 : Yes, that's correct.
Researcher : You get the result 452 and 4 × 45 using a long multiplication?
Subject 2 : Yes, I used a long multiplication.

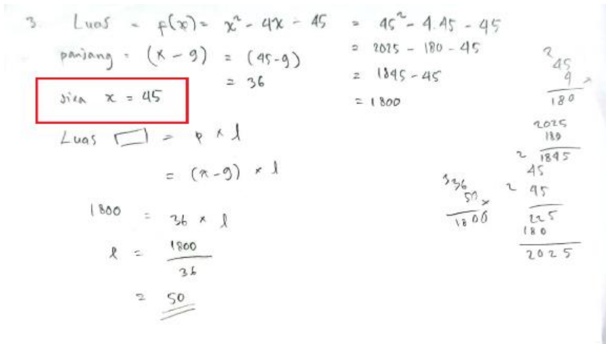


Figure 12. Subject 2's thinking in solving problem number 3

The results show that Subject 2 automatically substituted the value $x = 45$ without much deliberation, indicating the involvement of System 1 (see Figure 12). However, when performing arithmetic calculations such as $45 - 9$ and 4×45, Subject 2 used fingers and long multiplication, indicating the activation of System 2. Furthermore, the following interview excerpt illustrates how Subject 2 engaged in conceptual reasoning when performing the calculation steps.

Interview 12. System 2 cctivation in conceptual reasoning and verification
Researcher : Why does the expression $\frac{1800}{36}$ take that form?
Subject 2 : Since the area is already known, it can be directly substituted. The length is also known and substituted accordingly. I visualize it as a triangle where the area is at the top, while the length and width are at the bottom. To determine the width, cover the width and then divide the area by the length.
Researcher : Then, where does the 50 come from?
Subject 2 : It comes from $\frac{1800}{36}$, and then I checked it again using long multiplication

Subject 2 constructed a conceptual representation by using a triangle as an analogy for the rectangle area formula rather than relying solely on mechanical calculations. However, when Subject 2 stated that the result of $\frac{1800}{36}$ equals 50, she explained that the answer was derived from memorization but subsequently verified using long multiplication Furthermore, Subject 2's solution is presented in a triangular representation of the area formula, as shown in Figure 13.

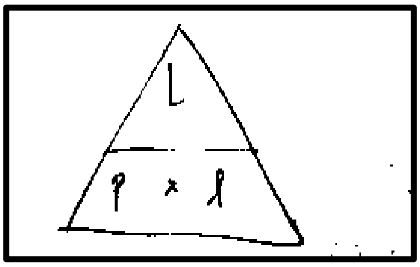


Figure 13. Triangle Area Formula

Subject 2's visual representation shows coordinated activation of System 1 and System 2. System 1 is involved in generating automatic responses, such as drawing a rectangle and recalling the computed result, whereas System 2 ensures the logical validity of the procedures undertaken.

Discussion

Activation of System 1 and System 2 in Subject 1

Subject 1 misidentified the coefficients despite the function being explicitly stated in the problem. This indicates reliance on System 1, characterized by fast, automatic, and unconscious thinking, which are the main characteristics of System 1. As explained by (Kahneman, 2020), System 1 is prone to perceptual bias and familiarity illusions, especially when individuals feel that they "recognize" the form of the problem or topic. This finding indicates that Subject actually possesses an understanding of the general form of a quadratic function; however, access to this information did not occur at the initial stage of problem solving. Familiarity with similar problems guided Subject 1's initial response. As a result, the student overlooked the negative sign of b. System 1 errors arise from rapid decisions without reflective verification, not lack of knowledge, but rather by decisions made too quickly without conscious verification.

This is consistent with dual process theory, which suggests that System 1 generates fast and cognitively efficient responses but is prone to error when not complemented by System 2 (Borodin, 2016) . In this context, although Student 1 was able to articulate the general form of a quadratic function during the interview, the student did not engage in reflective verification of the given information during the initial response. Therefore, the error did not stem from insufficient conceptual understanding, but from automatic processing that occurred without reflective control, a defining characteristic of System 1. Furthermore, when the subject was able to write the formula spontaneously without checking the result, this indicates the dominance of System 1, characterized by fast, automatic, and familiarity-based thinking. According to (Kahneman, 2020), System 1 is highly efficient in routine situations but prone to errors when dealing with information that requires careful attention, such as negative signs or inconsistencies in format. The error at the initial stage was not caused by a lack of knowledge, but rather by the failure of System 2 to intervene and verify the initial process. This condition indicates the absence of reflective checking associated with System 2 engagement. This stage plays an important role in ensuring that the result aligns with the initial conditions and in detecting possible errors in the solution process. In fact, problem solving does not end with obtaining an answer but also involves reviewing whether the solution is consistent with the initial conditions. The absence of this process causes initial errors to go undetected. This condition indicates that the reflective process, which should be part of System 2 activation, has not functioned optimally.

Thus, the correct final answer emerged only after delayed System 2 activation. However, the delayed activation of System 2 led to initial errors that could have been corrected if reflective processes had occurred from the beginning. This indicates that: (1) final success does not guarantee that the thinking process is optimal, and (2) analyzing internal processes is necessary to understand how the two thinking systems operate during problem solving. The case study of Subject 1 shows that the initial stage of problem solving was dominated by the intuitive processes of System 1, while System 2 was only activated after an external evaluative trigger (the researcher). The correctness of the final answer does not guarantee the accuracy of the thinking process from the outset; therefore, instructional approaches are needed that stimulate the reflective activation of System 2 from the early stages, not only during correction.

Activation of System 1 and System 2 in Subject 2

Subject 2's Activation of System 1 and System 2 in Solving Problem Number 1

Subject 2 was categorized as having relatively high academic ability and demonstrated correct solutions across all research problems. In line with dual process theory proposed by Kahneman (2011), which distinguishes two modes of human thinking, System 1 (fast, automatic thinking) and System 2 (conscious, reflective thinking). Subject 2's responses demonstrate the simultaneous involvement of both systems. System 1 and System 2 in Subject 2 appeared to be well coordinated, as shown by the student's ability to make quick decisions while simultaneously recalling formulas, matching prior learning experiences with relevant mathematical concepts, and verifying the obtained results. In line with dual process theory by Kahneman (2011), which distinguishes two modes of thinking, the activation of System 1 is evident in the emergence of rapid initial responses through the use of formulas and the identification of coefficients based on internalized knowledge. This process is then followed by the activation of System 2, which plays a role in rechecking and ensuring the correctness of the result. This indicates that intuitive decisions do not stand alone, but are accompanied by a process of matching with relevant mathematical concepts, enabling the subject to produce a correct answer. This finding is consistent with the results of (J. S. B. T. Evans, 2019; Gawronski et al., 2013; Leron & Hazzan, 2006), which show that solving mathematical problems involves an interaction between intuitive responses (System 1) and analytical-reflective processes (System 2), leading to accurate answers.

Subject 2's Activation of System 1 and System 2 in Solving Problem Number 2

The subject's awareness in thinking while solving the given mathematical problem reflects the activation of System 2, as the subject demonstrates analytical thinking in understanding the demands of the problem before sketching the graph. This is consistent with(Borodin, 2016; J. S. B. T. Evans, 2019; Safitri & Darmawan, 2022), who state that System 2 plays a role in conscious and analytical thinking when individuals engage in planning or decision-making. A rapid and confident response reflects System 1 activation based on internalized prior knowledge, where conceptual knowledge has been internalized and emerges automatically without the need for in-depth analysis. Meanwhile, Subject 2 began solving the problem by sketching a graph, which reflects the activation of System 2. This indicates that the subject was able to plan a solution strategy and select information relevant to the problem demands. Furthermore, the accuracy of the resulting sketch indicates strong spatial visualization ability, where high-ability students are able to imagine and represent objects accurately (Musriroh et al., 2021). Thus, the problem-solving process illustrates a collaboration between System 1 and System 2, in which rapid intuitive responses generated by System 1 are subsequently controlled and refined through analytical processes by System 2, resulting in efficient and accurate answers.

Subject 2's Activation of System 1 and System 2 in Solving Problem Number 3

The problem-solving process demonstrated by Subject 2 in Problem 3 reflects the coordinated involvement of System 1 and System 2, as described in Dual Process Theory (Kahneman, 2020), who explains that System 1 operates intuitively and spontaneously without requiring conscious thought. In line with (J. S. B. T. Evans, 2019), such responses are driven by experience and pattern recognition stored in long-term memory. This process reflects subjective-empirical processing, where Subject 2 uses visual intuition to represent information without in-depth analysis. This finding also supports (Darmawan et al., 2024) who state that students often rely on representations based on concrete experiences when dealing with real-world contextual problems.

On the other hand, the involvement of System 2 becomes evident when the student rechecks the obtained result. This process indicates a more conscious and controlled form of thinking. (Frankish, n.d.) explains that System 2 plays a role when individuals need to ensure the accuracy of results through analytical processes. In addition, according to (Leron & Hazzan, 2006), the transition from automatic thinking to conscious thinking is an important part of cognitive regulation in mathematics learning. However, the use of results derived from memorization can still lead to correct answers when accompanied by rechecking. This indicates that System 1 and System 2 work together. This is in line with (J. S. B. T. Evans, 2019), who states that System 1 and System 2 often operate collaboratively, where intuition produces quick responses and analysis ensures their correctness, enabling Subject 2 to solve the mathematical problem accurately. Furthermore, the process of rechecking the subject's answer also demonstrates reflective thinking ability, namely the ability to understand the relationships between mathematical concepts more deeply (Sukma, 2011). This indicates a shift from intuitive processing to reflective processing, where the student not only performs procedures mechanically but also consciously controls intuition and understands the solution steps (Parta, 2021).

From the perspective of problem solving according to Krulik and Rudnick (Hadi & Zaidah, 2020; Istiandaru, 2017), Subject 2 completed all problem-solving stages, including reflection, demonstrating System 2 engagement. This stage is evident in the subject's effort to recheck the obtained result and ensure that the solution aligns with the conditions of the problem. This indicates that Subject 2 not only arrived at a correct answer but also carried out an evaluative process, which is an essential part of problem solving. Thus, Subject 2 did not rely solely on memorization but also evaluated the solution steps through the coordination of System 1 and System 2 in Dual Process Theory, thereby supporting an optimal problem-solving process and producing an accurate answer. This is in line with (Kahneman, 2020), who explains that solving complex problems requires the simultaneous involvement of two thinking systems to produce decisions that are both efficient and accurate. Thus, the solution to problem number 3 by Subject 2 illustrates a dynamic interaction between System 1 and System 2, where intuition and analysis work complementarily. This finding is consistent with (Parta, 2021), who emphasizes that students' success in solving mathematical problems does not depend solely on analytical thinking, but also on the ability to use intuition Comparison of System Activation in Subject 1 and Subject 2 developed from prior learning experiences.

Comparison of Two Systems Activation between Subjects

Based on the researcher's analysis, both Subject 1 and Subject 2 initially relied on System 1 activation, particularly automatic thinking, to respond quickly to the given problems, as summarized in Table 2.

Table 2. Comparison of System 1 and System 2 Activation in Research Subjects

Problem Number	Subject	System 1	System 2	Coordination (S1–S2)
1	Subject 1	Clearly Appeared	Not Optimally Appeared	Appeared after researcher intervention
	Subject 2	Clearly Appeared	Clearly Appeared	Clearly Appeared
2	Subject 1	–	–	–
	Subject 2	Clearly Appeared	Clearly Appeared	Clearly Appeared
3	Subject 1	–	–	–
	Subject 2	Clearly Appeared	Clearly Appeared	Clearly Appeared

Based on the table 2, it can be seen that both subjects activated System 1 at the initial stage of problem solving. This is reasonable considering that both are Grade 10 high school students who have mastered basic mathematical operations, such as multiplication, division, and simple algebraic manipulation. However, a noticeable difference lies in the involvement and coordination of System 2. The activation of System 2 appears when the subjects match information across concepts, as well as when performing calculations that require accuracy and rechecking. This study shows that answers generated through System 1 are not always accurate because the process tends to be fast and intuitive without in-depth analysis, whereas the involvement of System 2 enables subjects to produce more accurate answers through reflective and evaluative processes. These findings are consistent with previous studies which state that the characteristics of System 2 being conscious, analytical, and careful play an important role in verification and deeper reasoning processes

(Augusto, 2024; Borodin, 2016; Darmawan, 2019; J. S. B. T. Evans, 2019; Gawronski et al., 2013; Kahneman, 2020; Kholifah & Darmawan, 2025; Safitri & Darmawan, 2022; Sukma, 2011; Wahyuni et al., 2023). The results of this study indicate that mathematical problem solving generally begins with System 1 activation, followed by System 2 involvement to support evaluation and verification processes. This finding indicates that successful mathematical problem solving does not solely depend on conceptual understanding, but also on students' ability to coordinate intuitive and reflective thinking processes in a balanced manner. The coordination of System 1 and System 2 underpins more accurate and reliable mathematical problem solving.

Implications

The findings of this study provide important implications for mathematics education, particularly in fostering students' mathematical reasoning during quadratic function problem solving. The results indicate that successful problem solving depends not only on conceptual knowledge but also on students' ability to coordinate intuitive and reflective thinking processes. Students with lower mathematical ability tended to rely more heavily on System 1, which often led to rapid but insufficiently verified responses. In contrast, students with higher mathematical ability demonstrated a more effective integration of System 1 and System 2, enabling them to evaluate and refine their initial judgments. These findings suggest that mathematics instruction should encourage students to move beyond automatic responses and engage in deliberate reflection throughout the problem-solving process. Teachers may support this development by incorporating scaffolding strategies, reflective questioning, and think-aloud activities that promote conscious evaluation of mathematical procedures and outcomes. Furthermore, learning tasks should be designed to create opportunities for students to examine and justify their intuitive responses before accepting them as correct. The study also highlights the importance of assessing students' reasoning processes rather than focusing exclusively on final answers. From a pedagogical perspective, understanding how System 1 and System 2 operate can help teachers identify the sources of students' errors and misconceptions more effectively. The findings further contribute to the growing application of Dual-Process Theory in mathematics education by demonstrating its relevance to algebraic problem solving. In addition, they emphasize the role of reflective reasoning in validating intuitive judgments and improving solution accuracy. Therefore, mathematics learning should aim to develop students' metacognitive awareness so that they can consciously regulate, monitor, and improve their reasoning during problem solving.

Limitations and Suggestions for Future Research

This study has several limitations that should be considered when interpreting the findings. First, the research involved only two tenth-grade students, which allowed for an in-depth exploration of reasoning processes but limited the transferability of the results to broader student populations. Second, the study focused exclusively on quadratic function problems, and therefore the identified patterns of System 1 and System 2 activation may not fully represent students' reasoning in other mathematical domains. Third, the findings were derived from written responses, video recordings, and interviews, all of which depend on participants' ability to express and communicate their thinking processes. Consequently, some cognitive activities may have remained implicit and were not completely captured during data collection. In addition, the study examined reasoning at a single point in time and did not investigate how students' cognitive processes might develop through instruction or repeated problem-solving experiences. Future research should involve a larger and more diverse sample to explore whether similar reasoning patterns emerge across different educational levels and mathematical ability groups. Further studies are also encouraged to examine the interaction between System 1 and System 2 in other mathematical topics, such as geometry, calculus, statistics, and mathematical modeling. Longitudinal investigations may provide valuable insights into how intuitive and reflective reasoning evolve over time. Researchers may additionally explore the role of instructional interventions, including scaffolding, metacognitive prompts, and reflective questioning, in strengthening System 2 engagement during problem solving. Comparative studies involving different problem-solving frameworks, such as those proposed by Polya or Krulik and Rudnick, may further enrich the understanding of dual-process reasoning. Future investigations could also integrate eye-tracking, think-aloud protocols, or digital learning analytics to capture cognitive processes more comprehensively. Such efforts would contribute to a deeper understanding of how intuitive and reflective reasoning interact and support effective mathematical problem solving.

CONCLUSION

This study examined students' mathematical reasoning in solving quadratic function problems through the lens of Dual-Process Theory. The findings demonstrate that both System 1 and System 2 were involved throughout the problem-solving process, although the nature and coordination of these systems differed according to students' mathematical ability levels. System 1 contributed to the generation of rapid responses based on prior experiences, pattern recognition, and memorized knowledge, whereas System 2 supported conscious evaluation, verification, and decision-making. Students with lower mathematical ability tended to rely more heavily on System 1, resulting in responses that were often automatic and insufficiently verified. Consequently, they were more likely to overlook important details and make errors despite possessing the necessary conceptual knowledge. In contrast, students with higher mathematical ability demonstrated a more balanced interaction between intuitive and reflective reasoning. Their problem-solving processes involved not only the rapid generation of ideas but also deliberate checking and refinement of those ideas through analytical thinking. The findings suggest that successful quadratic function problem solving depends on the effective coordination of System 1 and System 2 rather than on the dominance of either system alone. The study also indicates that mathematical errors may emerge from the absence of reflective verification rather than from a lack of conceptual understanding. These results highlight the importance of fostering reflective reasoning alongside intuitive thinking in mathematics classrooms. Encouraging students to evaluate, justify, and re-examine their solutions may help reduce errors and improve the quality of mathematical reasoning. Therefore, mathematics instruction should be designed to support the development of balanced cognitive processes that enable students to integrate intuition and reflection effectively during problem solving.

AUTHOR CONTRIBUTIONS STATEMENT

Evi Triani Puspitasari conceptualized the study, conducted data collection, performed data analysis, interpreted the findings, and prepared the original manuscript draft. Subanji contributed to the development of the research framework, provided methodological guidance, and critically reviewed the manuscript. Santi Irawati contributed to the validation of the research design, assisted in data interpretation, and reviewed the manuscript for intellectual content. Lathiful Anwar contributed to the mathematical content analysis, provided theoretical insights related to mathematical reasoning and dual-process theory, and participated in manuscript revision. Puguh Darmawan supervised the overall research process, contributed to the study conception and design, provided methodological and theoretical guidance, and reviewed and approved the final version of the manuscript. All authors discussed the results, contributed to the refinement of the manuscript, and approved the final version for publication.

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