



A convergent mixed-methods approach to enhancing learners' mathematical problem-solving skills in disadvantaged contexts

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Abstract

Background: Mathematical problem-solving is a fundamental competency emphasized in the South African Curriculum and Assessment Policy Statements (CAPS). However, learners in disadvantaged contexts continue to struggle with applying mathematical knowledge to unfamiliar problems, partly due to teaching practices characterized by low cognitive demand.

Aims: This study investigated the effects of a multi-component problem-centered teaching and learning (PCTL) intervention on Grade 8 learners' mathematical problem-solving performance and attitudes in a disadvantaged South African secondary school and explored the instructional mechanisms underlying observed changes.

Method: A convergent mixed-methods design involved 98 Grade 8 learners. Quantitative data were collected through pre- and post-intervention problem-solving tests and an attitude questionnaire, while qualitative data were obtained from classroom observations, reflective learner journals, and semi-structured interviews with learners and teachers. Both strands were integrated to explain the intervention outcomes.

Results: The experimental group demonstrated significant improvement in mathematical problem-solving performance from pre-test to post-test, $t(48) = 9.72$, $p < .001$, $d = 1.39$. Gain scores were also significantly higher than those of the comparison group, $t(96) = 8.54$, $p < .001$. Learners showed moderately improved attitudes toward mathematical challenges. Qualitative findings revealed a shift toward learner-centered discourse characterized by increased teacher wait time, productive struggle, active participation, and metacognitive reflection.

Conclusion: The multi-component PCTL intervention was associated with meaningful improvements in mathematical problem-solving performance and learner attitudes. The findings highlight metacognitive reflection and student-led discourse as important mechanisms for strengthening conceptual problem solving and provide implications for mathematics teacher professional development in resource-constrained settings.

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INTRODUCTION

Mathematical problem solving is a fundamental component of meaningful mathematics learning because it requires learners to interpret information, establish mathematical relationships, select appropriate strategies, and justify the solutions they construct. Beyond obtaining correct answers, effective problem solving requires learners to coordinate conceptual knowledge with strategic reasoning when familiar procedures cannot be applied directly. These competencies become increasingly important at the secondary-school level, where learners are expected to engage with mathematical tasks involving interpretation, reasoning, and application across unfamiliar situations. In South Africa, however, the development of these competencies remains challenging, particularly in historically disadvantaged schooling contexts characterized by unequal opportunities to learn and differences in access to instructional resources. Persistent inequalities in instructional

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quality, teacher preparation, and educational provision may influence learners' opportunities to participate meaningfully in mathematically demanding activities (Chirinda et al., 2023; Mpalami, 2022). Linguistic diversity can add further complexity when learners are required to interpret mathematical information and communicate reasoning through a language that may differ from the language used in their everyday interactions. Teachers must simultaneously address these learner differences while meeting curriculum requirements and preparing students for formal assessment, which can restrict the instructional time available for sustained engagement with non-routine problems (Chirinda, 2021). Consequently, mathematics instruction may continue to emphasize procedural demonstration and reproduction rather than allowing learners sufficient opportunities to investigate, discuss, justify, and refine their mathematical reasoning (Chirinda, 2021). Such instructional conditions can constrain the development of strategic competence because learners may become proficient in reproducing procedures without necessarily understanding when, why, and how particular strategies should be applied. Addressing these challenges therefore requires pedagogical environments that position learners as active mathematical thinkers and provide structured opportunities for reasoning, collaboration, reflection, and engagement with challenging problems.

Problem-Centered Teaching and Learning (PCTL) offers a potentially appropriate pedagogical response because it organizes mathematical learning around meaningful problems rather than beginning primarily with procedures to be reproduced. Within a problem-centered environment, learners are encouraged to activate prior knowledge, explore possible strategies, communicate mathematical ideas, and refine their reasoning as they work toward defensible solutions. Recent research on problem-centered instruction has increasingly examined instructional design, scaffolding, collaborative learning, and alternative approaches to conventional knowledge-transmission practices (Hall, 2022; Kim et al., 2020; Meng et al., 2025; Merrill, 2020). Evidence from problem-centered STEM instruction further suggests that scaffolding can support cognitive learning across individual and collaborative problem-solving arrangements, with metacognitive scaffolding showing particular relevance to group activity (Kim et al., 2020). In mathematics education, related studies have investigated problem-solving development through teaching-model interventions, cooperative learning, self-assessment, and metacognitive approaches (Barana et al., 2022; Klang et al., 2021; Sinaga et al., 2023; Son et al., 2020). These approaches share an emphasis on moving learners beyond passive reception toward active participation in constructing, examining, and communicating mathematical ideas. Problem-centered learning may also have an affective dimension because opportunities to participate actively in meaningful mathematical activity can shape learners' confidence, persistence, and attitudes toward problem solving. Recent evidence concerning student-centered mathematics pedagogies similarly indicates that cognitive outcomes and learners' confidence can both constitute important dimensions when evaluating instructional approaches. However, the successful implementation of such pedagogy cannot be assumed across contexts because classroom resources, teacher preparation, learner diversity, and established instructional practices may influence how problem-centered principles are enacted. PCTL should therefore be examined not only in terms of changes in measurable learning outcomes but also through the classroom processes that may explain how learners engage with and respond to challenging mathematical activity.

Recent scholarship provides important foundations for understanding these classroom processes, although the relevant evidence remains distributed across several strands of research. Productive struggle has received increasing attention as a means of sustaining engagement with cognitively demanding tasks, but studies identified in the literature include teacher education, science education, and educational settings beyond secondary mathematics classrooms (Blitz, 2025; Chen, 2022; Trinter & Hughes, 2021; Warshauer et al., 2021). Research on collaborative

mathematical activity has separately highlighted mathematical discourse, peer interaction, learner agency, and collaborative dynamics as important features of participation in mathematical meaning-making (Langer-Osuna et al., 2020; Moon et al., 2024; Nieminen et al., 2022). Metacognition research has likewise examined relationships among metacognitive processes, learner engagement, learning strategies, self-regulation, and academic performance (An et al., 2024; Li & Yuan, 2022; Wang et al., 2021). Although these research strands illuminate important components of effective learning, they frequently investigate problem-centered instruction, problem-solving outcomes, productive struggle, collaborative discourse, and metacognitive engagement independently or through only selected relationships among them. Research on problem-centered instruction itself illustrates this issue, as existing synthesis has identified the intersection between scaffolding and collaborative support as an area requiring explicit investigation rather than assuming that each component operates independently (Kim et al., 2020). Consequently, limited evidence explains how multiple classroom mechanisms operate together during PCTL and how they may accompany changes in both mathematical problem-solving performance and learners' attitudes. This limitation becomes particularly important in disadvantaged South African secondary schools, where contextual and resource constraints may influence how learner-centered instructional principles translate into everyday classroom practice. Examining teacher wait-time, productive struggle, collaborative discourse, and reflective journaling alongside learning outcomes can therefore provide a more process-oriented account of what occurs during PCTL implementation. Integrating these mechanisms with measures of mathematical problem-solving performance and attitudes may also help bridge the separation between outcome-focused studies and research concerned primarily with classroom processes. Such an integrated perspective provides the basis for investigating not merely whether PCTL is associated with learner change, but also how specific instructional and learner processes may help explain the patterns observed within a disadvantaged Grade 8 mathematics classroom.

Recent research has increasingly emphasized problem-centered instruction, mathematical problem solving, productive struggle, collaborative discourse, and metacognitive engagement as important dimensions of meaningful mathematics learning, yet these areas have largely developed as separate or only partially connected strands of inquiry. Studies on problem-centered instruction have examined instructional design, scaffolding, and alternative approaches to conventional knowledge-transmission practices, demonstrating growing interest in organizing learning around meaningful and challenging problems (Hall, 2022; Kim et al., 2020; Meng et al., 2025; Merrill, 2020). Other studies have investigated mathematical problem-solving performance through teaching-model interventions, cooperative learning, self-assessment, and metacognitive approaches, indicating that problem-solving development can be supported through diverse instructional strategies (Barana et al., 2022; Klang et al., 2021; Sinaga et al., 2023; Son et al., 2020). In parallel, productive struggle has received increasing scholarly attention as an important feature of learning through challenging tasks, although the identified literature frequently addresses teacher education, science education, or educational settings beyond secondary mathematics classrooms (Blitz, 2025; Chen, 2022; Trinter & Hughes, 2021; Warshauer et al., 2021). Research on collaborative mathematical activity has similarly highlighted classroom discourse, peer interaction, student agency, and collaborative dynamics as important elements of mathematical meaning-making (Langer-Osuna et al., 2020; Moon et al., 2024; Nieminen et al., 2022). Meanwhile, contemporary metacognition research has predominantly examined its relationships with learner engagement, learning strategies, self-regulation, and academic performance rather than positioning metacognitive engagement within a broader problem-centered mathematics environment (An et al., 2024; Li & Yuan, 2022; Wang et al., 2021). Taken together, these studies reveal a fragmented evidence base in which problem-centered instruction, mathematical problem-solving outcomes, productive struggle,

collaborative discourse, and metacognitive engagement are more commonly investigated independently than as interconnected components of the same instructional process. Consequently, limited evidence from the literature identified in the present review explains how these classroom mechanisms operate together during PCTL and how they may accompany changes in both learners' mathematical problem-solving performance and attitudes. This gap is particularly important in disadvantaged secondary-school settings, where contextual constraints may shape how learner-centered and problem-focused pedagogies are enacted, yet the studies identified above provide limited direct evidence from disadvantaged South African Grade 8 mathematics classrooms. Therefore, the present study addresses this gap by integrating quantitative evidence of learners' problem-solving performance and attitudes with qualitative examination of teacher wait-time, productive struggle, collaborative discourse, and reflective journaling, thereby providing a more context-sensitive account of both the outcomes and classroom processes associated with PCTL implementation.

Responding to these concerns, the present study aimed to investigate the influence of a Problem-Centered Teaching and Learning (PCTL) intervention on Grade 8 learners' mathematical problem-solving performance and attitudes within a disadvantaged South African secondary school. The study specifically examined the extent to which learners' mathematical problem-solving performance changed following participation in the PCTL intervention. It also investigated changes in learners' attitudes toward mathematical problem solving, recognizing both cognitive performance and affective dispositions as relevant dimensions of learners' engagement with challenging mathematical activity. Beyond these outcome measures, the study sought to examine classroom processes and instructional mechanisms that may help explain the patterns observed during the intervention. Particular attention was directed toward teacher wait-time, productive struggle, collaborative discourse, and reflective journaling as key features of the implemented problem-centered learning environment. A convergent mixed-methods design was employed to integrate quantitative evidence concerning problem-solving performance and attitudes with qualitative evidence derived from learners' and teachers' classroom experiences. This integration enabled the investigation to move beyond identifying whether changes occurred toward examining how those changes were manifested within classroom interactions and problem-solving activity. By situating the intervention within a disadvantaged South African secondary school, the study further sought to generate context-sensitive evidence reflecting the instructional, linguistic, and resource-related conditions surrounding mathematics learning. In doing so, the study addresses the identified separation between outcome-focused research and studies of classroom processes by examining learner outcomes and multiple instructional mechanisms within a single PCTL investigation. Ultimately, the study seeks to provide an empirically grounded understanding of how PCTL may support mathematical problem solving and learner attitudes while informing the development of context-responsive mathematics instruction in South Africa and comparable educational settings.

LITERATURE REVIEW

Mathematical problem solving extends beyond the execution of learned procedures because learners must interpret unfamiliar situations, identify relevant mathematical relationships, formulate representations, and determine appropriate solution strategies. Contemporary mathematics education therefore places increasing emphasis on learners' capacity to coordinate conceptual knowledge, procedural resources, and strategic reasoning when addressing non-routine tasks. From this perspective, problem-solving performance reflects not only whether learners obtain correct answers but also how effectively they formulate problems, select strategies, justify

procedures, and reconsider their reasoning (Kholifah et al., 2026; Oroh et al., 2025; Tuveri et al., 2025). Recent research has consequently examined instructional approaches designed to strengthen problem-solving performance through more active forms of mathematical engagement. These approaches include cooperative learning, self-assessment, metacognitive support, and instructional models that position learners as active participants in constructing solutions (Barana et al., 2022; Klang et al., 2021; Sinaga et al., 2023; Son et al., 2020). These developments can be interpreted framework of mathematical proficiency, which conceptualizes mathematical competence through the interrelated strands of conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. Strategic competence is particularly relevant to non-routine problem solving because learners must formulate and represent problems before identifying potentially appropriate solution pathways. Adaptive reasoning becomes equally important when learners justify strategies, evaluate mathematical arguments, identify inconsistencies, and determine whether solutions are reasonable. Productive disposition extends this perspective beyond cognitive performance by emphasizing learners' willingness to perceive mathematics as meaningful and themselves as capable of engaging with mathematical challenges. Accordingly, mathematical problem solving in the present study is conceptualized as a multidimensional process in which conceptual understanding, procedural fluency, strategic decision-making, adaptive reasoning, and learners' dispositions operate as interconnected rather than isolated dimensions of mathematical proficiency.

Problem-Centered Teaching and Learning (PCTL) provides an instructional environment through which these dimensions of mathematical proficiency can be developed by positioning meaningful problems as the organizing context of learning. Rather than beginning primarily with teacher demonstrations followed by procedural reproduction, problem-centered instruction creates opportunities for learners to investigate tasks, activate prior knowledge, formulate strategies, test mathematical ideas, and justify emerging solutions. Recent scholarship has examined instructional design, scaffolding, and collaborative arrangements as important components of problem-centered learning environments (Hall, 2022; Kim et al., 2020; Meng et al., 2025; Merrill, 2020). These instructional processes are theoretically Zone of Proximal Development (ZPD), which conceptualizes learning as a socially mediated process in which learners can engage with tasks beyond their current independent capabilities when appropriate assistance is available. Within PCTL, the ZPD positions teacher support not as the direct transmission of predetermined solutions but as responsive mediation that enables learners to progressively assume greater responsibility for mathematical reasoning. Scaffolding may consequently take the form of probing questions, strategic prompts, representations, peer explanations, or feedback that assists learners without prematurely reducing the intellectual demand of the problem. Research on problem-centered STEM instruction has similarly demonstrated the relevance of different forms of scaffolding across individual and collaborative problem-solving arrangements (Kim et al., 2020). From this perspective, challenging mathematical tasks alone are insufficient to characterize PCTL because the quality and timing of instructional mediation influence how learners engage with the mathematical demands embedded in those tasks. PCTL therefore connects the development of mathematical proficiency with socially mediated learning by creating conditions in which learners can explore increasingly challenging problems while receiving strategically adjusted instructional and peer support. This theoretical relationship makes the ZPD particularly useful for explaining how learning opportunities are mediated, while Kilpatrick et al.'s framework provides a complementary lens for interpreting what dimensions of mathematical proficiency are reflected in learners' performance and experiences.

Metacognitive engagement provides an important conceptual connection between these two theoretical perspectives because learners must increasingly regulate their own thinking as they assume greater responsibility for mathematical problem solving. Contemporary research has

connected metacognition with learning strategies, self-regulation, engagement, and academic performance across educational contexts (An et al., 2024; Li & Yuan, 2022; Wang et al., 2021). During mathematical problem solving, metacognitive activity may involve planning an approach, monitoring progress, identifying inconsistencies, reconsidering an ineffective strategy, and evaluating whether a final solution is mathematically plausible. Explicit metacognitive support can make these regulatory processes more visible by encouraging learners to articulate what they are doing, explain why a particular strategy was selected, and determine whether their reasoning remains coherent. Recent studies have therefore examined metacognitive prompts, self-assessment, and reflective activities as mechanisms for supporting learners' regulation of complex problem-solving activity (Barana et al., 2022; Sadykova et al., 2024). For example, reported both quantitative and qualitative changes following metacognition-oriented instruction, demonstrating the value of examining performance alongside learners' reflective activity (Sadykova et al., 2024). Within the integrated theoretical perspective adopted in this study, metacognitive engagement can contribute to strategic competence by supporting strategy selection and adjustment, while monitoring and evaluation may provide evidence associated with adaptive reasoning. It can simultaneously be interpreted through the ZPD because external prompts and socially mediated questioning may initially support regulatory activity that learners are expected to perform with increasing independence. Reflective journaling is particularly relevant to this transition because it provides learners with an opportunity to articulate difficulties, reconsider strategies, and examine their own responses to mathematical challenges. Metacognitive engagement therefore connects scaffolded participation with mathematical proficiency by illuminating how learners progressively develop greater awareness and control over their problem-solving processes.

Productive struggle and collaborative mathematical discourse further illustrate how the ZPD and mathematical proficiency can operate together within a problem-centered classroom. Research on productive struggle has increasingly examined how learners can remain engaged with cognitively demanding tasks when instructional support sustains reasoning without immediately removing the mathematical challenge (Blitz, 2025; Chen, 2022; Trinter & Hughes, 2021; Warshauer et al., 2021). From a ZPD perspective, productive struggle can be understood as engagement within a space where a task is sufficiently challenging to require cognitive effort but remains accessible through appropriately calibrated teacher or peer mediation. Teacher wait-time is relevant to this process because premature intervention may limit opportunities for learners to formulate ideas, reconsider strategies, and persist with uncertainty before receiving additional assistance. Collaborative mathematical discourse provides another form of social mediation by allowing learners to externalize reasoning, encounter alternative representations, negotiate meaning, and justify mathematical claims. Recent studies have emphasized discourse, peer interaction, learner agency, and participation as important features of collaborative mathematical meaning-making (Langer-Osuna et al., 2020; Moon et al., 2024; Nieminen et al., 2022). These interactions also intersect with Kilpatrick et al.'s framework because explaining and defending a strategy can provide evidence of adaptive reasoning, while selecting and comparing approaches can engage strategic competence. Productive disposition may likewise become visible through learners' persistence, willingness to participate, and readiness to continue engaging with challenging mathematical tasks. Reflective journaling complements these socially mediated processes by creating an individual space in which learners can reconsider the strategies, difficulties, and interactions experienced during collaborative problem solving. Thus, teacher wait-time, productive struggle, collaborative discourse, and reflective journaling are treated in the present study not as independent instructional variables but as interconnected classroom mechanisms through which problem-centered learning opportunities may be structured.

The operation of these mechanisms must nevertheless be interpreted within the context in which PCTL is implemented, particularly in disadvantaged South African secondary schools. Instructional resources, class size, language practices, teacher preparation, curriculum pacing, and assessment demands may influence the extent to which teachers can sustain learner-centered mathematical activity. Research concerning educational disadvantage has similarly emphasized the importance of linguistic and contextual accessibility when designing opportunities for meaningful mathematical participation (Mpalami, 2022). These contextual conditions are important from a ZPD perspective because the availability and quality of social mediation depend on how teachers organize interaction, questioning, collaboration, and instructional support within actual classroom constraints. They are equally relevant to mathematical proficiency because opportunities to develop strategic competence, adaptive reasoning, and productive disposition depend partly on whether learners are given sufficient opportunities to engage meaningfully with non-routine problems. The existing literature nevertheless remains relatively fragmented, with studies frequently examining problem-solving outcomes, metacognition, productive struggle, collaborative discourse, or particular instructional strategies as separate or only partially connected dimensions (An et al., 2024; Barana et al., 2022; Moon et al., 2024; Son et al., 2020; Trinter & Hughes, 2021).. Problem-centered instruction provides a conceptual space for bringing these processes together because challenging tasks, scaffolding, collaboration, reflection, and learner regulation can occur within the same instructional environment (Hall, 2022; Kim et al., 2020; Meng et al., 2025). However, comparatively limited evidence explains how these classroom mechanisms operate together and how they accompany changes in both mathematical problem-solving performance and learners' attitudes, particularly within disadvantaged secondary-school settings. The present study therefore integrates Vygotsky's ZPD and Kilpatrick et al.'s mathematical proficiency framework to examine both the socially mediated processes through which PCTL is enacted and the dimensions of learner development reflected in its outcomes. This integration provides the conceptual basis for aligning quantitative measures of problem-solving performance and attitudes with qualitative evidence from learner reflections, classroom observations, and interviews, as summarized in Table 1.

Table 1. Conceptual alignment of study instruments with the theoretical framework

Instrument	Primary Construct Assessed	Relevant Theoretical Lens	Associated Strand(s) / Mechanism(s)	Interpretive Note
Mathematical Problem-Solving Skills Inventory (MPSSI)	Learners' performance on non-routine mathematical tasks	Kilpatrick et al. (2001)	Strategic competence, procedural fluency, and aspects of adaptive reasoning	Scores were interpreted as indicators of learners' ability to formulate, represent, and solve unfamiliar problems, rather than as direct measures of individual strands in isolation.
Problem-Solving Attitudes Questionnaire (PSAQ)	Learners' confidence, persistence, and disposition toward challenge	Kilpatrick et al. (2001)	Productive disposition	Questionnaire responses were used to examine shifts in learners' willingness to engage with mathematical challenge and their beliefs about themselves as problem solvers.
Learner Reflection Journals	Learners' reflections on strategy use, difficulty, and self-monitoring	Vygotsky (1978); Kilpatrick et al. (2001)	Adaptive reasoning, productive disposition, metacognitive engagement	Journals provided insight into how learners monitored their thinking, justified strategies, and reflected on problem-solving experiences over time.

Classroom Observations	Classroom interaction, teacher scaffolding, learner participation	Vygotsky (1978)	ZPD mediation, teacher wait-time, productive struggle, collaborative discourse	Observation data were used to identify the interactional and pedagogical mechanisms through which learning opportunities were structured during the intervention.
Semi-Structured Learner Interviews	Learners' experiences, reasoning, confidence, and strategy awareness	Vygotsky (1978); Kilpatrick et al. (2001)	Adaptive reasoning, productive disposition, metacognitive engagement	Interviews provided explanatory insight into how learners made sense of tasks, interpreted challenges, and described changes in their approach to problem solving.

As shown in Table 1, the two theoretical perspectives perform complementary rather than interchangeable functions within the analytical framework. Vygotsky's ZPD primarily supports interpretation of how learning opportunities were socially mediated, whereas Kilpatrick et al.'s framework supports interpretation of the dimensions of mathematical proficiency reflected in learner outcomes and experiences. The alignment does not imply that individual instruments directly or exclusively measure particular theoretical constructs, because several sources of evidence illuminate overlapping dimensions of learning. The MPSSI primarily provides evidence concerning strategic competence, procedural fluency, and aspects of adaptive reasoning, while the PSAQ provides evidence associated principally with productive disposition. Learner journals and interviews extend these measures by illuminating strategy awareness, reflection, confidence, and metacognitive engagement. Classroom observations and the teacher interview provide complementary process evidence concerning scaffolding, wait-time, productive struggle, collaborative discourse, and instructional decision-making. Triangulating these sources enables quantitative changes in performance and attitudes to be interpreted alongside qualitative evidence concerning how learners and teachers experienced the PCTL environment. This alignment is particularly important to the convergent mixed-methods design because quantitative and qualitative evidence are not treated as parallel but unrelated datasets. Instead, they provide complementary perspectives on the relationship between learner outcomes and the classroom processes accompanying those outcomes. Consequently, the integrated framework establishes a conceptual bridge from the literature to the methodology by clarifying what the study examines, how its principal constructs are theoretically understood, and how the different instruments contribute evidence to their interpretation.

METHOD

Research Design

This study employed a convergent mixed-methods design (Creswell & Plano Clark, 2011) to investigate the influence of a Problem-Centered Teaching and Learning (PCTL) intervention on Grade 8 learners' mathematical problem-solving performance and attitudes toward mathematical problem solving. The quantitative strand used a quasi-experimental non-equivalent groups design, whereas the qualitative strand examined classroom processes, instructional practices, and learner experiences associated with the intervention. Quantitative and qualitative data were collected during the same overall study period, analyzed separately, and subsequently integrated to develop a comprehensive interpretation of the findings. The quantitative component focused on changes in mathematical problem-solving performance and attitudes, while the qualitative component examined processes that may help explain the observed patterns, particularly teacher wait-time,

productive struggle, collaborative discourse, reflective journaling, and learner engagement. This design was considered appropriate because mathematical problem solving involves cognitive, affective, and instructional dimensions that cannot be adequately represented through outcome measures alone (Sadykova et al., 2024). The integration of numerical and qualitative evidence therefore enabled the study to examine both changes in learner outcomes and the classroom conditions accompanying those changes.

The study was conducted at a co-educational government secondary school situated in a disadvantaged South African community. Many learners spoke isiZulu, Sesotho, or Setswana as their home languages, whereas English was the official language of learning and teaching. The setting was characterized by contextual challenges relevant to the implementation of learner-centered mathematics instruction, including large classes, limited instructional resources, and linguistic diversity. Within this setting, the intervention was implemented as a 10-week multi-component PCTL program informed by constructivist principles and previous work on problem-centered pedagogy. The intervention comprised three interrelated components: problem-centered lesson design using rich and contextually meaningful non-routine tasks, teacher mentoring and professional support, and learner reflection journaling as structured metacognitive support. Because these components were implemented together, the intervention was conceptualized as a combined instructional package rather than as a single-variable treatment, and the study did not attempt to estimate the independent effect of each component.

Participant

Participants were selected purposively from two intact Grade 8 classes at the participating school. The learner sample consisted of 98 Grade 8 learners, including 51 females and 47 males, aged between 13 and 15 years. One intact class served as the intervention group and the other as the comparison group. Learners were not randomly assigned to conditions; consequently, the quantitative component was treated as a quasi-experimental non-equivalent groups design rather than a randomized experiment. Grade 8 was selected because it represents an important transition within the South African schooling system in which learners are expected to demonstrate increasing independence and higher-order mathematical thinking.

Two mathematics teachers also participated. Teacher Anne, a pseudonym, taught the intervention group and implemented the PCTL program. She participated in three professional mentoring workshops conducted before and during implementation, focusing on questioning strategies, facilitation techniques, extended wait-time, productive struggle, and discussion-oriented mathematics instruction. Teacher Ben, also a pseudonym, taught the comparison group and continued routine instructional practice without receiving parallel professional development during the study period. This arrangement provided a naturalistic comparison between instructional conditions but introduced a potential teacher and professional-development effect. Accordingly, between-group differences cannot be attributed exclusively to the PCTL pedagogical component because the intervention condition incorporated both instructional redesign and professional support for the teacher.

Instrument and Data Collection

Multiple quantitative and qualitative instruments were used to capture complementary dimensions of mathematical problem solving and the implementation of PCTL. Quantitative data were collected using the Mathematical Problem-Solving Skills Inventory (MPSSI) and the Problem-Solving Attitudes Questionnaire (PSAQ). The researcher-developed MPSSI comprised 10 non-routine mathematical problems aligned with Grade 8 Curriculum and Assessment Policy Statement (CAPS) topics. The items were designed to require learners to interpret information, formulate strategies, and provide solutions rather than merely reproduce routine procedures. Content validity was

established through review by experienced mathematics educators, who examined alignment with CAPS content, cognitive demand, and item clarity. A pilot study involving 20 Grade 8 learners from a neighboring school was subsequently conducted to evaluate item clarity and difficulty, after which minor modifications were made to item wording and structure. Basic item analysis was also conducted to examine the distribution of item difficulty and discrimination.

The MPSSI was scored using an analytic rubric that awarded partial credit for strategy use, accuracy, and justification. To strengthen scoring reliability, a subset of learner responses was independently scored by a second rater, with a high level of agreement observed. Internal consistency was also examined to determine whether the instrument demonstrated sufficient reliability for research use. The MPSSI was administered to both groups before and after the intervention to measure changes in mathematical problem-solving performance. The PSAQ, provided in Appendix A, assessed learners' confidence, persistence, and enjoyment in relation to mathematical problem solving. It was adapted original instrument and contextualized for the South African setting. Pilot testing indicated strong internal consistency, with a Cronbach's alpha coefficient of 0.87. Like the MPSSI, the PSAQ was administered to both groups at pre-test and post-test.

Qualitative evidence was obtained from classroom observations, learner reflection journals, semi-structured learner interviews, and teacher interviews. Classroom observations were conducted twice weekly using a structured observation guide focusing on teacher questioning, learner participation, group interaction, and the enactment of PCTL practices. Detailed field notes and audio recordings supplemented the structured observations. The observation guide also served as an intervention-fidelity monitoring tool by tracking four core PCTL practices: (a) learner collaboration in pairs or groups, (b) extended teacher wait-time and probing questions, (c) use of learner reflection journals, and (d) whole-class sharing and comparison of solution strategies. Learners in the intervention group completed weekly reflection journals documenting strategies, difficulties, adjustments, and insights generated during problem-solving activities. Semi-structured interviews were conducted with both participating teachers and a stratified sample of 12 learners from each group, focusing on experiences of mathematical problem solving, instructional practices, strategy awareness, confidence, and perceived barriers to learning. Together, these sources provided complementary evidence concerning learner outcomes, classroom processes, and the implementation of the intervention.

Data Analysis

Quantitative data were analyzed using SPSS version 28. Means and standard deviations were calculated to summarize mathematical problem-solving performance and attitude scores at pre-test and post-test. Because the quantitative component addressed a limited set of a priori research questions, the principal inferential analyses were treated as planned comparisons. Paired-samples t-tests were used to examine within-group changes between pre-test and post-test, whereas independent-samples t-tests on gain scores were used to compare improvement between the intervention and comparison groups. The same analytical strategy was applied to changes in learner attitudes. Cohen's d was calculated to estimate the magnitude of observed differences in addition to statistical significance. More detailed analyses by problem type—procedural, conceptual, and non-routine modeling—were treated as supplementary rather than primary hypothesis tests. A Bonferroni-adjusted significance threshold was applied where relevant to reduce inflation of Type I error across these related comparisons.

Qualitative data were analyzed using thematic analysis following Braun and Clarke (2006). The analysis involved familiarization with the data, initial coding, category development, theme review, theme definition and naming, and selection of representative excerpts. Coding was conducted inductively, using meaning units such as sentences or short passages as the unit of analysis. An initial

codebook was developed from a subset of the data and iteratively refined as analysis progressed. Two independent coders participated in the coding process, producing an initial inter-coder agreement of 89% for the sampled data. Cohen's kappa was subsequently calculated to account for chance agreement and indicated substantial agreement ($\kappa = 0.76$). Coding discrepancies were discussed and resolved through consensus, after which the codebook was refined before final coding.

Following the separate quantitative and qualitative analyses, the two strands were integrated using a joint display matrix. Quantitative trends were compared with qualitative themes and illustrative evidence to identify convergence, complementarity, and explanatory connections across the datasets. For example, changes in problem-solving performance were considered alongside classroom observations, learner journal entries, and interview evidence concerning strategy use, productive struggle, collaborative discourse, and instructional support. The qualitative evidence was used to contextualize and help explain quantitative patterns rather than to establish that particular classroom mechanisms statistically mediated the observed outcomes. This integration enabled the final interpretation to address both what changed following the intervention and what classroom processes accompanied those changes.

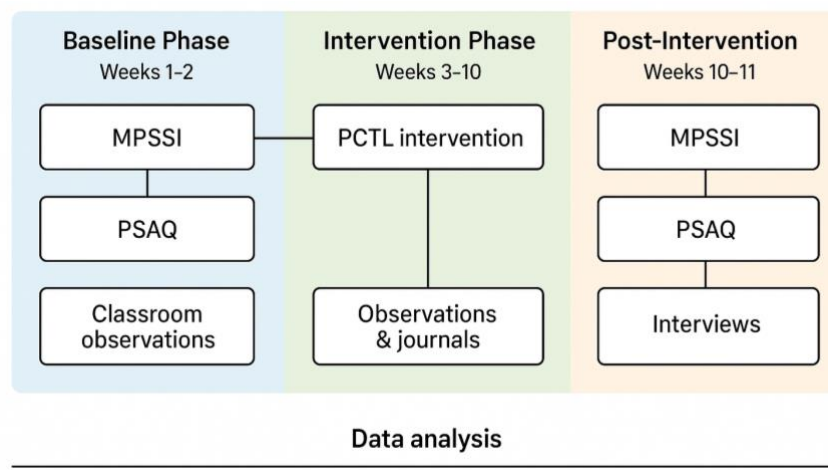


Figure 1. Flowchart of the convergent mixed-methods study design showing the three phases

Research Procedure

The study was implemented across three sequential phases within an 11-week data-collection period. During the baseline phase (Weeks 1–2), learners in both groups completed the MPSSI and PSAQ pre-tests, while initial classroom observations were undertaken to document baseline instructional practices. During the intervention phase (Weeks 3–10), the intervention group participated in the multi-component PCTL program, whereas the comparison group continued with routine mathematics instruction. Problem-centered lessons began with contextually meaningful, non-routine tasks drawn from situations such as budgeting, sports statistics, and environmental data. Tasks were designed to permit multiple solution strategies, and learners worked collaboratively before participating in whole-class discussions in which alternative approaches were presented, compared, and discussed. Metacognitive prompts encouraged learners to plan, monitor, and evaluate their reasoning, while weekly reflection journals provided structured opportunities to document strategies, challenges, and insights. Teacher Anne received three professional mentoring workshops before and during implementation as part of the intervention package. The workshops emphasized probing questions, facilitative teaching, extended wait-time, management of productive struggle, and reduced reliance on directive instruction. Classroom observations were conducted throughout implementation to document classroom processes and monitor the presence of the intended PCTL

practices. Teacher coaching and learner journal submissions also continued during this phase. Teacher Ben did not receive equivalent mentoring and maintained routine instructional practices with the comparison group. Because teacher professional support was embedded within the intervention, it was treated as one component of the overall PCTL package rather than as a separate experimental factor.

During the post-intervention phase (Weeks 10–11), learners in both groups completed the MPSSI and PSAQ post-tests. Semi-structured interviews were subsequently conducted with the two teachers and the selected learners from each group to obtain explanatory accounts of their experiences and perceptions. Quantitative and qualitative datasets were then analyzed separately before being integrated through the joint display. Figure 1 summarizes the sequence of baseline measurement, intervention implementation, post-intervention assessment, and concurrent collection of qualitative evidence across the study phases. Ethical approval was obtained from the host institution before data collection commenced. Written informed consent was obtained from the school principal, participating teachers, and learners' parents or guardians, while learners provided assent. Pseudonyms were used for teachers and learners to protect confidentiality and anonymity. Quantitative methodological quality was supported through expert review, pilot testing, reliability assessment, and scoring procedures, while qualitative trustworthiness was strengthened through data-source triangulation, teacher member checking, and an audit trail documenting analytic decisions.

RESULTS AND DISCUSSION

Results

This study investigated the effects of a PCTL intervention on Grade 8 learners' mathematical problem-solving performance, attitudes toward mathematical problem solving, and the classroom processes associated with these changes. A convergent mixed-methods design was employed to capture both the measurable outcomes of the intervention and the rich contextual details of how these outcomes emerged. The findings are presented in relation to the study's three analytic concerns: changes in mathematical problem-solving performance, changes in learner attitudes, and the instructional mechanisms that appeared to mediate these outcomes.

Quantitative Findings

Improvement in Overall Problem-Solving Performance

The pre- and post-tests were used to measure learners' ability to solve non-routine mathematical problems before and after the intervention. Descriptive statistics showed that the two groups performed similarly at baseline, suggesting initial equivalence. The experimental group obtained a mean pre-test score of 42.3% (SD = 9.4), while the comparison group obtained a mean pre-test score of 41.6% (SD = 8.7). At post-test, the experimental group improved to a mean score of 64.8% (SD = 10.1), representing a gain of 22.5 percentage points, whereas the comparison group increased only slightly to 44.1% (SD = 9.2), representing a gain of 2.5 percentage points. To examine within-group change over time, a paired-samples t-test was conducted separately for each group. The experimental group showed a statistically significant increase in problem-solving performance from pre-test to post-test, $t(48) = 9.72$, $p < .001$, $d = 1.39$. In contrast, the comparison group showed no statistically significant improvement, $t(48) = 1.28$, $p = .21$, $d = 0.18$. To compare intervention effects across groups, gain scores (post-test minus pre-test) were calculated and analyzed using an independent-samples t-test. This analysis showed that the experimental group's improvement was significantly greater than that of the comparison group, $t(96) = 8.54$, $p < .001$. These show that the experimental group demonstrated substantially greater improvement in problem-solving

performance than the comparison group. Figure 2 illustrates this pattern, showing a marked upward shift in the experimental group relative to the comparison group over the 10-week intervention.

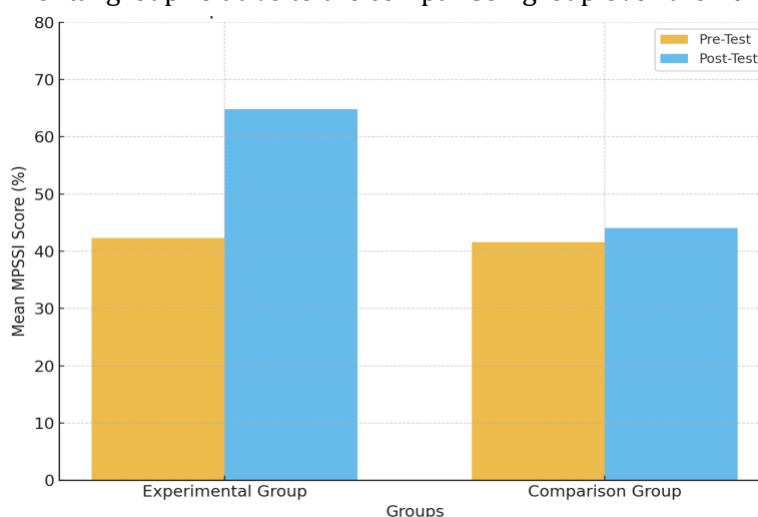


Figure 2. Comparison of mean pre-test and post-test MPSSI scores for experimental and comparison groups.

Performance by Problem Type

The MPSSI included three categories of problems: procedural, conceptual, and non-routine, real-world modeling tasks. Table 2 summarizes the mean percentage scores by category.

Table 2. Pre-test and post-test MPSSI scores for experimental and comparison groups

Problem Type	Pre-Test Mean (Experimental)	Post-Test Mean (Experimental)	Mean Gain	Pre-Test Mean (Comparison)	Post-Test Mean (Comparison)
Procedural Problems	52% (SD = 8.1)	63% (SD = 9.0)	11%	51% (SD = 8.0)	52% (SD = 8.4)
Conceptual Problems	40% (SD = 7.9)	62% (SD = 8.7)	22%	41% (SD = 7.6)	43% (SD = 8.1)
Non-Routine Modeling	35% (SD = 9.2)	65% (SD = 9.8)	30%	34% (SD = 8.9)	37% (SD = 9.3)

The largest descriptive gain in the experimental group was observed in non-routine modeling tasks, in which learners had to apply mathematical concepts to unfamiliar real-world scenarios. This category showed a mean increase of 30 percentage points in the experimental group compared to a modest 3-point increase in the comparison group. These results indicated greater improvement on non-routine modeling tasks in the experimental group than in the comparison group.

Changes in Learners' Attitudes Toward Problem Solving

The Problem-Solving Attitudes Questionnaire (PSAQ) measured shifts in learners' confidence, persistence, and enjoyment of problem solving. At baseline, both groups reported similar attitudes, with mean scores around 3.1 (SD = 0.45) on a 5-point scale. After the intervention, the experimental group's mean increased significantly to 3.9 (SD = 0.51), indicating a notable improvement in positive dispositions ($t(48) = 7.38$, $p < .001$, Cohen's $d = 1.05$). In contrast, the comparison group's mean remained statistically unchanged at 3.0 (SD = 0.43) ($t(48) = 0.62$, $p = .54$). Further analysis revealed a moderate positive correlation between post-test confidence scores and problem-solving performance ($r = .56$, $p < .01$). This indicated a moderate positive correlation between post-test attitude scores and problem-solving.

Regression Analysis

To examine whether learners' attitudes toward mathematical problem solving were associated with their end-of-intervention mathematical performance, a linear regression was conducted using post-test MPSSI scores as the dependent variable and post-test PSAQ attitude scores as the predictor. The model was statistically significant, $F(1, 96) = 43.72, p < .001$, indicating that attitude was significantly associated with post-test problem-solving performance. Post-test attitude scores accounted for 31% of the variance in post-test MPSSI scores ($R^2 = .31$). This suggests that learners who reported more positive attitudes toward mathematical problem solving also tended to perform better on the final non-routine problem-solving assessment. The finding highlights the close relationship between affective dispositions and mathematical performance, suggesting that confidence, persistence, and willingness to engage with challenge may be important correlates of successful problem solving.

Interpretation of Quantitative Findings

The quantitative results demonstrate that the PCTL intervention had a transformational impact on learners' mathematical problem-solving skills, particularly for complex, non-routine tasks. These findings align with prior studies, which have shown that problem-based learning improves learners' ability to reason, apply strategies flexibly, and engage in mathematical modelling (Shongwe, 2024). The strong link between attitudes and performance suggests that improving learner dispositions is as crucial as teaching strategies. This aligns with framework, which identifies productive disposition as a key strand of mathematical proficiency.

Qualitative Findings

Thematic analysis of observation notes, learner journals, and interviews revealed four overarching themes, each with sub-themes that provide deeper insights into the instructional mechanisms and classroom processes that appeared to mediate changes in learners' mathematical problem-solving performance and attitudes during the intervention.

Increased Engagement and Persistence

At the outset, many learners displayed low confidence and quickly gave up when confronted with unfamiliar problems. Observations during the first two weeks showed that most learners spent minimal time grappling with tasks before seeking teacher assistance. By mid-intervention, this pattern had shifted dramatically. Learners began working collaboratively, discussing possible approaches, and demonstrating a willingness to persevere through difficulty. Teacher Anne described this transformation:

Initially, learners would stop at the first sign of confusion. By the end, they were pushing each other to try different ways before asking me for help. (Teacher Interview, Week 10)

Journal entries corroborated these observations. One learner wrote:

Before, I hated hard problems because I always got stuck. Now I don't mind because I know I can figure it out if I keep trying. (Learner Journal, Week 9)

This shift reflects a move toward productive struggle, a hallmark of problem-centered learning.

Development of Strategic Flexibility

Early observations revealed that learners relied almost entirely on memorized algorithms. When problems deviated from familiar formats, they were unsure how to proceed. Over time, learners began to adopt a variety of heuristic strategies, including drawing diagrams, creating tables, estimating, and working backward. For instance, during a lesson on budgeting, one group used a table to systematically track expenses, while another drew a visual diagram to represent relationships between quantities. Teacher Anne highlighted this change:

I was amazed to see learners using multiple strategies and even arguing about which one was better. That never happened before.

This finding aligns with international research showing that problem-based learning fosters strategic competence and creativity (Zhang et al., 2024).

Growth in Metacognitive Awareness

Metacognitive development was evident in learners' journals and class discussions. Initially, journal entries were brief and procedural, often stating, "I followed the steps the teacher showed." By Week 10, entries reflected deeper reflection:

I started by planning what the problem was asking. When I got stuck, I reviewed my steps and tried an alternative method. At the end, I looked back to see if my answer made sense."

Teacher Anne observed that learners began using prompts independently, such as "What is the question asking?" and "Does my solution make sense?" This suggests that the intervention fostered self-regulated learning, a critical skill for lifelong mathematical competence (Sadykova et al., 2024).

Teacher Transformation and Beliefs

Teacher Anne initially expressed doubts about whether Grade 8 learners were ready for open-ended problem solving. As the intervention progressed, her beliefs shifted. She reported gaining confidence in facilitating rather than directing learning: *I learned to step back and let learners struggle a bit. It was hard at first, but I saw them grow in ways I never expected."*

In contrast, Teacher Ben, who taught the comparison group, maintained a traditional approach and remained skeptical about the feasibility of PCTL given time and curriculum pressures. This divergence underscores the role of teacher beliefs as both barriers and enablers of instructional reform. Table 3 presents a direct comparison of numerical trends and descriptive themes.

Table 3. Joint display linking quantitative results and qualitative themes

Quantitative finding	Qualitative theme	Supporting qualitative evidence (example)	Integrated interpretation
Significant improvement in problem-solving scores (experimental group)	Increased engagement and persistence	Learners reported "not giving up" and working through difficult problems in journals	Improved performance is associated with greater persistence and willingness to engage in productive struggle
Largest gains in non-routine modelling tasks	Development of strategic flexibility	Observations showed learners using diagrams, tables, and multiple strategies	Exposure to non-routine tasks supported flexible strategy use
Positive shift in attitudes toward problem solving	Growth in confidence and metacognitive awareness	Learners reflected on planning, checking, and evaluating their solutions	Improved attitudes appear linked to increased metacognitive control
No significant change in comparison group	Teacher-directed instruction remained dominant	Limited discussion and reliance on procedures observed	Lack of pedagogical change corresponds with minimal performance gains

Discussion

The findings indicate that the multi-component PCTL intervention was associated with substantial improvements in Grade 8 learners' mathematical problem-solving performance and more positive attitudes toward mathematical problem solving. Rather than attributing these outcomes to a single instructional element, the quantitative and qualitative evidence suggests that they emerged alongside the combined implementation of problem-centered tasks, metacognitive support, collaborative discourse, teacher wait-time, productive struggle, and reflective journaling. During the intervention, the teacher's role increasingly shifted from direct knowledge transmission toward facilitation through probing questions, strategic hints, and structured opportunities for

collaboration. Observational and journal evidence further showed that learners became less dependent on immediate teacher direction and increasingly engaged in peer dialogue, self-questioning, strategy monitoring, and evaluation of their solutions. These patterns are consistent with recent research emphasizing the contribution of metacognitive support to learners' capacity to regulate and evaluate their mathematical thinking (Sadykova et al., 2024; Zhang et al., 2024). The particularly large improvement in non-routine modeling tasks also corresponded with qualitative evidence showing greater use of diagrams, tables, estimation, and alternative solution strategies. Learners increasingly justified their approaches, compared alternative strategies, and reconsidered unsuccessful solutions, indicating development in strategic flexibility and reflective mathematical reasoning. Such findings are consistent with recent evidence indicating that problem-centered and related learner-active approaches can support strategic reasoning and flexible engagement with mathematical problems (Zhang et al., 2024). At the same time, improvements in confidence, persistence, and willingness to engage with challenging tasks suggest that changes were not restricted to cognitive performance. The combined evidence therefore indicates that PCTL was associated with interconnected changes in learners' problem-solving performance, strategic engagement, metacognitive awareness, and dispositions toward mathematical challenge.

The contrast between the intervention and comparison groups provides further insight into the instructional conditions accompanying these changes. Whereas the comparison group demonstrated minimal change in problem-solving performance and attitudes, learners in the intervention group showed substantial quantitative improvement alongside qualitative changes in persistence, strategic flexibility, collaboration, and reflective engagement. Several learners initially experienced open-ended problems as difficult or overwhelming but gradually demonstrated greater willingness to persist as they engaged in collaborative problem solving and received strategically timed teacher support. The positive association between post-intervention attitudes and problem-solving performance further suggests that affective dispositions and mathematical performance were closely related within the study. Learners who reported stronger confidence, persistence, and willingness to engage with mathematical challenges also tended to achieve higher problem-solving scores. This pattern is consistent with recent research highlighting the importance of learner attitudes and engagement within problem-centered and problem-based mathematical learning environments (Shongwe, 2024). Qualitative evidence also showed increasing metacognitive awareness, particularly as learners began independently asking questions about task requirements, strategy selection, and the reasonableness of their solutions. These findings correspond with recent studies emphasizing planning, monitoring, reflection, and self-evaluation as important features of metacognitively supported learning (Sadykova et al., 2024; Zhang et al., 2024). Importantly, however, these classroom processes should not be interpreted as statistically established mediators of the observed gains. Instead, teacher wait-time, productive struggle, collaborative discourse, and reflective journaling represent salient implementation processes that accompanied the quantitative improvements and may help explain how learners experienced the PCTL environment.

The findings also emphasize the importance of teacher preparation and instructional support when implementing PCTL in disadvantaged mathematics classrooms. Teacher Anne initially experienced difficulty relinquishing instructional control but gradually became more confident in allowing learners time to struggle, discuss alternatives, and construct solutions before receiving direct assistance. In contrast, Teacher Ben maintained predominantly teacher-directed practices and identified curriculum coverage and classroom constraints as barriers to implementing more learner-centered instruction. These contrasting experiences indicate that implementing PCTL involves not only introducing non-routine mathematical tasks but also reconsidering teachers' facilitation practices, beliefs, and responses to learner difficulty (Mumu et al., 2025). This interpretation is consistent with recent research emphasizing sustained mentoring and professional support when

introducing pedagogical innovations in challenging educational contexts (Soforon et al., 2024). The findings should nevertheless be interpreted in light of the multi-component nature of the intervention because the intervention teacher received professional mentoring while the comparison teacher did not receive equivalent support. Consequently, the observed between-group differences cannot be attributed exclusively to problem-centered tasks, reflective journaling, teacher facilitation, or professional development as independent components. Instead, the findings support interpretation of PCTL as a combined instructional package in which these components operated together. Integrating quantitative and qualitative evidence further enabled measurable changes in performance and attitudes to be considered alongside classroom observations, learner reflections, and teacher experiences. Overall, the study provides context-sensitive evidence that a multi-component PCTL environment may support mathematical problem solving and positive learner dispositions while highlighting the importance of teacher facilitation, metacognitive support, and classroom interaction in understanding how such interventions are enacted.

Implications

These findings suggest that successful implementation of PCTL in disadvantaged Grade 8 classrooms requires practical classroom structures, not only general endorsement of problem solving. First, teacher professional development should include CAPS-aligned banks of non-routine tasks linked to topics such as patterns, ratios, algebra, geometry, and data handling, with guidance on likely strategies and misconceptions. Second, brief reflection journaling should be embedded in weekly classroom routines using prompts such as: *What strategy did I try first? Where did I get stuck?*, and *How do I know my answer makes sense?* Third, teachers need questioning routines that sustain cognitive demand, for example: *What is the problem asking? What have you tried?* and *Can you explain why your method works?* In practical terms, a feasible lesson structure in large classes would include approximately 10 minutes for task launch, 20–25 minutes for collaborative problem solving, 10–15 minutes for discussion of strategies, and 5 minutes for reflection. These findings suggest that PCTL is more likely to be sustainable when supported by explicit task resources, structured reflection, and manageable facilitation routines.

Limitations and Suggestions for Future Research

This study was conducted at a single school with two teachers, which limits generalizability. The short intervention period (10 weeks) may have constrained the depth of the observed changes. Future research should include larger and more diverse samples across multiple schools and regions, as well as longitudinal designs to examine whether improvements in problem-solving performance and learner dispositions are maintained over time. A further limitation relates to the multi-component nature of the intervention. Because the intervention included problem-centered instruction, teacher professional development, and structured learner reflection, it is not possible to attribute observed improvements to any single component. In addition, only the intervention teacher received mentoring and support, introducing a potential teacher effect that may have contributed to differences between groups. Future research should seek to disentangle these components through more controlled designs.

CONCLUSION

The integration of quantitative and qualitative findings demonstrates that a well-structured multi-component PCTL intervention can produce meaningful improvements in learners' mathematical problem-solving performance, strategic competence, and attitudes toward mathematical problem solving. By combining rich problem contexts, metacognitive scaffolding, and

teacher support, this approach addresses both the cognitive and affective dimensions of learning, offering a viable pathway for improving mathematics education in South African secondary schools. The findings further suggest that these improvements are not attributable to task design alone but are also mediated by specific classroom processes, particularly learner discourse, teacher wait time, productive struggle, and reflective journaling.

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