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The thinking processes of prospective mathematics teachers in reversibly translating nets into geometric shapes

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Abstract

Background: Spatial reasoning is a fundamental component of geometry learning, particularly in translating between two-dimensional nets and three-dimensional geometric shapes. For prospective mathematics teachers, the ability to perform such spatial translations is essential because it influences conceptual understanding and future instructional practices. However, correct answers in spatial tasks do not always indicate genuine conceptual understanding.

Aims: This study aims to explore and describe the thinking processes of prospective mathematics teachers when translating spatial representations, specifically from nets to geometric shapes and from geometric shapes to nets. The scope focuses on identifying qualitative characteristics of their thinking processes in spatial translation tasks.

Methods: This research employed a qualitative approach involving 12 prospective mathematics teachers enrolled in the Mathematics Education Study Program, Faculty of Mathematics, Natural Sciences, and Technology, Universitas PGRI Pontianak. Participants were selected using purposive sampling. Data were collected through spatial translation tasks followed by semi-structured interviews. Interview data were audio-recorded, transcribed verbatim, and analyzed qualitatively to identify patterns in participants' thinking processes.

Result: The findings revealed five distinct characteristics of thinking processes: (1) correct and understanding the concept, (2) correct but not understanding the concept, (3) correct through reflection, (4) wrong but understanding the concept, and (5) wrong and not understanding the concept. The results indicate that answer correctness does not necessarily reflect conceptual understanding, and that reflection plays a crucial role in reconstructing spatial concepts.

Conclusion: This study emphasizes the importance of conceptual understanding and reflective thinking in geometry instruction and provides insights for mathematics teacher education programs in designing learning experiences that develop prospective teachers' spatial reasoning skills.

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INTRODUCTION

Spatial reasoning is a fundamental component of mathematical thinking, particularly in geometry learning. It enables learners to visualize, manipulate, and mentally transform two-dimensional and three-dimensional objects (Lowrie & Logan, 2023; Supli & Yan, 2024). A substantial body of research has demonstrated that spatial reasoning plays a critical role not only in mathematics learning but also across STEM-related disciplines, positioning it as a core cognitive competence rather than a peripheral skill (Li & Wang, 2021; Zhu et al., 2023). In geometry, one important

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manifestation of spatial reasoning is the ability to translate between spatial figures and their nets. This translation process requires coordination between imagination, conceptual understanding, and reversible thinking (Mee-Kwang & Byung-Soo, 1999), as learners must mentally reconstruct three-dimensional structures from two-dimensional representations and vice versa.

For prospective mathematics teachers, mastery of this ability is essential because it directly influences how geometric concepts are understood, explained, and taught in classroom practice. Research has shown that geometry learning which explicitly emphasizes spatial reasoning contributes significantly to students' mathematical achievement and conceptual understanding (Lowrie & Logan, 2018). However, despite its recognized importance, many learners experience difficulties when required to move flexibly between different geometric representations, particularly in tasks involving reversible transformations. These difficulties suggest that spatial reasoning is often underdeveloped or treated implicitly in instruction rather than as an explicit learning objective (Mix, 2018).

Several studies further indicate that students may produce correct answers in geometric tasks without fully understanding the underlying concepts. Such responses frequently rely on visual similarity, surface-level cues, or intuitive recognition rather than deep conceptual reasoning (Vinner, 1997; Vinner & Dreyfus, 1989). This phenomenon highlights the limitation of evaluating learning outcomes based solely on correctness and underscores the importance of examining students' thinking processes when solving geometric problems, especially those involving complex spatial translations. Research in mathematics education has consistently emphasized the role of spatial reasoning as a foundational competence supporting learning in mathematics and STEM-related disciplines (Lowrie & Logan, 2018; Wai et al., 2009). Spatial reasoning has been shown to enhance learners' capacity to interpret visual information, construct and manipulate mental images, and solve geometric problems effectively (Lowrie et al., 2020; Newcombe, 2010). These findings position spatial reasoning not merely as an auxiliary skill, but as a central component of mathematical thinking that underpins conceptual understanding in geometry.

Within geometry learning, a substantial body of research has investigated learners' abilities to manipulate spatial objects and translate between different representations, particularly between two-dimensional nets and three-dimensional figures (Clements & Sarama, 2011; Uttal & Cohen, 2012). Such translation tasks require learners to coordinate visualization, orientation, and reversible transformations, making them cognitively demanding and revealing of underlying reasoning processes. Studies suggest that difficulties in these tasks often arise not from a lack of procedural knowledge, but from challenges in coordinating multiple representations simultaneously. From a cognitive and semiotic perspective, research on mathematical representation has highlighted that understanding geometry involves the ability to move flexibly between different representational systems. Gomez et al (2021) argues that mathematical comprehension depends on learners' capacity to perform conversions between representations rather than on mastery of a single form. Similarly, research on visualization emphasizes that visual representations function as tools for reasoning only when they are conceptually controlled, rather than interpreted perceptually (Eberhard, 2023; Schloss, 2025).

Several studies have further examined learners' reasoning processes in geometry and identified patterns of pseudo-conceptual or pseudo-analytical thinking, in which learners produce correct responses without possessing stable conceptual understanding (Subanji, 2007; Vinner, 1997). Such findings reveal a critical limitation of assessments that focus primarily on correctness, as correct answers may mask fragile or incomplete reasoning. In response, researchers have emphasized the importance of examining learners' thinking processes to gain deeper insight into how mathematical understanding is constructed (Kooloos et al., 2022; Wakhata et al., 2023). More recently, research on reflective thinking has suggested that reflection plays a crucial role in enabling

learners to reconstruct initial reasoning and develop new conceptual understanding after encountering errors (Subanji & Supratman, 2015). Reflection allows learners to identify inconsistencies in their reasoning and reorganize their conceptual structures, particularly in complex tasks involving spatial transformation.

Despite extensive research on spatial reasoning and geometric visualization, existing studies predominantly focus on school students or emphasize learning outcomes rather than examining the qualitative characteristics of learners' thinking processes. In particular, research that systematically investigates how prospective mathematics teachers translate between spatial figures and their nets remains limited. Moreover, previous studies have not sufficiently explored the distinction between answer correctness and conceptual understanding in reversible spatial translation tasks. Learners may produce correct solutions without stable conceptual grounding, or conversely, demonstrate emerging understanding despite incorrect answers. However, few studies have explicitly categorized these variations in thinking processes or examined the role of reflection in mediating transitions between incorrect and correct reasoning. Additionally, existing research often treats spatial reasoning as a static ability rather than as a dynamic process shaped by visualization, conceptual coordination, and reflective control. As a result, there is a lack of integrative frameworks that connect spatial reasoning, representation, pseudo-understanding, and reflective thinking within a single analytical model, particularly in the context of prospective mathematics teachers as future educators.

Although international research has firmly established the importance of spatial reasoning, its systematic development remains relatively neglected in many educational contexts, particularly in Indonesia. Spatial reasoning is often treated as a secondary outcome rather than a core instructional objective, despite its crucial role in supporting mathematical understanding and interdisciplinary competence. This study is guided by the following research questions: (1) What qualitative characteristics of thinking processes are demonstrated by prospective mathematics teachers in reversible spatial translation tasks?; (2) How are these thinking processes related to answer correctness and conceptual understanding?; (3) What role does reflection play in revising incorrect reasoning?

METHOD

Research Design

This study employed a qualitative research design aimed at exploring in depth the thinking processes of prospective mathematics teachers in solving spatial translation problems. A qualitative approach was chosen because it allows researchers to interpret participants' reasoning, representations, and reflections that cannot be captured adequately through quantitative measures (Cresswell, 2014). The focus of the study was not on measuring performance scores, but on understanding how prospective teachers think when translating between spatial figures and their nets.

Participant

The participants of this study were 12 prospective mathematics teachers enrolled in the Mathematics Education Study Program at Universitas Pontianak. The participants were selected using a purposive sampling technique, with the following criteria: (1) they had completed introductory geometry courses, (2) they were familiar with basic concepts of three-dimensional figures and nets, and (3) they were willing to participate in problem-solving tasks and interviews. These criteria were used to ensure that participants had sufficient background knowledge to engage meaningfully with the given spatial translation tasks.

Instrument

Two main instruments were used in this study: spatial translation tasks and semi-structured interview guidelines. The spatial translation tasks consisted of problems requiring participants to translate spatial figures into nets and nets into spatial figures. These tasks were designed to reveal participants' abilities in visualization, imagination, and reversible thinking. The semi-structured interviews were conducted after participants completed the tasks. The interview questions focused on participants' reasoning processes, the basis for their answers, and the strategies they used when solving the problems. The use of semi-structured interviews allowed flexibility for probing deeper into participants' thinking while maintaining consistency across interviews (Carlson & Thompson, 2005).

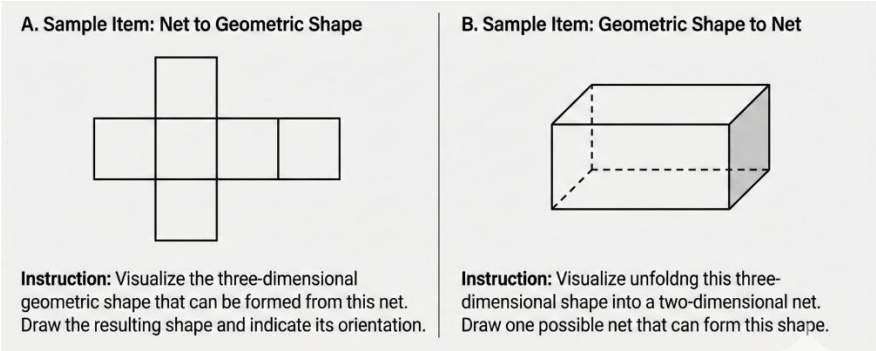

Figure 1. Example Items of Spatial Translation Tasks

Data Analysis

The semi-structured interviews were conducted after participants completed the tasks. The interview data were audio-recorded and transcribed verbatim**.** Data analysis was conducted through several systematic stages. First, participants' written responses and interview recordings were transcribed verbatim. Second, the transcripts were coded to identify observable indicators of participants' reasoning during spatial translation tasks. Examples of initial codes included "face correspondence via adjacency," "orientation inferred from folding direction," "guessing by visual similarity," "trial-and-error folding," and "reflective correction after inconsistency recognition."

Third, these codes were compared, grouped, and mapped to thinking characteristics based on the relationship between answer correctness and the nature of reasoning. For example, codes indicating consistent use of adjacency and orientation with coherent justification were categorized as correct and understanding the concept, whereas codes reflecting correct answers based primarily on visual similarity without conceptual explanation were categorized as correct but not understanding the concept. Instances in which participants initially produced incorrect answers but subsequently revised their reasoning through reflection were classified as correct through reflection. This categorization resulted in five thinking characteristics: (1) correct and understanding the concept, (2) correct but not understanding the concept, (3) correct through reflection, (4) wrong but understanding the concept, and (5) wrong and not understanding the concept.

This analytical process was guided by theoretical perspectives on pseudo-thinking, reflective reasoning, and conceptual understanding in mathematics learning (Subanji, 2007; Subanji & Supratman, 2015; Vinner, 1997). To enhance the credibility of the analysis, consistency between participants' written responses and interview explanations was systematically examined, and ambiguous cases were re-evaluated by returning to the original data.

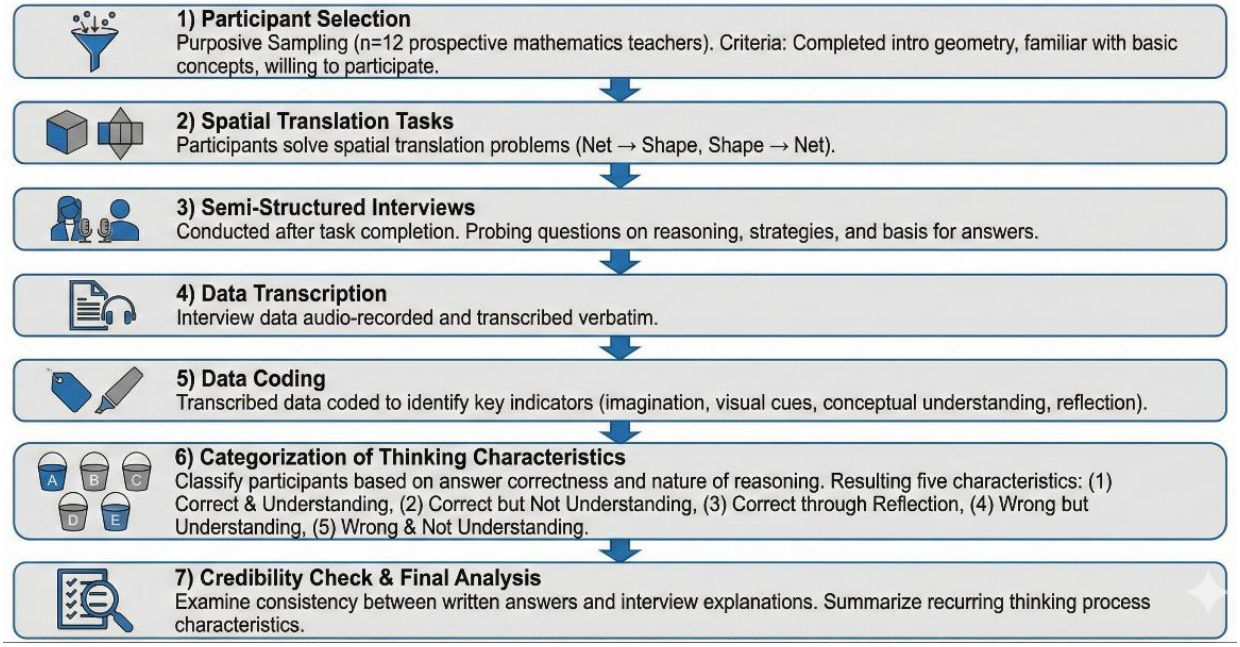

Figure 2. Research Procedure Flowchart

RESULTS AND DISCUSSION

Results

The analysis of prospective mathematics teachers' written responses and interview data revealed clear patterns in the way participants translated between spatial figures and their nets. Rather than presenting raw classifications or individual response codes, the results are summarized based on recurring characteristics of thinking processes identified across participants.

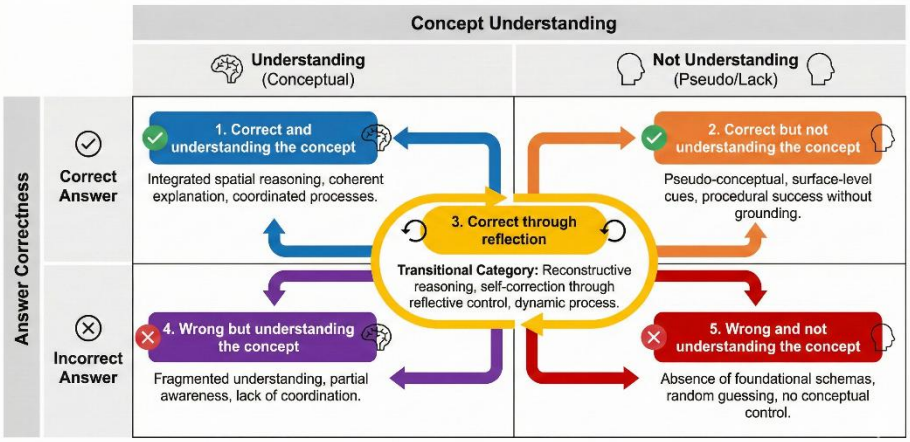

Figure 3. Conceptual Matrix of Thiking Process Characteristics

Analysis of participants' written responses and interview data revealed five qualitatively distinct thinking characteristics demonstrated by prospective mathematics teachers in reversibly translating nets into geometric shapes. These characteristics were identified based on the relationship between answer correctness and the nature of reasoning, particularly the use of visualization, conceptual understanding, and reflection. To ensure transparency and plausibility of interpretation, each category is illustrated using a short vignette, an interview excerpt (translated from Indonesian), and a description of the participant's written work.

1. Correct and Understanding the Concept

One prospective mathematics teacher produced a correct translation between the net and the geometric shape and provided a coherent explanation grounded in spatial relationships among faces, orientation, and adjacency.

"I checked which faces are adjacent first, then imagined folding the net step by step. When the base is fixed, these two faces must meet, so this net fits the shape," (P2, translated from Indonesian).

The participant's written response showed a correct selection of the net accompanied by annotations indicating face correspondence and orientation. The explanation explicitly referenced adjacency relationships and the folding sequence, demonstrating conceptually regulated visualization.

2. Correct but Not Understanding the Concept

One prospective mathematics teacher produced a correct answer in translating a net into a geometric shape; however, the reasoning relied primarily on surface-level visual cues rather than conceptual understanding.

"I chose this net because it looks the same when folded, but I didn't really think about which faces touch each other," (P4, translated from Indonesian).

The written response showed a correct selection of the net without explanations related to face correspondence, orientation, or adjacency. The correctness of the answer was not supported by conceptual justification.

3. Correct Through Reflection

One prospective mathematics teacher initially produced an incorrect net–shape translation but revised the answer after reflecting on inconsistencies between the selected net and the spatial structure of the geometric shape.

"At first I thought this net would work, but when I imagined folding it, the faces didn't meet properly. Then I realized the side faces should be adjacent like this, so I changed my answer," (P6, translated from Indonesian).

The initial written response showed an incorrect selection without clear consideration of face adjacency. After reflection during the interview, the participant revised the answer by identifying correct face correspondence and orientation, resulting in a correct spatial translation.

4. Wrong but Understanding the Concept

One prospective mathematics teacher demonstrated awareness of relevant geometric relationships but failed to integrate these elements into a coherent transformation, leading to an incorrect final answer.

"I know these faces should be next to each other, but I got confused about the direction when folding, so I wasn't sure how they meet in the shape," (P5, translated from Indonesian).

The written response showed partial identification of corresponding faces and adjacency relationships; however, incorrect orientation prevented successful spatial translation.

5. Wrong and Not Understanding the Concept

One prospective mathematics teacher was unable to produce a correct translation and showed no evidence of conceptual understanding of the spatial relationships involved.

"I'm not really sure how this net becomes the shape. I just guessed because I couldn't imagine the folding," (P3, translated from Indonesian).

The written response showed an incorrect selection without annotations or explanations related to face correspondence, adjacency, or orientation, indicating random guessing rather than conceptually guided visualization.

Table 1 synthesizes the five thinking characteristics by comparing answer accuracy, key indicators of reasoning, and typical reasoning patterns. The table highlights that correctness alone does not distinguish levels of understanding and clarifies the role of reflection as a transitional mechanism in spatial reasoning.

Table 1. Summary of Prospective Teachers' Thinking Characteristics in Spatial Translation Tasks

Thinking Category	Answer Accuracy	Key Indicators of Reasoning	Typical Reasoning Pattern
Correct and Understanding the Concept	Correct	Face adjacency, orientation, folding sequence	Conceptually regulated visualization
Correct but Not Understanding the Concept	Correct	Visual similarity, lack of justification	Surface-level matching
Correct Through Reflection	Initially incorrect, then correct	Inconsistency recognition, revision	Reflective correction
Wrong but Understanding the Concept	Incorrect	Partial adjacency, orientation errors	Fragmented conceptual coordination
Wrong and Not Understanding the Concept	Incorrect	Guessing, confusion	Absence of conceptual control

Discussion

The results of this study indicate that prospective mathematics teachers' thinking processes in spatial translation tasks are qualitatively diverse and cannot be adequately interpreted through correctness of answers alone. The five identified thinking characteristics demonstrate that spatial reasoning involves complex interactions between visualization, conceptual understanding, and reflective control, rather than a single linear ability.

Correct and Understanding the Concept

Prospective mathematics teachers in the correct and understanding the concept category demonstrated an integrated form of spatial reasoning, in which correct answers were consistently supported by coherent explanations. Visualization functioned as a conceptually regulated reasoning tool rather than mere perceptual recognition, enabling accurate orientation, identification of corresponding faces, and reversible transformations between spatial figures and their nets. Cognitively, this reflects effective coordination of visualization, orientation, and transformation processes, supporting the view that spatial reasoning involves controlled manipulation of mental representations (Bishop, 1980) and that reversibility is central to higher-order spatial cognition (Uttal & Cohen, 2012).

Epistemologically, this category represents convergence between correctness and understanding, where stable knowledge structures integrate visual representations with conceptual meaning. Such understanding reflects coherence between concept image and concept definition (Tall & Vinner, 1981), supported by epistemic control over representations, including the ability to distinguish relevant from misleading visual features (Vinner, 1997; Vinner & Dreyfus, 1989). From a theoretical perspective, this understanding aligns with Vinner (2020) dual view, as participants flexibly coordinated process and object perspectives in spatial reasoning. Pedagogically, this category represents the ideal profile of prospective mathematics teachers, highlighting the importance of fostering integrated spatial reasoning in teacher education programs (Clements & Sarama, 2011).

Correct but Not Understanding the Concept

The correct but not understanding the concept category highlights a critical epistemological separation between correctness and understanding in spatial reasoning. Although participants produced correct answers, their explanations relied on surface-level visual cues or trial-and-error strategies, indicating the absence of stable conceptual grounding. Cognitively, their reasoning reflected visual recognition rather than conceptual visualization, consistent with pseudo-conceptual thinking in which mental images are detached from formal conceptual structures (Vinner, 1997).

This phenomenon can be explained through the distinction between concept image and concept definition (Tall & Vinner, 1981). Participants' correct responses were guided by intuitive concept images without meaningful connection to formal definitions, resulting in superficial and

context-dependent reasoning. Epistemologically, correctness in this category represents procedural or accidental success rather than genuine understanding, as participants lacked epistemic control over representations and were unable to justify structural relationships (Vinner & Dreyfus, 1989).

From a theoretical perspective, this category aligns with Vinner (2020) dual view of mathematical conceptions, where process-oriented actions fail to develop into object-level conceptualization. The limitations of such reasoning are particularly evident in spatial translation tasks that require reversible thinking and coordination of representations, supporting the argument that correctness alone obscures essential aspects of reasoning (Carlson & Thompson, 2005).

Pedagogically, this category is concerning for teacher education, as equating correctness with understanding may reinforce procedural learning and discourage conceptual reasoning. Therefore, assessment and instruction must emphasize explanation, justification, and reflection to foster meaningful geometric understanding (Clements & Sarama, 2011).

Correct Through Reflection

The correct through reflection category represents a distinctive and theoretically significant thinking characteristic in spatial translation tasks. Participants initially produced incorrect answers but were able to revise their reasoning and arrive at correct solutions through reflective engagement, indicating that understanding may emerge through reflective control rather than immediate procedural execution. Cognitively, reflection functioned as a metacognitive mechanism that enabled participants to monitor, evaluate, and regulate their reasoning by re-examining mental representations and identifying inconsistencies (Flavell, 1979; Schön, 1983).

Epistemologically, this category challenges a static view of understanding by revealing it as dynamic and reconstructive. Participants reconstructed their conceptual frameworks through reflection, aligning with reflective reasoning as a mechanism for conceptual rebuilding (Subanji, 2007) and illustrating accommodation through reorganization of prior knowledge (Subanji & Supratman, 2015). This category occupies a transitional position within the spatial reasoning model, as reflection bridged fragmented understanding toward more coherent conceptualization.

Pedagogically, the findings suggest that errors should be treated as opportunities for conceptual development when reflective processes are supported. Reflection-centered learning designs provide deeper insight into learners' reasoning than correctness alone (Carlson & Thompson, 2005) and are particularly important in geometry tasks involving reversible transformations. Overall, this category highlights reflection as a key instructional lever in developing spatial reasoning.

Wrong but Understanding the Concept

The wrong but understanding the concept category reflects a nuanced form of spatial reasoning in which prospective teachers possess partial conceptual awareness but fail to produce correct solutions. Participants were able to recognize relevant geometric features, yet were unable to integrate visualization, orientation, and transformation into a coherent reversible process, resulting in incorrect answers despite emerging understanding. Cognitively, this pattern represents fragmented coordination of spatial processes, consistent with the view that spatial reasoning involves dynamic interaction among multiple components rather than a single skill (Lowrie et al., 2020).

Epistemologically, this category challenges a dichotomous view of understanding as present or absent. Learners demonstrated genuine but insufficient understanding that failed at the level of integration, reflecting a mismatch between image-based intuitions and formal structure (Vinner, 1997). Unlike correct but not understanding the concept, participants showed conceptual sensitivity without procedural success, underscoring that conceptual awareness alone is insufficient without operational coordination (Vinner & Dreyfus, 1989).

Pedagogically, this category highlights the need for instruction that emphasizes coordination and integration of spatial processes rather than isolated concepts. Learners in this group represent high potential for conceptual consolidation through targeted scaffolding, such as guided transformations and representational integration, which are essential for developing robust spatial reasoning in teacher education contexts (Clements & Sarama, 2011).

Wrong and Not Understanding the Concept

The wrong and not understanding the concept category represents the most fundamental breakdown of spatial reasoning. Participants were unable to produce correct answers or meaningful explanations, indicating the absence of both conceptual understanding and controlled visualization. Cognitively, this reflects a failure to activate essential spatial processes such as correspondence, orientation, and adjacency, suggesting the absence of foundational spatial schemas required for reversible spatial tasks (Battista, 2007; Bishop, 1980).

Consistent with Battista (2007), the lack of coordinated mental structures prevented participants from interpreting or manipulating geometric representations meaningfully, resulting in reasoning disconnected from the mathematical structure of the task. Epistemologically, this category signifies the absence of epistemic access to spatial meaning; neither concept image nor concept definition functioned productively (Vinner, 1997). Unlike cases of accidental correctness, this category reflects structural failure rather than surface-level success.

From a representational perspective, visual representations failed to serve as tools for reasoning due to the absence of conceptual control (Arcavi, 2003). Pedagogically, this category is particularly concerning for teacher education, as foundational deficits in spatial reasoning may transfer into future classroom practice. Therefore, targeted foundational reconstruction through guided visualization and concrete experiences is essential, reinforcing that spatial thinking can be systematically developed through appropriate instruction (Clements & Sarama, 2011; Uttal & Cohen, 2012).

Conceptual Model of Prospective Teachers' Spatial Thinking

Prospective teachers' spatial thinking in translation tasks can be understood as a dynamic and non-linear model rather than a rigid hierarchical structure. Each category reflects a particular cognitive position that emerges from the interplay between visualization ability, conceptual understanding, and reflective control. This perspective highlights that spatial reasoning develops through complex interactions among these components, rather than progressing in a simple, step-by-step sequence.

The category of wrong and not understanding the concept represents a condition in which foundational spatial schemas are not yet established. At this stage, prospective teachers struggle to form meaningful mental representations, leading to incorrect interpretations of spatial transformations. Meanwhile, the category of wrong but understanding the concept illustrates a partial or fragmented understanding. In this case, individuals may grasp certain conceptual elements but fail to coordinate them effectively, resulting in errors during the translation process.

On the other hand, the category of correct but not understanding the concept reflects procedural success without deeper epistemic control. Prospective teachers may arrive at correct answers by following rules or patterns, yet lack a clear conceptual basis for their reasoning. This condition emphasizes that correctness alone does not guarantee true understanding, as noted by Carlson and Thompson (2005) and Vinner (1997), where procedural fluency can exist independently from conceptual comprehension.

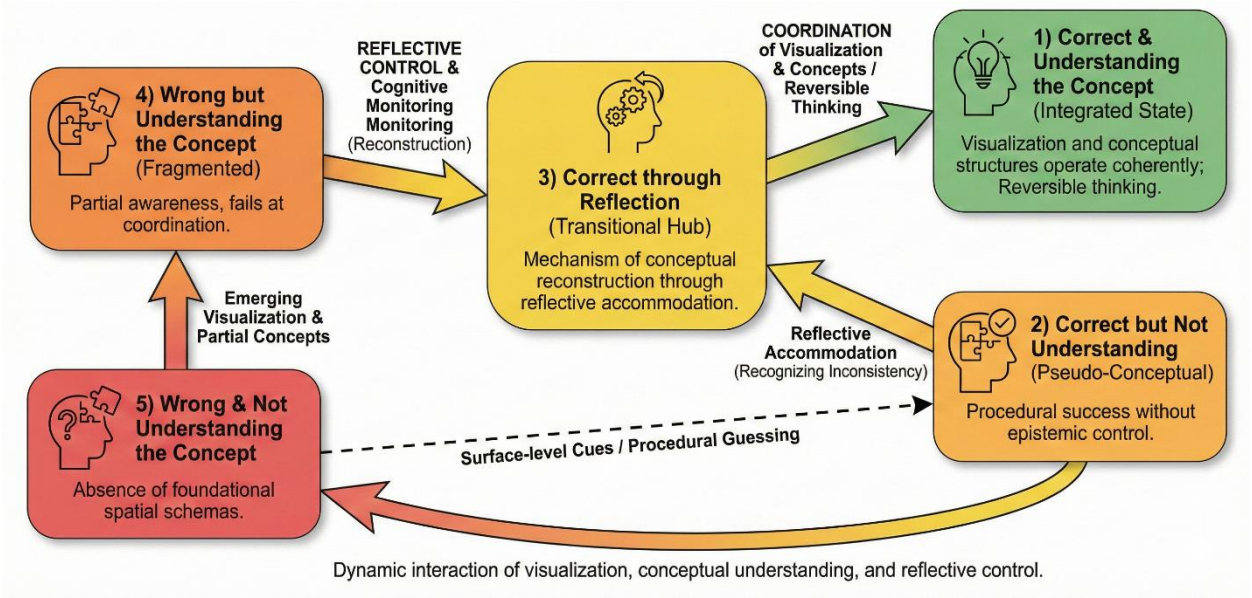

Figure 4. A Dynamic Conceptual Model of Prospective Teachers' Spatial Thinking Trajectories

The correct through reflection category represents a distinct cognitive position characterized by conceptual reconstruction through reflective accommodation (Subanji, 2007; Subanji & Supratman, 2015). In contrast, the correct and understanding the concept category reflects an integrated reasoning profile in which visualization and conceptual structures operate coherently, supported by reversible thinking (Bishop, 1980; Uttal & Cohen, 2012).

Overall, this model conceptualizes prospective teachers' spatial thinking as a pattern space of qualitatively different reasoning profiles, shaped by varying degrees of conceptual control and reflection. Rather than implying developmental movement over time, the model captures observable positions of reasoning manifested during spatial translation tasks and interviews, highlighting how reflection and coordination of spatial processes differentiate these positions.

Implications

The findings have important implications for mathematics teacher education. First, teacher education programs should emphasize conceptual understanding and reflective thinking rather than focusing solely on correct answers. Second, instructional practices should explicitly support the coordination of visualization, orientation, and reversible thinking in geometry learning (Clements & Sarama, 2011). Third, reflective activities such as guided questioning and discussion should be integrated to support conceptual reconstruction, particularly for prospective teachers who initially demonstrate incorrect reasoning.

Limitations

This study involved a small number of participants from a single institution, limiting the generalizability of the findings. In addition, the study focused only on spatial translation tasks involving nets and geometric shapes, without examining other forms of spatial reasoning such as rotation or perspective-taking. Data were also limited to written responses and interviews.

Suggestions

Future research should involve larger and more diverse samples across multiple institutions to validate the identified thinking characteristics. Further studies could also explore prospective teachers' thinking processes in other spatial reasoning tasks and examine instructional interventions designed to promote reflective thinking and conceptual integration. Incorporating additional data sources such as think-aloud protocols or classroom observations may provide deeper insight into the dynamics of spatial reasoning development.

CONCLUSION

This study demonstrates that prospective mathematics teachers exhibit qualitatively diverse thinking processes when translating between spatial figures and their nets, indicating that correct answers do not necessarily reflect conceptual understanding. The identified thinking characteristics highlight spatial reasoning as a non-linear reasoning profile shaped by visualization, conceptual coordination, and reflective control, with reflection functioning as a key mechanism for revising incomplete or incorrect reasoning. These findings have concrete implications for mathematics teacher education. First, assessment practices in geometry should move beyond answer correctness by incorporating rubrics that explicitly require explanations of face correspondence, orientation, and folding sequences. Second, instructional tasks should include structured reflective prompts that encourage prospective teachers to examine inconsistencies in their reasoning and revise their solutions. Third, diagnostic spatial translation tasks can be used to identify distinct reasoning profiles, enabling instructors to provide targeted feedback rather than uniform remediation. Together, these implications suggest that effective geometry instruction should systematically attend to thinking processes, not merely final answers, in preparing future teachers to support meaningful spatial reasoning.

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AUTHOR CONTRIBUTIONS STATEMENT

SA conceived the research idea, designed the study, developed the research instruments, conducted data collection, and performed data analysis. SF contributed to the development of the theoretical framework and supported data interpretation related to spatial reasoning. DR reviewed the methodology, validated the findings, and provided critical revisions to improve the academic quality of the manuscript. AN contributed to the literature review, data organization, and assisted in refining the discussion and conclusions. All authors contributed to the final manuscript and approved the submitted version.

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