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Development of a cartesian board learning media to enhance students' problem-solving in translation and reflection

Nurul Ilmi Badrud Dhujjah
Universitas Negeri Malang, INDONESIA

Abd. Qohar*
Universitas Negeri Malang, INDONESIA

Hery Susanto
Universitas Negeri Malang, INDONESIA

Puguh Darmawan
Universitas Negeri Malang, INDONESIA

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Abstract

Background: Learning geometric transformation, particularly translation and reflection, often relies heavily on symbolic explanations, leading to students' difficulties in spatial visualization and problem-solving. While mathematics manipulatives provide concrete interactions to bridge abstract concepts, their specific development for transformation geometry remains limited.
Aims: This study aims to develop a Cartesian board learning media and evaluate its validity, practicality, and effectiveness in enhancing Grade IX students' problem-solving abilities in translation and reflection.
Method: This study employed a Research and Development (R&D) approach using the ADDIE (Analysis, Design, Development, Implementation, Evaluation) model. The subjects were Grade IX students. Data were collected using expert validation sheets, student response questionnaires, classroom observation sheets, and problem-solving tests (pretest and posttest).
Results: Expert validation results indicated that the Cartesian board learning media is highly valid with an average score of 3.70. The media also proved to be highly practical, achieving an average score of 3.59 based on positive student responses and classroom observations. Furthermore, the media effectively improved students' problem-solving skills, as evidenced by the increase in the average test score from 71.30 to 89.78 (paired t-test, $p < .001$), yielding a moderate N-Gain score of 0.64. The student response questionnaire also demonstrated high internal consistency (Cronbach's Alpha = 0.91).
Conclusion: The developed Cartesian board is a valid, practical, and effective learning manipulative that significantly enhances students' problem-solving abilities in geometric transformation.

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INTRODUCTION

Problem-solving ability is widely recognized as a fundamental competence in mathematics education because it enables students to apply mathematical concepts in meaningful contexts and develop logical reasoning skills. In contemporary curricula, students are expected not only to perform procedural calculations but also to interpret problems, construct strategies, and evaluate solutions systematically. However, international assessments indicate that many students still experience difficulties in solving mathematical problems, particularly those involving visual representation and spatial reasoning. The Programme for International Student Assessment (PISA) reports that students often struggle to interpret geometric representations and connect them with symbolic mathematical expressions (Moreno et al., 2011; Rau, 2017). This challenge becomes more evident in geometry topics that require strong visualization skills and spatial understanding. Consequently, improving students' problem-solving ability in geometry remains an important concern in mathematics education research.

One topic that frequently causes learning difficulties is geometric transformation, particularly translation and reflection on the Cartesian plane. These concepts require students to understand

* **Corresponding author:**
Abd. Qohar, Universitas Negeri Malang, INDONESIA
abd.qohar.fmipa@um.ac.id

positional changes of objects and to interpret coordinate relationships through spatial reasoning and visual representation. In many classrooms, however, instruction tends to rely heavily on symbolic explanations and textbook exercises, with limited use of visual or manipulative support. As a result, students often experience misconceptions and difficulties when attempting to visualize how objects move or change position in the coordinate plane. Research in mathematics education suggests that the integration of visual representations and concrete learning tools can help students bridge the gap between abstract mathematical ideas and conceptual understanding (Moreno et al., 2011; Rau, 2017). Therefore, the use of appropriate instructional media becomes essential to support students' comprehension of transformation geometry concepts.

In this context, mathematics manipulatives have been widely acknowledged as effective tools for supporting students' understanding of abstract concepts. Manipulative media allow students to interact physically with mathematical representations, thereby facilitating the transition from concrete experiences to abstract reasoning. Previous studies have demonstrated that concrete mathematics manipulatives are generally perceived as useful and acceptable by students in supporting mathematical learning activities (Carbonneau et al., 2013; Uttal et al., 1997). In addition, research has shown that well-designed manipulative products can foster higher-order thinking skills and enhance students' engagement in learning mathematics (Abrahamson et al., 2020; Hidayah et al., 2020; Yan et al., 2018). Despite these positive findings, many studies mainly focus on students' perceptions, usability, or general cognitive outcomes of manipulatives. Consequently, their direct contribution to improving students' mathematical problem-solving ability in specific mathematical topics remains insufficiently explored.

Meanwhile, recent developments in mathematics education have increasingly utilized digital learning media to support students' understanding of geometric transformation. For example, STEAM-based instructional videos and other technology-integrated learning resources have been shown to improve students' conceptual understanding of transformation concepts (Seymour, 2024; Ulya et al., 2025). Although digital tools provide dynamic visualization and interactive learning experiences, they do not always offer the same level of concrete interaction that physical manipulatives can provide. Concrete manipulative media allow students to directly observe and manipulate objects, which can strengthen spatial reasoning and conceptual connections. However, research investigating the development of concrete manipulative tools specifically designed to support students' problem-solving processes in transformation geometry remains relatively limited. This limitation indicates the need for further studies that integrate manipulative media with problem-solving-oriented learning approaches.

Based on these considerations, there is a need to develop instructional media that can provide concrete visualization while simultaneously supporting students' problem-solving processes in transformation geometry learning. One potential solution is the use of a Cartesian board as a manipulative learning medium that enables students to directly observe and explore translation and reflection processes on the coordinate plane. Through this medium, students can visualize the movement of points and shapes more clearly, which may help them construct systematic strategies when solving transformation problems. Therefore, this study aims to develop a Cartesian board learning media and examine its validity, practicality, and effectiveness in improving Grade IX students' problem-solving abilities in translation and reflection topics. By integrating concrete representation with problem-solving oriented learning, this research is expected to contribute to the improvement of geometry instruction and provide an alternative instructional medium for mathematics classrooms.

Previous studies have highlighted the potential of mathematics manipulatives in supporting students' understanding of abstract mathematical concepts. For example, research on concrete mathematics manipulatives has shown that such tools are generally perceived as useful and acceptable for facilitating mathematical learning activities among students (Hidayah et al., 2020). In addition, other studies have emphasized the role of manipulative products in fostering students' higher-order thinking skills and improving learning engagement (Hidayah et al., 2020). These findings indicate that manipulative media can provide meaningful learning experiences by bridging the gap between concrete representations and abstract mathematical reasoning. However, most of these studies focus on students' perceptions, product development, or general cognitive outcomes, without specifically examining their effectiveness in improving students' problem-solving ability in particular mathematical topics.

Meanwhile, recent studies on geometric transformation learning have primarily explored the use of digital learning media to enhance students' conceptual understanding. For instance, STEAM-based instructional videos have been used to support students' comprehension of transformation concepts and improve learning outcomes (Ulya et al., 2025). Although digital tools offer dynamic visualization, research investigating the use of concrete manipulative media that allow students to directly observe and explore transformation processes remains limited. In particular, there is still a lack of studies that develop concrete manipulatives designed to improve students' problem-solving ability in transformation geometry topics such as translation and reflection. Therefore, this study aims to address this gap by developing a Cartesian board learning media and examining its validity, practicality, and effectiveness in improving Grade IX students' problem-solving abilities.

Based on field observations, the teaching of geometric transformation, particularly translation and reflection, is still heavily dominated by the rote memorization of algebraic formulas (e.g., coordinate shifts) without providing adequate spatial visualization. Students often struggle to mentally manipulate shapes and solve complex spatial problems. Previous studies have attempted to address this issue by utilizing digital media or video-based learning, such as the integration of STEAM-based videos (Ulya et al., 2025), which successfully improved conceptual understanding. However, digital interventions often lack the tactile, hands-on experience crucial for cognitive development and are heavily dependent on school infrastructure. Furthermore, while the use of concrete manipulatives in mathematics has long been advocated (Uttal et al., 1997), the specific development of physical learning media designed to systematically guide the problem-solving stages in translation and reflection remains largely underexplored. Therefore, the novelty of this study lies in the development of a physical Cartesian board explicitly designed not only to visualize abstract geometric transformations but also to provide structured scaffolding for Grade IX students' problem-solving skills. Unlike generic graphing boards, this media offers a practical, highly interactive, and accessible solution that bridges the gap between symbolic mathematical formulas and concrete spatial reasoning in a real classroom setting. This study aims to develop a Cartesian board learning media designed to support students' understanding of geometric transformation concepts, particularly translation and reflection. The development of this media is intended to provide a concrete manipulative tool that helps students visualize positional changes of objects on the Cartesian plane and facilitates the construction of systematic problem-solving strategies. Specifically, this study seeks to examine the quality of the developed Cartesian board media in terms of its validity, practicality, and effectiveness in improving students' mathematical problem-solving ability. Through a Research and Development approach using the ADDIE model, the study evaluates whether the Cartesian board media can serve as an effective instructional tool to enhance Grade IX students' problem-solving performance in transformation geometry learning.

METHOD

Research Design

This study employed a Research and Development (R&D) approach aimed at developing a Cartesian board learning media to support students' problem-solving abilities in geometric transformation topics, particularly translation and reflection. The development process followed the ADDIE model, which consists of five systematic stages: analysis, design, development, implementation, and evaluation (Adriani et al., 2020; Spatioti et al., 2023; Wahira et al., 2023). This model was selected because it provides a structured framework for designing, developing, and evaluating instructional products in educational research.

At the analysis stage, the researchers identified learning needs, analyzed students' characteristics, and examined curriculum requirements related to transformation geometry topics, particularly translation and reflection. This stage aimed to determine the learning problems experienced by students and the type of instructional media required to support their understanding of geometric transformations. The design stage involved preparing the initial concept of the Cartesian board media, including determining the coordinate layout, visual representation of points and shapes, and instructional scenarios for classroom implementation. In addition, research instruments such as validation sheets, problem-solving ability tests, student response questionnaires, and observation sheets were also designed during this stage.

The development stage focused on producing the initial prototype of the Cartesian board media. The developed media was then evaluated through expert validation involving a subject matter expert and a media expert to assess aspects such as conceptual accuracy, visual representation, and usability of the media. Revisions were made based on the feedback provided by the validators in order to improve the quality of the product before classroom implementation.

The implementation stage involved a limited trial of the Cartesian board media with Grade IX students. During this stage, students participated in learning activities using the developed media. A pretest was administered before the learning intervention to measure students' initial problem-solving ability, and a posttest was conducted after the learning activities to evaluate improvement. In addition, during the learning process, classroom observation sheets were used to document the implementation of activities and student engagement. At the end of the learning sessions, a student response questionnaire was also administered to evaluate the practicality of the developed media from the students' perspective. Finally, the evaluation stage aimed to determine the overall quality of the developed media in terms of validity, practicality, and effectiveness. The evaluation results were used to refine the final product and determine whether the Cartesian board media could serve as an effective instructional tool for supporting students' problem-solving abilities in transformation geometry learning. The overall research and development framework based on the ADDIE model used in this study is presented in Figure 1.

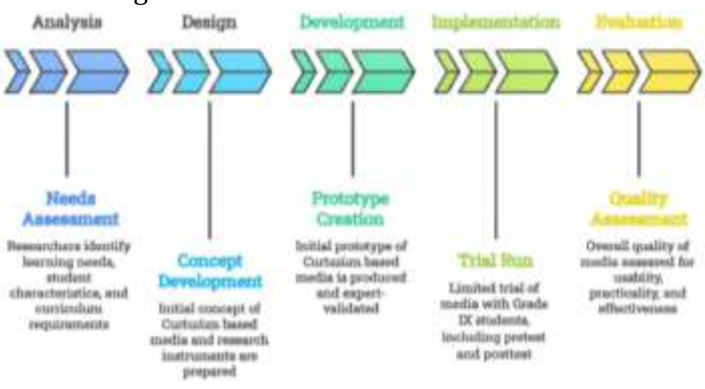

Figure 1. ADDIE Model Cartesian Board Learning Media Development

At the analysis stage, learning needs, student characteristics, and curriculum requirements based on the Merdeka Curriculum were examined to identify instructional problems in transformation geometry learning. The design stage involved preparing the prototype of the

Cartesian board media, developing learning scenarios, and designing the research instruments used in the study. During the development stage, the initial product was produced and subsequently validated by experts to assess its conceptual accuracy, media appearance, and usability, followed by revisions based on the feedback obtained. The implementation stage consisted of piloting the media with students using a pretest–posttest design to evaluate its impact on students' problem-solving ability. Finally, the evaluation stage assessed the overall quality of the developed media in terms of its validity, practicality, and effectiveness. The design of the developed Cartesian board media is illustrated in Figure 2.

Figure 2. Design of the Cartesian Board Learning Media

The developed media was then used as the main instructional tool during the classroom implementation stage to support students in visualizing translation and reflection processes on the Cartesian plane.

Participant

The participants in this study consisted of expert validators and Grade IX students involved in the implementation stage of the developed learning media. The expert validators included a mathematics education expert and a media expert who were responsible for evaluating the feasibility of the Cartesian board learning media. Their evaluation focused on several aspects, including the suitability of the mathematical content, conceptual clarity of translation and reflection concepts, accuracy of Cartesian representation, visual design, and usability of the media.

In addition to expert validation, the implementation stage involved Grade IX students from a junior secondary school who participated in a limited classroom trial of the developed Cartesian board media. A total of 23 students participated in this stage. The participants were selected using purposive sampling to represent typical classroom learning conditions. These students had previously learned basic coordinate concepts but had not yet received intensive instruction on translation and reflection using manipulative learning media.

The students participated in learning activities that integrated the Cartesian board as a concrete learning tool to support visualization of geometric transformations. During the implementation stage, students completed a pretest before the learning intervention and a posttest after the learning activities to measure their problem-solving ability related to translation and reflection. The involvement of both expert validators and student participants allowed the researchers to evaluate the developed media from both pedagogical feasibility and classroom practicality perspectives.

Instrument

Several research instruments were used to collect data during the development and implementation stages. First, a media validation sheet was used by expert validators to assess the feasibility of the Cartesian board learning media. The validation instrument evaluated aspects such as content suitability, conceptual clarity, Cartesian representation, visual appearance, and ease of use using a four-point Likert scale. Second, a problem-solving ability test was administered as both a pretest and posttest to measure students' performance in solving translation and reflection problems on the Cartesian plane. The test consisted of essay-type questions designed to assess students' problem-solving processes, including understanding the problem, planning a strategy, executing the solution, and verifying the final answer.

Third, a student response questionnaire was used to evaluate the practicality of the developed media from the students' perspective. The questionnaire employed a four-point Likert scale to measure students' perceptions of clarity, engagement, ease of use, and learning interest when using the Cartesian board media. The reliability of the questionnaire was examined using Cronbach's Alpha, while the validity of each item was examined using the corrected item-total correlation coefficient. Finally, an observation sheet was used to document the implementation of learning activities and students' engagement during classroom instruction using the Cartesian board media.

Data Analysis

Data analysis in this study was conducted to evaluate the quality of the developed Cartesian board learning media in terms of validity, practicality, and effectiveness. The analysis involved both descriptive statistical techniques and learning improvement analysis based on the results obtained from expert validation, student responses, classroom observations, and students' problem-solving test scores.

First, the validity of the Cartesian board media was determined based on the results of expert validation conducted by a subject matter expert and a media expert. The validators assessed several aspects, including content suitability, conceptual clarity, Cartesian representation, visual appearance, and ease of use using a four-point Likert scale. The average score obtained from the validation results was calculated and interpreted using predetermined validity criteria to determine whether the developed media met the feasibility requirements as an instructional learning medium.

Second, the practicality of the developed media was analyzed based on the results of the student response questionnaire and classroom observation sheets. The student questionnaire data were analyzed using descriptive statistics by calculating the mean score of each indicator to determine students' perceptions regarding clarity, engagement, and ease of use of the Cartesian board media. Meanwhile, the observation data were used to describe the implementation of learning activities and students' engagement during the use of the developed media in classroom learning.

Third, the effectiveness of the Cartesian board media was evaluated by analyzing students' problem-solving ability before and after the learning intervention. Students were given a pretest before the implementation of the learning activities and a posttest after the use of the Cartesian board media. The improvement in students' problem-solving ability was measured using the normalized gain (N-gain) formula as proposed (Negara et al., 2022), which is commonly used to measure learning improvement in pretest–posttest educational studies.

$$N - gain = \frac{Posttest\ -\ Pretest}{Maximum\ Score - Pretest}$$

The N-gain values were then categorized into three levels of improvement: high ($g \geq 0.70$), moderate ($0.30 \leq g < 0.70$), and low ($g < 0.30$). An inferential statistical test was also conducted to examine whether the pretest-posttest difference was statistically significant, using a Shapiro-Wilk normality test to select between a paired-samples t-test and the non-parametric Wilcoxon signed-rank test. Through these analyses, the study aimed to determine whether the developed Cartesian board media was valid, practical, and effective in improving students' problem-solving abilities in translation and reflection topics on the Cartesian plane.

RESULTS AND DISCUSSION

Results

The development process resulted in a Cartesian board learning media designed to support the visualization of translation and reflection on the Cartesian plane.

1. Development Result and Validity of the Cartesian Board Media

The development stage produced a Cartesian board learning media intended to assist students in visualizing translation and reflection processes on the Cartesian plane. The developed media was subsequently validated by a subject matter expert and a media expert to evaluate the feasibility of its content, conceptual clarity, and visual presentation. The results of the expert validation of the Cartesian board media are presented in Table 1.

Table 1. Validation Results of the Cartesian Board Media

Assessment Aspect	Average Score	Category
Content suitability	3.75	Highly Valid
Concept clarity	3.67	Highly Valid
Cartesian representation	3.83	Highly Valid
Media appearance	3.58	Valid
Ease of use	3.67	Highly Valid
Overall Average	3.70	Highly Valid

Based on the validation results presented in Table 1, the Cartesian board media obtained an overall average score of 3.70, which falls into the highly valid category. Among the evaluated aspects, the Cartesian representation obtained the highest score, indicating that the media is able to clearly demonstrate changes in object positions according to the principles of transformation geometry.

These findings indicate that the developed media fulfills both content validity and construct feasibility as an instructional medium. In addition, several suggestions from the validators, particularly regarding the visual presentation of the media, were used as a basis for revising and improving the product before its implementation in classroom learning.

These results indicate that conceptually valid manipulative and visual media can support students in constructing geometric understanding through accurate and consistent representations (Del Ponte et al., 2023; Lavicza et al., 2023; Žakelj & Klančar, 2022). Media that meet validity criteria are also considered capable of helping students connect abstract mathematical concepts with visual learning experiences (Wahyudi & Mz, 2022).

2. Practicality of the Cartesian Board Media

The practicality of the Cartesian board media was evaluated based on students' responses and the implementation of learning when the media was used in the classroom. Students' responses were collected through a practicality questionnaire, while the implementation of learning activities was assessed through classroom observation.

The results of students' responses toward the use of the Cartesian board media are presented in Table 2.

Table 2. Practical Results of the Media Based on Students' Responses

Assessment Indicator	Average Score	Category
Clarity of the Cartesian board media	3.75	Practical
Ease of understanding the material	3.60	Practical
Student engagement in learning	3.40	Practical
Students' interest in learning	3.60	Practical
Overall Average	3.59	Practical

Based on the results presented in Table 2, the Cartesian board media obtained an overall average score of 3.59, which falls into the practical category. Students reported that the media helped them understand the concepts of translation and reflection more easily and made the learning process more engaging. Cronbach's Alpha values were 0.76 (ease of use), 0.91 (interest), 0.87 (understanding), and 0.91 overall, indicating high internal consistency. The corrected item-total correlation for all items ranged from 0.40 to 0.86 (> 0.30), confirming that all items were valid. In addition to the questionnaire results, the practicality of the media was also evaluated through observation of the learning implementation. The summary of the observation results is presented in Table 3.

Table 3. Results of the Observation of Learning Implementation

Observation Aspect	Maximum Score	Obtained Score	Category
Lesson Planning	15	13	Very Good
Use of Media	18	15	Good
Learning Interaction	15	13	Very Good
Closing and Evaluation	12	8	Good
Total	60	49	Very Good

The results presented in Table 3 indicate that the learning process using the Cartesian board media was implemented in the very good category, with a total score of 49 out of a maximum possible score of 60. This finding suggests that the media is easy to use for both teachers and students and can be effectively integrated into classroom learning activities.

The use of concrete manipulative media such as the Cartesian board allows students to directly observe changes in object positions on the coordinate plane. This learning experience supports students' understanding of transformation concepts and creates a more interactive learning environment.

3. Effectiveness of the Cartesian Board Media

The effectiveness of the Cartesian board media was evaluated based on the improvement in students' problem-solving abilities related to translation and reflection topics. This improvement was analyzed using the results of the pretest and posttest administered before and after the learning intervention. The average pretest and posttest scores are presented in Table 4.

Table 4. Average Pretest and Posttest Scores of Problem-Solving Ability

Test	Average Score
Pretest	71.30
Posttest	89.78

Based on the results presented in Table 4, the average pretest score of 71.30 (SD = 9.66) increased to 89.78 (SD = 5.82) in the posttest. A Shapiro-Wilk test on the difference scores indicated a normal distribution ($W = 0.919$, $p = .063$), so a paired-samples t-test was applied, showing a statistically significant improvement, $t(22) = 6.19$, $p < .001$, with a large effect size (Cohen's $d = 1.29$). A Wilcoxon signed-rank test was also run as a robustness check given the ceiling effect in the raw posttest scores, and likewise confirmed the significant improvement ($Z = 4.11$, $p < .001$). This indicates that students' ability to solve transformation geometry problems improved significantly after using the developed media.

To determine the level of improvement more precisely, the N-gain score was calculated. The results of the N-gain analysis are presented in Table 5.

Table 5. N-Gain Scores of Problem-Solving Ability

N-Gain	Category
0.64	Moderate

The N-gain value of 0.64 falls into the moderate category (approaching the high category threshold of 0.70), indicating that the Cartesian board media has a positive effect on improving students' problem-solving abilities. This finding is further supported by the statistically significant result of the paired-samples t-test reported above, providing stronger evidence that the observed improvement did not occur by chance. This improvement occurred because the media enables students to visualize the processes of translation and reflection in a gradual and concrete manner, helping them develop more systematic strategies for solving transformation problems.

These findings indicate that the developed Cartesian board media is effective in supporting students' understanding of geometric transformation concepts and improving their problem-solving performance.

Discussion

The results of this study indicate that the Cartesian board learning media contributes positively to improving students' problem-solving abilities in geometric transformation topics, particularly translation and reflection. The moderate N-gain score, supported by a statistically significant paired-samples t-test result, obtained in this study suggests that the use of concrete visual media can facilitate students' understanding of coordinate transformations and support the development of structured problem-solving strategies. This improvement occurs because the Cartesian board allows students to directly observe how the position of points or shapes changes on the coordinate plane. Through this visual interaction, students are able to connect symbolic mathematical procedures with spatial representations, which helps them interpret transformation processes more systematically. Previous studies have shown that visual and manipulative learning tools can significantly improve students' mathematical reasoning and problem-solving performance by making abstract concepts more accessible and meaningful (Siller & Ahmad, 2024).

The high validity score obtained from expert validation also indicates that the developed media meets both content and construct feasibility requirements. In particular, the highest score obtained in the Cartesian representation aspect highlights the importance of accurate visual representation in transformation geometry learning. When the representation of coordinates, points, and movements is displayed clearly, students can more easily interpret the direction and magnitude of transformations such as translation and reflection. This supports the idea that well-designed manipulative media function as an effective bridge between abstract mathematical concepts and concrete learning experiences. Previous research has emphasized that visual and manipulative representations play a crucial role in strengthening students' conceptual understanding in geometry learning and help reduce misconceptions related to spatial transformations (Alamian et al., 2020; Roberts et al., 2020).

Furthermore, the practicality results indicate that the Cartesian board media is easy to use and can be effectively implemented in classroom learning. Students reported that the media helped them understand transformation concepts more clearly and increased their engagement during learning activities. The observation results also showed that the learning process using the Cartesian board was conducted smoothly, suggesting that the media can be integrated into regular mathematics instruction without requiring complicated teaching procedures. These findings are consistent with studies indicating that concrete manipulative tools promote active student participation and provide opportunities for learners to explore mathematical concepts through direct interaction with learning objects (Byrne et al., 2023; Roberts et al., 2020). In addition, manipulatives have been reported to enhance students' engagement and motivation in mathematics learning environments, particularly when dealing with topics that require spatial visualization and conceptual interpretation (Moyer-Packenham & Westenskow, 2013).

Overall, the findings of this study demonstrate that the Cartesian board media not only meets validity and practicality criteria but also shows promising effectiveness in improving students' problem-solving abilities in transformation geometry. The improvement in students' performance indicates that the integration of visual and manipulative learning tools can support the development of spatial reasoning and mathematical understanding simultaneously. This finding aligns with previous research suggesting that concrete representations in geometry learning help students develop more systematic reasoning when solving transformation problems (Borden et al., 2025; Mulligan, 2015). Therefore, the Cartesian board media can serve as an alternative instructional medium that supports more meaningful and interactive learning experiences in geometry instruction, particularly in helping students understand coordinate transformations through concrete visualization.

Implications

The findings of this study have several important implications for mathematics education, particularly in the teaching of geometric transformation concepts. The positive impact of the Cartesian board media indicates that the use of concrete visual and manipulative learning tools can effectively support students' problem-solving abilities in topics that require strong spatial reasoning and visualization skills. By enabling students to directly observe and manipulate the positions of points and shapes on the Cartesian plane, the media helps bridge the gap between abstract mathematical concepts and concrete learning experiences. This supports previous research suggesting that visual and manipulative instructional media can enhance students' conceptual understanding and problem-solving performance in geometry learning (Guan et al., 2024; Lee & Chen, 2014).

From a pedagogical perspective, the Cartesian board media provides teachers with an alternative instructional tool that can facilitate more interactive and meaningful learning activities in the classroom. The media allows teachers to demonstrate transformation processes more clearly and encourages students to actively explore mathematical relationships through visual representation. In addition, the improvement in students' problem-solving abilities observed in this study indicates that integrating manipulative media into geometry instruction can contribute to more effective learning outcomes. These results reinforce previous findings that well-designed instructional media can play a significant role in improving students' engagement and understanding in mathematics learning environments (Cevikbas & Kaiser, 2022; Chen et al., 2021).

Limitations

Despite the positive findings obtained in this study, several limitations should be acknowledged. First, the implementation of the Cartesian board media was conducted on a small scale involving only 20 Grade IX students from a single school. As a result, the generalizability of the findings to broader educational contexts remains limited. Second, the effectiveness analysis relied on a pretest–posttest design without the inclusion of a control group. Although this design can provide preliminary evidence of learning improvement, it limits the ability to establish stronger causal conclusions regarding the impact of the developed media on students' problem-solving abilities (Aslan & Duruhan, 2021; Crippen & Earl, 2007).

Based on these limitations, several directions for future research are recommended. Future studies could involve larger samples across multiple schools to enhance the generalizability of the findings. In addition, the use of quasi-experimental or experimental research designs with control groups would allow researchers to more rigorously examine the comparative effectiveness of the Cartesian board media. Further investigations may also explore the long-term impact of the media on students' retention of geometric transformation concepts and the development of higher-order thinking skills. Moreover, integrating the Cartesian board media with digital learning platforms, such as GeoGebra, may provide opportunities to combine concrete manipulative experiences with dynamic visualization, thereby creating more interactive and technologically enriched mathematics learning environments.

Suggestions

Future research is recommended to implement the Cartesian board media on a broader scale involving students from different schools and educational contexts to examine the consistency of its effectiveness across diverse classroom environments and student characteristics. More rigorous research designs, such as quasi-experimental or experimental methods with control groups, are also needed to provide stronger evidence regarding the comparative effectiveness of the media in improving students' problem-solving abilities compared with conventional instruction. In addition, future studies may investigate the long-term impact of the Cartesian board media on students' retention of transformation geometry concepts and the development of higher-order thinking skills to better understand the sustainability of learning outcomes achieved through manipulative-based instruction. The integration of digital technology, including dynamic geometry software and interactive learning platforms, is likewise recommended to enrich learning experiences and support more interactive mathematics instruction, particularly in topics related to spatial reasoning and geometric transformations.

CONCLUSION

This study developed a Cartesian board learning media to support the teaching of translation and reflection topics at the junior secondary school (SMP/MTs) level. The development process followed the stages of analysis, design, development, implementation, and evaluation to ensure that the media met instructional and pedagogical requirements. The results of expert validation indicated that the developed media satisfied the criteria of content suitability, conceptual clarity, and accurate Cartesian representation. In addition, students' responses and classroom observations demonstrated that the Cartesian board media is practical and easy to implement in mathematics learning activities.

In terms of effectiveness, the use of the Cartesian board media contributed to improving students' problem-solving abilities in transformation geometry, particularly in translation and reflection topics. The moderate improvement observed in students' learning outcomes indicates that the media helps students visualize coordinate transformations more clearly and supports the development of systematic problem-solving strategies. Overall, the developed media meets the criteria of validity, practicality, and effectiveness, suggesting that it can serve as an alternative instructional tool for teaching geometric transformations. These findings highlight the potential of concrete manipulative media to enhance students' conceptual understanding and problem-solving skills in mathematics learning.

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AUTHOR CONTRIBUTIONS STATEMENT

The first author was responsible for research conception and design, data collection, data analysis, and manuscript preparation. The second and third authors contributed as academic supervisors by providing continuous guidance, critical feedback, and direction throughout the research process, including research design refinement, instrument validation, data interpretation, and manuscript revision. All authors reviewed and approved the final version of the manuscript.

REFERENCES

Abrahamson, D., Nathan, M. J., Williams-Pierce, C., Walkington, C., Ottmar, E. R., Soto, H., & Alibali, M. W. (2020). The future of embodied design for mathematics teaching and learning. *Frontiers in Education, 5*, Article 147. https://doi.org/10.3389/feduc.2020.00147

Adriani, D., Lubis, P., & Triono, M. (2020). Teaching material development of educational research methodology with ADDIE models. In *Proceedings of the 3rd International Conference Community Research and Service Engagements (IC2RSE 2019)*. https://doi.org/10.4108/eai.4-12-2019.2293793

Alamian, V., Barati, A., & Habibi, M. (2020). The effects of visual representations and manipulatives on reduction of algebraic misconceptions of ninth-grade students. *Archives of Pharmacy Practice, 11*(4), 49–58.

Aslan, S. A., & Duruhan, K. (2021). The effect of virtual learning environments designed according to problem-based learning approach on students' success, problem-solving skills, and motivations. *Education and Information Technologies, 26*(2), 2253–2283. https://doi.org/10.1007/s10639-020-10354-6

Borden, M. L., Belcher, M., Krupa, E. E., & Anderson, R. K. (2025). Designing to transform: High school students' reasoning with geometric transformations in an authentic real-world project. *The Journal of Educational Research*, 1–14. https://doi.org/10.1080/00220671.2025.2557296

Byrne, E. M., Jensen, H., Thomsen, B. S., & Ramchandani, P. G. (2023). Educational interventions involving physical manipulatives for improving children's learning and development: A scoping review. *Review of Education, 11*(2), e3400. https://doi.org/10.1002/rev3.3400

Carbonneau, K. J., Marley, S. C., & Selig, J. P. (2013). A meta-analysis of the efficacy of teaching mathematics with concrete manipulatives. *Journal of Educational Psychology, 105*(2), 380–400. https://doi.org/10.1037/a0031084

Cevikbas, M., & Kaiser, G. (2022). Student engagement in a flipped secondary mathematics classroom. *International Journal of Science and Mathematics Education, 20*(7), 1455–1480. https://doi.org/10.1007/s10763-021-10213-x

Chen, Y.-C., Lu, Y.-L., & Lien, C.-J. (2021). Learning environments with different levels of technological engagement: A comparison of game-based, video-based, and traditional instruction on students' learning. *Interactive Learning Environments, 29*(8), 1363–1379. https://doi.org/10.1080/10494820.2019.1628781

Crippen, K. J., & Earl, B. L. (2007). The impact of web-based worked examples and self-explanation on performance, problem solving, and self-efficacy. *Computers & Education, 49*(3), 809–821. https://doi.org/10.1016/j.compedu.2005.11.018

Del Ponte, A., Masiliūnas, A., & Lim, N. (2023). Information about historical emissions drives the division of climate change mitigation costs. *Nature Communications, 14*, Article 1408. https://doi.org/10.1038/s41467-023-37130-7

Guan, H., Li, J., Rao, Y., Chen, R., & Xu, Z. (2024). Comparative effects of dynamic geometry system and physical manipulatives on inquiry-based math learning for students in junior high school. *Education and Information Technologies, 29*(16), 21477–21499. https://doi.org/10.1007/s10639-024-12663-6

Hidayah, I., Isnarto, Masrukan, Asikin, M., & Margunani. (2020). Quality management of mathematics manipulative products to support students' higher order thinking skills. *International Journal of Instruction, 14*(1), 537–554. https://doi.org/10.29333/IJI.2021.14132A

Lavicza, Z., Abar, C. A. A. P., & Tejera, M. (2023). Spatial geometric thinking and its articulation with the visualization and manipulation of objects in 3D. *Educação Matemática Pesquisa, 25*(2), 258–277. https://doi.org/10.23925/1983-3156.2023v25i2p258-277

Lee, C.-Y., & Chen, M.-J. (2014). The impacts of virtual manipulatives and prior knowledge on geometry learning performance in junior high school. *Journal of Educational Computing Research, 50*(2), 179–201. https://doi.org/10.2190/EC.50.2.b

Moreno, R., Ozogul, G., & Reisslein, M. (2011). Teaching with concrete and abstract visual representations: Effects on students' problem solving, problem representations, and learning perceptions. *Journal of Educational Psychology, 103*(1), 32–47. https://doi.org/10.1037/a0021995

Moyer-Packenham, P. S., & Westenskow, A. (2013). Effects of virtual manipulatives on student achievement and mathematics learning. *International Journal of Virtual and Personal Learning Environments, 4*(3), 35–50. https://doi.org/10.4018/jvple.2013070103

Mulligan, J. (2015). Looking within and beyond the geometry curriculum: Connecting spatial reasoning to mathematics learning. *ZDM Mathematics Education, 47*(3), 511–517. https://doi.org/10.1007/s11858-015-0696-1

Negara, H., Nurlaelah, E., & Herman, T. (2022). Improving students' mathematical reasoning abilities through social cognitive learning using GeoGebra. *International Journal of Emerging Technologies in Learning, 17*(18), 118–135. https://doi.org/10.3991/ijet.v17i18.32941

Rau, M. A. (2017). Conditions for the effectiveness of multiple visual representations in enhancing STEM learning. *Educational Psychology Review, 29*(4), 717–761. https://doi.org/10.1007/s10648-016-9365-3

Roberts, J., Phipps, S., Subeeksingh, D., Jaggernauth, S. J., Ramsawak-Jodha, N., & Dedovets, Z. (2020). Reflection on the effects of concrete mathematics manipulatives on student engagement and Problem-Solving in three secondary schools in Trinidad and Tobago. *Caribbean Curriculum, 27*.

Rusdi, M., Sirajuddin, H., & Alfah, R. (2022). Implementation of the ADDIE model in PHP-based e-learning in the era of pandemic. *Jurnal Teknologi Informasi Universitas Lambung Mangkurat, 7*(1), 49–56. https://doi.org/10.20527/jtiulm.v7i1.74

Seymour, M. (2024). Enhancing the online student experience through the application of universal design for learning to research methods teaching. *Education and Information Technologies, 29*(3), 2767–2785. https://doi.org/10.1007/s10639-023-11948-6

Siller, H.-S., & Ahmad, S. (2024). The effect of concrete and virtual manipulative blended instruction on mathematical achievement for elementary school students. *Canadian Journal of Science, Mathematics and Technology Education, 24*(2), 229–266. https://doi.org/10.1007/s42330-024-00336-y

Spatioti, A. G., Kazanidis, I., & Pange, J. (2022). A comparative study of the ADDIE instructional design model in distance education. *Information, 13*(9), Article 402. https://doi.org/10.3390/info13090402

Spatioti, A., Kazanidis, I., & Pange, J. (2023). Educational design and evaluation models of learning effectiveness in the e-learning process: A systematic review. *Turkish Online Journal of Distance Education, 24*(4), 318–347. https://doi.org/10.17718/tojde.1177297

Ulya, N. H. A., Ishartono, N., Razak, R. binti A., Halili, S. H. binti, Khomeini, M., & Mubarokah, L. (2025). Integration of Steam-Based Video with Timestamp to Improve Student's Understanding of Geometric Transformation. AIP Conference Proceedings, 3142(1). https://doi.org/10.1063/5.0262009

Uttal, D. H., Scudder, K. V., & DeLoache, J. S. (1997). Manipulatives as symbols: A new perspective on the use of concrete objects to teach mathematics. Journal of Applied Developmental Psychology, 18(1). https://doi.org/10.1016/S0193-3973(97)90013-7

Wahira, W., Ansar, A., & Tolla, I. (2023). Analysis of the needs for developing the competence of elementary school supervisors through the ADDIE model. *Kasetsart Journal of Social Sciences, 44*(4), 1303–1312. https://doi.org/10.34044/j.kjss.2023.44.4.28

Wahyudi, D., & Mz, Z. A. (2022). Development of video learning media assisted by Sparkol Videoscribe to facilitate students' understanding of mathematical concepts. *International Journal of Trends in Mathematics Education Research, 5*(3), 306–314. https://doi.org/10.33122/ijtmer.v5i3.165

Yan, Z., Chai, C. S., & So, H.-J. (2018). Creating tools for inquiry-based mathematics learning from technological pedagogical content knowledge perspectives. *Australasian Journal of Educational Technology, 34*(4). https://doi.org/10.14742/ajet.3755

Žakelj, A., & Klančar, A. (2022). The role of visual representations in geometry learning. *European Journal of Educational Research, 11*(3), 1393–1411. https://doi.org/10.12973/eu-jer.11.3.1393