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Profiling students' mathematical reasoning in the alkhairaat context using rasch analysis: A mixed-methods study

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Abstract

Background: Students' mathematical reasoning is often limited by their difficulty in linking abstract mathematical concepts with meaningful real-life contexts. In many classrooms, mathematics is perceived as disconnected from students' sociocultural experiences, which can hinder the development of deeper reasoning processes.
Aims: This study aims to profile students' mathematical reasoning abilities within the Alkhairaat sociocultural context and to examine how contextual integration influences reasoning across different ability levels.
Method: A sequential explanatory mixed-methods design was employed involving 33 eighth-grade students in a madrasah. Quantitative data were collected through a mathematical reasoning test embedded in the Alkhairaat context and analyzed using the Rasch measurement model (Ministep 5.10.4.0) to evaluate the instrument's psychometric quality and map students' reasoning abilities. Qualitative data were then obtained through clinical interviews to explore students' cognitive processes when solving contextual problems.
Results: Rasch analysis indicated acceptable psychometric properties, with item reliability of 0.67 and person reliability of 0.58. Wright Map analysis categorized students into three ability levels: high (12.12%), medium (66.67%), and low (21.21%). The findings show that the Alkhairaat context functions differently across ability levels: as a cognitive tool for abstraction among high-ability students, as procedural scaffolding for medium-ability students, and primarily as an affective anchor for low-ability students.
Conclusion: Integrating sociocultural contexts such as Alkhairaat values can support mathematical reasoning by providing meaningful entry points for abstraction. However, its effectiveness depends on students' initial proficiency, indicating the need for additional instructional support for lower-ability learners.

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INTRODUCTION

Mathematical reasoning is widely recognized as a fundamental component of meaningful mathematics learning (Engelbrecht & Borba, 2024). Through reasoning, students are able to interpret relationships among mathematical concepts, formulate conjectures, justify procedures, and construct logical conclusions. These abilities are essential for developing higher-order thinking skills,

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particularly when students begin to encounter more abstract mathematical ideas at the secondary school level (Hidajat, 2021; Kwangmuang et al., 2021).

Despite its importance, many students still experience difficulties in demonstrating strong mathematical reasoning. In classroom practice, learners often concentrate on applying formulas or computational procedures without fully understanding the conceptual relationships underlying a problem (Shin et al., 2021). Consequently, correct answers may sometimes be obtained through routine procedures rather than through well-developed logical reasoning.

This difficulty is partly influenced by the way mathematical problems are commonly presented in instruction (Fanchamps et al., 2021; Kaitera & Harmoinen, 2022). Mathematical tasks are frequently introduced in abstract symbolic forms that appear distant from students' everyday experiences. When learners struggle to relate mathematical ideas to familiar contexts, translating real-world situations into mathematical representations becomes more challenging (Sevinc, 2023).

In madrasah environments, students' learning experiences are closely connected to religious traditions and sociocultural values. Within such contexts, integrating culturally meaningful elements into mathematics learning may provide a bridge between abstract mathematical ideas and students' lived experiences (Kolovou, 2023). Contextual learning that draws upon familiar values may help students interpret mathematical situations more meaningfully while supporting the development of reasoning processes. One sociocultural context that plays a significant role in Islamic education in Eastern Indonesia is the educational tradition of Alkhairaat. As one of the largest Islamic educational organizations in the region, Alkhairaat has shaped the educational and cultural identity of many students in madrasah institutions. Although Alkhairaat values are commonly taught through subjects related to institutional history and religious education, these values are rarely integrated into mathematics learning.

A growing body of research highlights the importance of contextual learning approaches in supporting students' mathematical reasoning (Basid et al., 2024; Hunter et al., 2023). Studies in ethnomathematics indicate that cultural contexts can serve as cognitive entry points that help students interpret mathematical ideas through experiences that are already familiar to them. When mathematical tasks are connected to meaningful contexts, students tend to engage more actively in reasoning and are better able to construct relationships between mathematical representations and real-life situations (Manfreda Kolar & Hodnik, 2021). Alongside contextual approaches, developments in educational measurement have provided more robust methods for analyzing students' cognitive abilities. One widely used approach in educational research is the Rasch measurement model (Tennant & Küçükdeveci, 2023). This model allows researchers to evaluate the quality of assessment instruments and to map students' abilities and item difficulties on a common measurement scale. By calibrating these elements simultaneously, Rasch analysis provides a more objective representation of students' performance (Chan et al., 2021). Furthermore, studies investigating mathematical reasoning increasingly employ mixed-methods designs to capture both quantitative patterns and qualitative insights (Buchholtz & Vollstedt, 2024). Quantitative analyses can identify variations in students' ability levels, while qualitative investigations allow researchers to explore the cognitive processes underlying students' reasoning strategies. The integration of these approaches therefore enables a more comprehensive understanding of students' reasoning development.

Although previous research has examined contextual learning and culturally responsive approaches in mathematics education, several limitations remain (Nolan & Xenofontos, 2023). Many studies investigating cultural contexts in mathematics primarily focus on traditional cultural practices such as crafts, architecture, or community activities. In contrast, the integration of religious-based sociocultural values into mathematics learning has received relatively limited empirical attention (Ma'ruf & Juhaidi, 2025). In addition, a large proportion of existing studies emphasize evaluating the effectiveness of specific instructional models rather than examining students' reasoning profiles in detail. While such research contributes valuable insights into teaching strategies, it often provides limited information about how students' reasoning abilities vary across different levels of mathematical proficiency. Another limitation concerns the methodological approaches used in previous research. Studies that combine rigorous psychometric measurement with qualitative exploration of students' reasoning processes remain relatively scarce (Uher, 2021). Without integrating these approaches, understanding how contextual learning environments influence students' reasoning development becomes less comprehensive. Considering these limitations, examining students' mathematical reasoning within a sociocultural learning context becomes an important research direction (Aksu & Zengin, 2022; Wang, 2025). The Alkhairaat context provides a culturally meaningful environment that may help students interpret mathematical problems through references that are already familiar in their educational experiences. By incorporating this context into mathematical reasoning tasks, it becomes possible to explore how students connect contextual narratives with formal mathematical representations. At the same time, employing Rasch analysis enables the objective mapping of students' reasoning abilities, while qualitative investigation provides insight into the cognitive processes underlying their responses.

Based on the considerations described above, this study aims to profile students' mathematical reasoning abilities within the Alkhairaat sociocultural context using Rasch analysis within a mixed-methods framework. The study examines the psychometric quality of a mathematical reasoning instrument embedded in the Alkhairaat context, maps students' reasoning ability levels through Rasch measurement, and explores how contextual elements influence students' reasoning processes across different levels of mathematical proficiency.

METHOD

Research Design

This study employed a sequential explanatory mixed-methods design to obtain a comprehensive understanding of students' mathematical reasoning abilities within the Alkhairaat context. In this design, the quantitative phase was conducted first to identify patterns in students' reasoning performance, followed by a qualitative phase aimed at explaining the cognitive processes underlying the quantitative findings. During the quantitative stage, students completed a mathematical reasoning test embedded in contextual situations related to Alkhairaat values. The responses were analyzed using the Rasch measurement model to examine the psychometric quality of the instrument and to map students' reasoning ability levels. After the quantitative analysis was completed, the qualitative phase was carried out through clinical interviews with selected students. This stage aimed to explore how students interpreted contextual problems and how the Alkhairaat context influenced their reasoning strategies. The overall structure of the sequential explanatory procedure is illustrated in Figure 1, which shows the relationship between the quantitative and qualitative stages of the research.

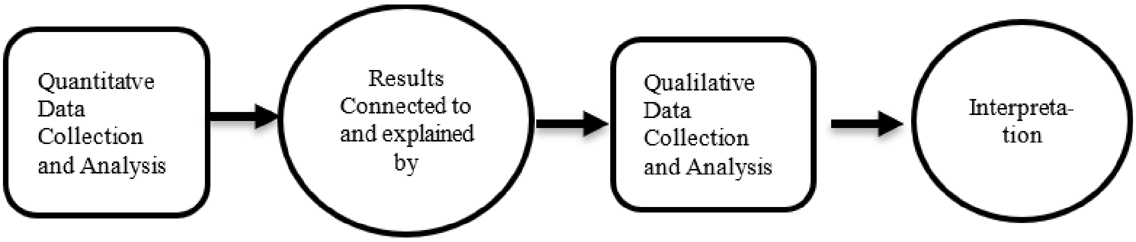

Figure 1. Sequential-explanatory mixed-methode

Participants

The participants in this study were eighth-grade students from Madrasah Tsanawiyah (MTs) Alkhairaat. A total of 33 students were involved in the quantitative phase of the study. All students were enrolled in the same class and had previously studied the topic of systems of linear equations in two variables. Participants for the quantitative phase were selected using total sampling, as the study aimed to examine the reasoning profile of students within a specific educational context. For the qualitative phase, purposive sampling was applied in order to select students representing different levels of mathematical reasoning ability. Based on the Rasch person measure results, students were grouped into three ability categories: high, medium, and low. Two students from each category were then selected as key informants for the interview stage. This selection allowed the researchers to investigate variations in reasoning processes across different levels of mathematical proficiency.

Research Procedure

The research was conducted in two main stages in accordance with the sequential explanatory mixed-methods design. In the first stage, the quantitative phase was carried out by administering a mathematical reasoning test that incorporated contextual problems related to Alkhairaat values. Students completed the test individually during a regular classroom session. The responses were then scored using an analytical rubric and analyzed using the Rasch measurement model to determine the characteristics of the test items and the distribution of students' reasoning abilities. In the second stage, the qualitative phase was conducted after the quantitative results had been obtained. Students selected from each ability category participated in semi-structured clinical interviews. During the interviews, students were asked to explain how they interpreted the contextual problems and how they constructed their mathematical solutions. The interviews were conducted using a guided think-aloud approach, allowing students to verbalize their reasoning processes while discussing their answers. Each interview lasted approximately 20–30 minutes and was audio-recorded. The recordings were later transcribed verbatim for further analysis.

Instruments

The primary instrument used in this study was a mathematical reasoning test in essay format designed around contextual situations derived from Alkhairaat sociocultural values. The test focused on problems related to systems of linear equations in two variables (SPLDV). The contextual tasks incorporated elements from Alkhairaat historical narratives and sociocultural practices in order to create meaningful problem situations. One example item required students to translate information from a historical narrative about Guru Tua into mathematical variables and determine the solution using algebraic reasoning. To ensure the validity of the instrument, content validation was conducted by three experts, consisting of a mathematics education expert, an educational evaluation specialist, and a scholar familiar with the historical and cultural context of Alkhairaat. The validation process evaluated the clarity of the items, their alignment with mathematical reasoning indicators, and the appropriateness of the contextual elements.

The content validity of the instrument was calculated using the Content Validity Index (CVI). The results showed that all items achieved I-CVI = 1.00 and S-CVI = 1.00, indicating excellent agreement among the validators.

Table 1. Calculation of Item-Level Content Validity Index (I-CVI) and Universal Agreement (UA)

Assessed aspects	Expert 1	Expert 2	Expert 3	Expert In agreement	I-CVI	Category	UA
1	1	1	1	3	1	Valid	1
2	1	1	1	3	1	Valid	1
3	1	1	1	3	1	Valid	1

Assessed aspects	Expert 1	Expert 2	Expert 3	Expert In agreement	I-CVI	Category	UA
4	1	1	1	3	1	Valid	1
5	1	1	1	3	1	Valid	1
6	1	1	1	3	1	Valid	1
7	1	1	1	3	1	Valid	1
8	1	1	1	3	1	Valid	1
Total Every Expert	1	1	1				
S-CVI/Average	1						

Table 2. S-CVI and UA Values for Content Validity

Sum of I-CVI	**8**	**Sum of UA**	**8**
S-CVI/ Average	1	Sum of UA	1
Category	Valid	Category	Valid

Each test item was also aligned with specific indicators of mathematical reasoning, including constructing conjectures, performing mathematical abstraction, and drawing logical conclusions. These indicators were integrated with contextual situations derived from Alkhairaat values.

Table 3. Integrated Operational Mapping of Mathematical Reasoning Indicators Based on Alkhairaat Values

No	Reasoning Indicator	Item No.	Alkhairaat Value Context	Focus of Problem	Max Score	Reference
1	Constructing Conjectures	1	Socio-religious interaction during the Haul of Guru Tua (transaction logic).	Estimation of geometric variables in contextual situations.	4	Saleh Haji (2019)
		2	Socio-religious interaction during the Haul of Guru Tua (transaction logic).	Logical estimation of prices and comparison of items.	4	Saleh Haji (2019)
2	Mathematical Abstraction	3	Al-Qur'an surah classification within the Madrasah curriculum.	Algebraic modeling of religious facts into SPLDV.	4	Surya & Andriana Putri (2017)
		4	Qiyamul Lail practices (Tahajjud and Witir) among santri.	Determination of variables derived from sunnah-based values.	4	Surya & Andriana Putri (2017)
3	Drawing Logical Conclusions	5	Historical biography of Sayyid Idrus bin Salim Al-Jufri (Guru Tua) and significant life events.	Verification of historical facts through mathematical logic.	4	Zebua (2020); Surya & Andriana Putri (2017)
		6	Lessons from the Prophet Muhammad's (PBUH) resilience and strategy during the Hijrah (Cave of Thawr context).	Critical analysis and decision-making based on SPLDV results.	4	Zebua (2020)

Students' responses were evaluated using an analytical scoring rubric with a polytomous scale ranging from 0 to 4. This rubric allowed the researchers to distinguish different levels of reasoning performance, ranging from incomplete responses to well-structured logical explanations.

Data Analysis

Quantitative Analysis

Quantitative data were analyzed using the Rasch measurement model in order to estimate students' reasoning ability and item difficulty on a common logit scale. The analysis was conducted using Ministep software, which provides several outputs relevant to educational measurement. The Rasch analysis produced indicators such as person reliability, item reliability, item difficulty estimates, and person ability measures. These results were used to evaluate the psychometric quality of the instrument and to classify students' reasoning abilities into different categories. The Wright Map generated from the Rasch model was also used to visualize the relationship between students' ability levels and item difficulty.

Qualitative Analysis

Qualitative data obtained from the interviews were analyzed using the interactive analysis model proposed by Miles and Huberman, which involves three main stages: data condensation, data display, and conclusion drawing. During the analysis, students' written responses were compared with their verbal explanations obtained during the interviews. This process allowed the researchers to examine how students interpreted contextual problems and how the Alkhairaat context influenced their reasoning processes. Finally, the findings from the quantitative and qualitative analyses were integrated in the discussion section to construct a comprehensive profile of students' mathematical reasoning.

RESULTS AND DISCUSSION

Results

Quantitative Profile of Students' Mathematical Reasoning

The quantitative phase of this study aimed to describe students' mathematical reasoning abilities using Rasch measurement. The analysis was conducted on responses from 33 students who completed six contextual essay items designed to assess mathematical reasoning within situations related to Alkhairaat values. The Rasch analysis produced several indicators describing the measurement quality of the instrument as well as the distribution of students' abilities. The results indicate that the person reliability coefficient was 0.58, while the item reliability coefficient reached 0.67. These values suggest that the instrument demonstrates an adequate level of consistency in distinguishing students with different reasoning abilities. Although the person reliability value indicates moderate separation among participants, the measurement results remain sufficient to describe patterns of reasoning performance within the sample. One advantage of Rasch measurement is that both item difficulty and student ability can be estimated on the same logit scale, allowing a clearer interpretation of how well the test items represent the range of students' reasoning abilities.

Item Difficulty and Model Fit

An item analysis was conducted to examine the difficulty level and statistical fit of each test item within the Rasch model. The results show that the difficulty of the six items ranges from −0.85 logits to +0.80 logits, indicating that the instrument contains items of varying levels of challenge. The fit statistics further demonstrate that all items fall within the acceptable range for Infit and Outfit Mean Square (MNSQ) values. This finding indicates that the items function consistently in measuring the intended construct of mathematical reasoning. None of the items exhibited substantial misfit, suggesting that each item contributes meaningfully to the overall measurement.

In addition, the Point Measure Correlation values for all items were positive, which indicates that the items are aligned with the construct being measured. Among the six items, Item E5 appears as the most demanding task, while Item E2 is identified as the least difficult. The presence of items with different levels of difficulty suggests that the instrument is capable of capturing variations in students' reasoning performance.

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TABLE 13.1 analisis penalaran siswa              ZOU359WS.TXT  Jan 07 2026 21:44
INPUT: 33 Person  6 Item  REPORTED: 33 Person  6 Item  4 CATS  MINISTEP 5.10.4.0
--------------------------------------------------------------------------------
Person: REAL SEP.: 1.17  REL.: .58 ... Item: REAL SEP.: 1.44  REL.: .67

         Item STATISTICS:  MEASURE ORDER

--------------------------------------------------------------------------------
----------
|ENTRY   TOTAL  TOTAL    JMLE    MODEL|   INFIT   |  OUTFIT   |PTMEASUR-AL|EXACT
MATCH|       |
|NUMBER  SCORE  COUNT  MEASURE  S.E. |MNSQ  ZSTD|MNSQ  ZSTD|CORR.  EXP.| OBS%
EXP%| Item |
|-------------------------------------+----------+----------+-----------
+-----------+------|
|     5    56     23      .80     .32|1.17   .68|1.17   .70|  .58   .62| 26.1
51.3| E5   |
|     1    81     33      .39     .27| .84  -.64| .82  -.73|  .72   .65| 57.6
54.7| E1   |
|     6    52     20      .15     .34|1.22   .78|1.22   .78|  .51   .53| 55.0
55.0| E6   |
|     4    73     28      .08     .29| .97  -.01| .99   .04|  .68   .64| 75.0
55.4| E4   |
|     3    83     29     -.56     .30| .68 -1.30| .70 -1.19|  .69   .61| 65.5
59.7| E3   |
|     2    94     32     -.85     .29|1.17   .74|1.08   .37|  .52   .62| 62.5
59.6| E2   |
|-------------------------------------+----------+----------+-----------
+-----------+------|
| MEAN   73.2   27.5      .00     .30|1.01   .04|1.00   .00|           | 56.9
56.0|        |
| P.SD   14.9    4.6      .56     .02| .20   .79| .19   .73|           | 15.2
2.9|       |
--------------------------------------------------------------------------------
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```

Figure 2. item fit

Distribution of Students' Reasoning Ability

The Rasch person measures were used to classify students' mathematical reasoning abilities into three categories: high, medium, and low. The categorization was based on the logit values obtained from the Rasch analysis.

Table 4. Distribution of Students' Mathematical Reasoning Ability Levels based on Person Measure Logit (n=33)

Ability Category	Logit Range (Person Measure)	Number of Students (f)	Percentage (%)
High	> +1.25 logit	4	12.12%
Medium	-0.75 to +1.25 logit	22	66.67%
Low	< -0.75 logit	7	21.21%
Total		33	100%

The distribution presented in Table 4 shows that the majority of students fall into the medium reasoning category, accounting for 66.67% of the participants. A smaller group of students demonstrates high reasoning ability (12.12%), while 21.21% of the students are categorized as having relatively low reasoning ability. These results indicate that most students are able to engage with contextual mathematical problems and apply appropriate procedures to reach solutions. However, many of them still experience difficulty when required to provide detailed logical explanations that justify their mathematical steps.

The relatively small number of students in the high category suggests that advanced reasoning skills, such as constructing generalized arguments and articulating formal justifications, remain challenging for a large proportion of learners.

Person–Item Distribution Based on the Wright Map

To further examine the relationship between students' abilities and item difficulty, the Rasch analysis generated a Wright Map, also known as a Person–Item Map.

Figure 3. Wright Map of Person Ability and Item Difficulty Levels

The Wright Map shows that students' ability levels extend from approximately −3 logits to +4 logits, with the highest concentration of students located between −1 and +1 logits. This pattern is consistent with the earlier classification indicating that most participants belong to the medium reasoning category. The map also provides insight into the relative difficulty of the test items. For example, Item 5, with a difficulty value of approximately +0.80 logits, appears as the most challenging item in the instrument. This task requires students to interpret historical contextual information related to Guru Tua and transform it into algebraic representations, which demands a higher level of abstraction.

In contrast, Item 2, located at −0.85 logits, is identified as the easiest item. The contextual situation in this problem is closely related to everyday experiences, making it easier for students to represent the situation mathematically. Another observation from the Wright Map concerns the distribution of student abilities relative to item difficulty. A small number of students appear at the extreme ends of the scale, suggesting that additional items with broader levels of difficulty could further improve the sensitivity of the instrument.

Profiles of Mathematical Reasoning Across Ability Levels

Beyond identifying ability levels, this study also examines how the Alkhairaat context interacts with students' reasoning processes across different categories of ability.

Table 5. Comparing Alkhairaat Context's Function at Different Ability Levels

Subject Category	Role of Alkhairaat Context	Cognitive Outcomes
High	Cognitive Instrument	Strong Abstraction & Modeling
Moderate	Bridge for Procedures (Scaffolding)	Execution of Fluent Algorithms
Low	Affective Anchor: Surface Context	Restricted to Knowledge and Love

As summarized in Table 5, the contextual elements appear to play different roles depending on students' reasoning proficiency. Students in the high ability group tend to use the contextual information as a cognitive tool that supports abstraction and mathematical modeling. These students are able to identify relevant quantitative relationships within the narrative context and translate them into formal mathematical expressions. Students in the medium ability group demonstrate a different pattern. For them, the contextual situation often serves as a procedural bridge that helps initiate the problem-solving process. While they are generally able to apply mathematical procedures such as substitution or elimination correctly, their explanations frequently remain procedural rather than conceptual.

A contrasting pattern is observed among students in the low ability group. In this group, the contextual narrative primarily functions as a surface-level reference that provides familiarity but does not substantially support the construction of mathematical reasoning. Although the context appears to increase engagement, many students still struggle to transform contextual information into mathematical representations. These differences suggest that the role of contextual learning environments may vary depending on students' existing levels of mathematical proficiency.

These differences suggest that the role of contextual learning environments may vary depending on students' existing levels of mathematical proficiency. To further illustrate these variations in reasoning processes, examples of students' written responses are presented in the following figures.

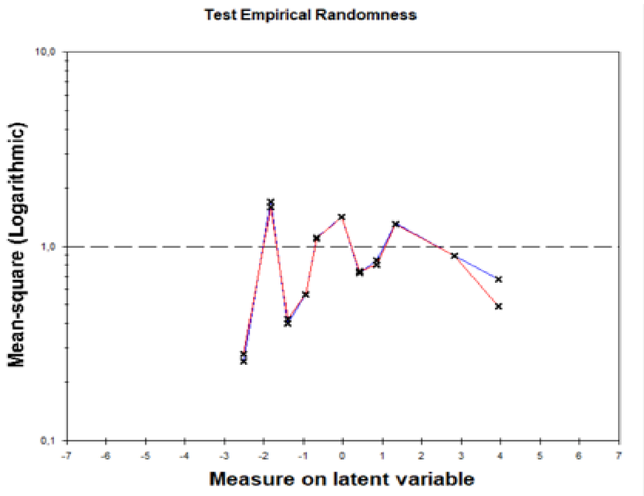

Figure 4. DIF Technical Data

Figure 4 shows an example of a response produced by a student categorized in the high reasoning group. In this response, the student successfully identifies relevant information from the contextual narrative and translates it into appropriate mathematical expressions. The solution process demonstrates a coherent reasoning structure, including the formulation of equations and the logical derivation of the final result.

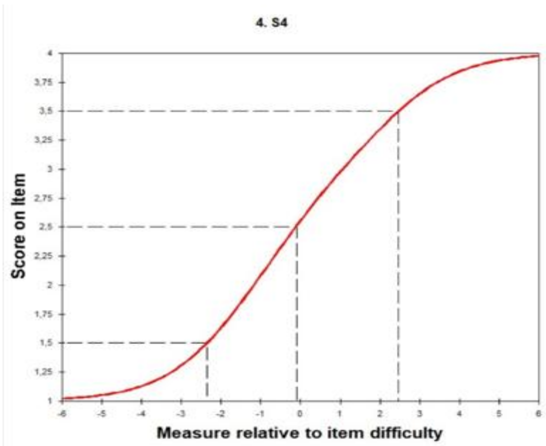

Figure 5. ICC Visualization

In contrast, Figure 5 illustrates the work of a student from a lower reasoning category. Although the student attempts to apply a mathematical procedure, several steps in the reasoning process remain incomplete or insufficiently justified. This example highlights how some students struggle to transform contextual information into formal mathematical representations.

Discussion

The present study was conducted to examine how students demonstrate mathematical reasoning when solving contextual problems grounded in the Alkhairaat sociocultural environment. By combining Rasch measurement with qualitative insights from students' responses, the findings provide a more detailed view of how learners interpret contextual information and transform it into mathematical representations (Ramadhani et al., 2022).

The quantitative results show that most students fall within the medium reasoning category, while only a small number demonstrate highly developed reasoning abilities. This pattern suggests that many students are capable of applying mathematical procedures to reach correct answers, yet their reasoning explanations often remain incomplete (Rohati et al., 2023). In many cases, students rely on familiar procedures without clearly articulating the underlying relationships among mathematical concepts. Such a pattern reflects a common situation in mathematics classrooms, where procedural knowledge tends to develop earlier than students' ability to construct formal reasoning and justification.

The Wright Map analysis further illustrates how students' abilities are distributed relative to the difficulty of the contextual problems. Most students appear in the middle range of the ability scale, indicating that the instrument is generally suitable for capturing differences in reasoning performance within the group. Nevertheless, the presence of a few students at both the highest and lowest ends of the scale suggests that reasoning abilities among learners remain highly varied. This variation highlights the importance of instructional strategies that address differences in students' conceptual understanding (Banda & Nzabahimana, 2021).

An important aspect revealed in this study concerns the role played by the Alkhairaat contextual setting in supporting students' reasoning processes. For students categorized in the high reasoning group, the contextual narrative appears to function as a meaningful reference that helps them organize mathematical ideas (Arnesen & Rø, 2024). These students are able to identify relevant information from the contextual description and transform it into mathematical expressions. In this situation, the sociocultural context does not merely provide background information but becomes part of the cognitive process through which mathematical abstraction is constructed.

Students in the medium reasoning category display a slightly different pattern. For them, the contextual situation often acts as an entry point that facilitates the initiation of problem solving (Raisch & Fomina, 2025). The narrative helps students recognize relevant quantities and relationships, which then leads them to apply familiar algebraic procedures. However, their reasoning explanations frequently remain focused on computational steps rather than on the conceptual structure of the problem.

A contrasting tendency can be observed among students with lower reasoning ability. Although the contextual narrative appears to increase students' interest and familiarity with the problem situation, many of them encounter difficulties when attempting to translate the contextual information into mathematical form (Manfreda Kolar & Hodnik, 2021). As a result, the context functions primarily as a source of familiarity rather than as a cognitive tool for constructing mathematical reasoning. Taken together, these findings suggest that the integration of sociocultural contexts such as the Alkhairaat tradition can enrich mathematics learning by making abstract ideas more meaningful for students. However, the effectiveness of contextual approaches appears to depend on students' existing mathematical proficiency. Contextual familiarity may help students engage with the problem situation, but additional instructional guidance is often necessary to support the development of deeper reasoning processes. In this sense, contextual learning environments should not be viewed solely as motivational tools. Instead, they should be accompanied by instructional strategies that guide students in identifying mathematical relationships embedded within contextual narratives (citation needed). When combined with appropriate pedagogical support, sociocultural contexts may provide valuable opportunities for strengthening students' mathematical reasoning.

Implications

The findings of this study offer several implications for mathematics education, particularly in learning environments where sociocultural values are closely connected to students' educational experiences. The results indicate that contextual situations derived from the Alkhairaat tradition can help students engage with mathematical problems by providing narratives that are familiar within their learning context. For students with stronger reasoning ability, the contextual information appears to support the process of identifying mathematical relationships and constructing formal representations. However, the findings also show that contextual familiarity alone does not automatically lead to deeper reasoning development. Many students still require explicit instructional guidance in order to translate contextual information into mathematical models. These results suggest that the integration of sociocultural contexts in mathematics learning should be accompanied by structured reasoning activities that guide students in connecting contextual narratives with mathematical structures. When implemented thoughtfully, culturally meaningful contexts may provide valuable opportunities to strengthen students' mathematical reasoning in madrasah education.

Limitations

Several limitations should be considered when interpreting the results of this study. First, the participants were drawn from a single madrasah with a relatively limited number of students, which may restrict the extent to which the findings can be generalized to other educational settings. Second, the reasoning tasks used in this study focused on contextual problems related to systems of linear equations in two variables. Although this topic provided an appropriate context for examining students' reasoning processes, different mathematical topics may produce different patterns of reasoning performance. In addition, the qualitative phase involved a limited number of interview participants selected to represent different ability levels. While this approach allowed for deeper insight into students' reasoning processes, further studies involving larger samples and broader contexts would help provide a more comprehensive picture of how sociocultural learning environments influence mathematical reasoning.

Suggestions

Future studies may extend this line of research in several directions. Investigations involving larger and more diverse samples would help determine whether similar patterns of reasoning emerge in different madrasah or school contexts. Further research may also explore the integration of sociocultural contexts in other areas of mathematics learning beyond linear equations, including topics that require different forms of reasoning. In addition, future studies could examine instructional approaches that explicitly guide students in transforming contextual information into formal mathematical representations. Such efforts may contribute to a deeper understanding of how culturally meaningful learning environments can be systematically integrated into mathematics instruction in order to strengthen students' reasoning abilities.

CONCLUSION

This study examined students' mathematical reasoning by situating problem-solving activities within the sociocultural context of Alkhairaat and analyzing the results through Rasch measurement combined with qualitative exploration. The findings indicate that students' reasoning abilities are distributed across different levels, with most students demonstrating a moderate level of reasoning performance. The Rasch analysis shows that the instrument used in this study is capable of distinguishing variations in students' reasoning abilities, while the Wright Map provides a clearer picture of the relationship between students' ability levels and the relative difficulty of the contextual tasks. The qualitative findings reveal that the Alkhairaat context plays different roles in students' reasoning processes depending on their level of mathematical proficiency. Students with stronger reasoning ability are generally able to use the contextual information to structure mathematical ideas and develop formal representations. In contrast, students with lower reasoning ability tend to engage with the contextual narrative at a more surface level and often encounter difficulties when transforming contextual information into mathematical expressions. Taken together, these findings suggest that incorporating sociocultural contexts into mathematics learning may help make mathematical ideas more accessible and meaningful for students. However, the results also indicate that contextual familiarity alone is not sufficient to foster deeper reasoning. Effective instructional support remains necessary to help students connect contextual narratives with formal mathematical structures and to gradually strengthen their reasoning abilities.

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AUTHOR CONTRIBUTIONS STATEMENT

Dewi Sri Wahyuni contributed to conceptualization, methodology, data collection, formal analysis, and preparation of the original manuscript draft. Tahril contributed to the development of the research methodology, instrument validation, and supervision of the research process. Dasa Ismaimuza provided theoretical guidance and contributed to the development of the conceptual framework and manuscript review. Pathuddin participated in instrument design, data interpretation, and critical revision of the manuscript. Mustamin Idris contributed to the validation of the Alkhairaat contextual content and supported the qualitative interpretation of the findings. Windia Hadi provided methodological consultation, assisted in Rasch analysis interpretation, and contributed to the final editing of the manuscript.

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