



Developing a PMRI-guided learning trajectory for elementary students' understanding of percentages within the melewang cultural context

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Abstract

Background: Understanding percentages poses persistent challenges for elementary students, particularly when the percent symbol is interpreted as a standalone number rather than as a relational expression of "per hundred." Instruction that is detached from meaningful contexts often restricts students' ability to connect percentages with fractions and decimals.

Aims: Responding to this issue, the study seeks to develop and validate a PMRI-guided learning trajectory that situates percentage concepts within the local cultural practice of melewang.

Method: Employing design research of the validation study type, the study involved 28 elementary students and was conducted in three phases: preliminary design to formulate a Hypothetical Learning Trajectory (HLT), teaching experiment to implement and observe the instructional sequence, and retrospective analysis to refine the HLT into an empirically grounded Actual Learning Trajectory (ALT). Data from classroom observations, student work, interviews, and video recordings were analyzed qualitatively to examine students' conceptual development.

Results: The findings indicate that engaging students with the melewang context enabled them to interpret percentage as a comparison to one hundred. Through a structured progression from concrete representations to bar models and 100-grids, students established meaningful connections among fractions, decimals, and percentages.

Conclusion: The resulting learning trajectory demonstrates how culturally grounded PMRI-based instruction can support coherent conceptual development and inform context-sensitive mathematics teaching practices.

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INTRODUCTION

Understanding percentages is a central component of proportional reasoning in elementary mathematics (Spitzer et al., 2025; Yao et al., 2021). As a concept, percentage serves as a crucial bridge between fractions and decimals while also functioning as a practical tool for interpreting everyday quantitative situations such as price discounts, comparisons, and data representation (Joutsenlahti & Perkkilä, 2024, 2024). Because of its broad application in real life, a stable understanding of percentage at the elementary level is essential for students' future development in rational number reasoning and various mathematical domains.

Despite its importance, percentage learning continues to present conceptual challenges for many students (Chew & Cerbin, 2021; Gillen-O'Neel, 2021). A common misconception is interpreting the percent symbol as a standalone number, rather than as a relational expression meaning "per hundred" (Davis, 2023). This interpretation leads students to rely on procedural

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engagement without fully grasping the relational understanding. While students may successfully apply formulas or convert percentages, they often struggle to explain what a percentage represents in terms of part-whole relationships. Such issues suggest that the difficulty lies not only in computation but also in conceptualization of percentage.

These challenges often arise from instructional approaches that prioritize symbolic manipulation before students have established a clear understanding of "the whole" (Cohen et al., 2025; Gamage et al., 2022). When learning begins with rules and conversion procedures, the relational meaning of percentage remains implicit (Hadjiosif et al., 2021; Thomas et al., 2023). As a result, connections among fractions, decimals, and percentages are formed mechanically, not relationally. Strengthening conceptual understanding therefore requires experiences that explicitly define "the whole" and allow students to explore its partitioning in meaningful, contextual ways.

The development of mathematical understanding is widely recognized as a gradual progression from concrete experience to visual representation and ultimately to formal abstraction (Andriatna et al., 2025; Gula & Lovric, 2024). In percentage learning, this progression is particularly critical because the concept of "one hundred" serves as both a quantitative benchmark and a structural reference point (Farkas & Németh, 2022). Students benefit when this understanding emerges from tangible, concrete contexts rather than from abstract symbolic definitions alone.

In the context of Indonesian education, the framework of Pendidikan Matematika Realistik Indonesia (PMRI) aligns well with these developmental principles (Sari et al., 2025). PMRI emphasizes using meaningful contexts as starting points for learning and supporting students' progression through guided reinvention, model construction, and classroom interaction. By embedding instruction within culturally familiar practices, PMRI helps enhance students' engagement and fosters clearer conceptual understanding, providing a more comprehensive approach to percentage learning.

Research on rational number learning consistently highlights the importance of representational fluency in supporting conceptual understanding (Jäder & Johansson, 2025; Schulz, 2024). Visual representations such as bar models and hundred grids are effective tools in helping learners coordinate different numerical forms and recognize the relationships among fractions, decimals, and percentages (Barbosa & Vale, 2021; Gorman, 2025). These models do not merely serve as illustrative tools; rather, they act as cognitive mediators, connecting concrete reasoning with more formal symbolic representations. The theoretical foundation of Realistic Mathematics Education emphasizes that mathematical knowledge should emerge from meaningful contexts through a process of guided reinvention (Fredriksen, 2021; Sevinc & Lesh, 2022). In PMRI, which is Indonesia's adaptation of RME, the key components include contextual grounding, model progression, student contribution, and social interaction. These principles are particularly effective in addressing the relational challenges students face when learning percentage concepts, as they emphasize both the local context and structured development of mathematical thinking. From a methodological perspective, design research has been recognized as a productive approach for developing and refining learning trajectories (Kenney & Ntow, 2024; Kubsch et al., 2022). By engaging in iterative cycles of design, implementation, and retrospective analysis, researchers can document how students' understanding evolves and fine-tune the learning trajectory based on empirical observations (Bhimdiwala et al., 2022). This approach is particularly valuable for percentage learning, where effective instructional sequences must guide students through complex representational transitions.

While numerous studies have explored contextual instruction and the use of representational tools in teaching percentages, several gaps remain in the literature (Friesen & Kuntze, 2021). First, much of the existing research focuses on post-instructional outcomes or comparative effectiveness, without providing detailed insights into how students' reasoning develops across instructional

phases. The transition from initial contextual engagement to formal abstraction is often not sufficiently documented or explored in-depth. Second, while PMRI-based studies have demonstrated positive outcomes in various areas of mathematics education, relatively few investigations have systematically constructed and validated a learning trajectory specifically for percentage concepts in elementary classrooms. Percentage learning involves unique relational and representational challenges that require carefully designed learning activities, which are still under-explored in the PMRI context. Third, although culturally responsive teaching has gained significant attention (Chang & Viesca, 2022; Ladson-Billings, 2021), the use of local cultural practices within PMRI-based learning trajectories has been limited. Often, cultural contexts are used as introductory examples or isolated cases, rather than being integrated as a central structural element throughout the learning process. The potential of a culturally embedded context to organize and sustain conceptual progression in percentage learning remains largely unexplored, and this study seeks to address this gap. These gaps indicate a need for research that integrates contextual grounding, representational progression, and empirical validation within a coherent instructional design for percentage learning.

In response to these gaps, the present study proposes the use of the local cultural practice of *melemang* as a meaningful context through which students can explore the concept of percentage. *Melemang*, a traditional food, offers a relatable way to illustrate the whole and its partitioning into parts of one hundred. This context is not merely an introductory example, but serves as a central, ongoing reference throughout the learning sequence. The study will integrate the *melemang* context within a PMRI-guided framework, allowing students to progressively reconstruct the concept of percentage through hands-on, culturally relevant experiences. Design research methodology will be employed to iteratively develop and refine the learning trajectory. By observing how students engage with the *melemang* context and trace their progression from concrete experiences to formal abstraction, the study will provide an empirically validated description of how percentage concepts can be effectively taught through cultural context. This research will contribute both theoretically to the understanding of percentage learning and pedagogically by offering an adaptable model for culturally responsive mathematics instruction.

The purpose of this study is to develop and validate a PMRI-guided learning trajectory for elementary students' understanding of percentages, utilizing the *melemang* cultural context. The study aims to construct a Hypothetical Learning Trajectory (HLT) based on PMRI principles, observe students' reasoning during classroom implementation, and refine the HLT into an Actual Learning Trajectory (ALT) that documents the progression from contextual understanding to formal mathematical abstraction. The study seeks to provide a validated model that integrates culturally relevant contexts with structured mathematical reasoning, offering valuable insights for mathematics instruction in elementary schools.

METHOD

Research Design

This study adopts a design research methodology, specifically a validation study type, which is ideal for developing and refining educational interventions through iterative cycles. The main objective of this research is to create and validate a learning trajectory focused on teaching percentages in elementary mathematics, incorporating PMRI principles while integrating the local cultural context of *melemang*. The design research process follows a structured sequence in three key phases: the Preliminary Design, the Teaching Experiment, and the Retrospective Analysis. In the Preliminary Design phase, the Hypothetical Learning Trajectory (HLT) was developed, drawing from theoretical frameworks and initial classroom observations. During the Teaching Experiment, the HLT was tested in real classroom settings, allowing students to actively participate in the learning activities. The Retrospective Analysis phase then involved analyzing the collected data student

responses, classroom observations, and artifacts—to refine the HLT into an Actual Learning Trajectory (ALT), capturing the students' development from hands-on experiences to formal mathematical understanding. This iterative approach ensures that the trajectory is not only theoretically sound but also grounded in actual classroom dynamics.

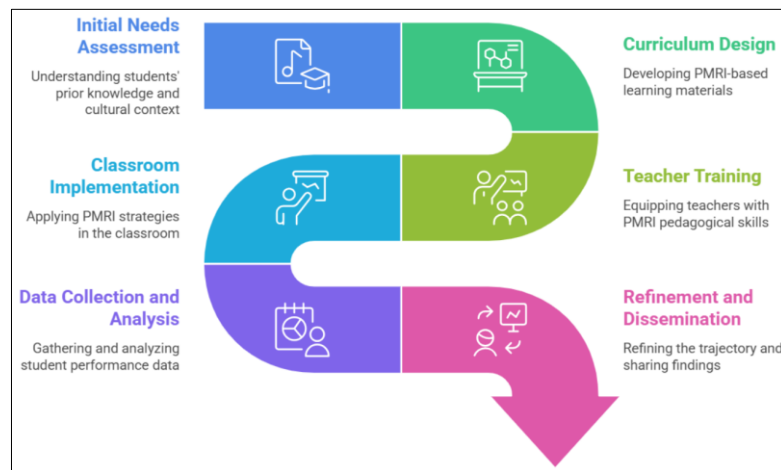


Figure 1. Research Design Process Flow

Participants

The study involved 28 fifth-grade elementary students from SD Negeri 1 Lawang Kidul, a school located in Muara Enim, Indonesia. This school was chosen due to its incorporation of local cultural practices, including *melemang*, which made it an ideal setting for embedding cultural context into mathematics instruction. The students were selected based on their grade level and the readiness to engage in activities that connect local culture with mathematical reasoning. These participants took part in the teaching experiment, which was conducted over three instructional sessions. The classroom teacher also played an important role as a co-researcher, assisting in the facilitation of the learning activities, guiding student engagement, and providing feedback. The teacher's involvement was crucial for ensuring that the learning process ran smoothly and that students received appropriate support throughout the experiment.

Research Procedure

The procedure followed in this research was divided into three main phases, aligned with design research principles. The Preliminary Design phase involved the development of the Hypothetical Learning Trajectory (HLT), which was based on PMRI principles and the *melemang* context. The HLT outlined the progression of learning activities, starting with tangible experiences using *melemang* and gradually progressing toward more formal representations of percentages using bar models and 100-grids. In the Teaching Experiment phase, the HLT was implemented in the classroom across three instructional sessions. These sessions allowed students to engage with the *melemang* context, and they progressively built their understanding of percentages. Initially, students partitioned the *melemang* into parts, and later used visual tools such as bar models and 100-grids to represent these parts as percentages. During the experiment, both the teacher and researcher observed the students' reasoning, documenting key moments of insight, struggles, and strategies in solving percentage problems. Finally, in the Retrospective Analysis phase, the data collected from the teaching experiment student work, classroom observations, video recordings, and interviews was analyzed. The aim was to refine the initial Hypothetical Learning Trajectory (HLT) into a more accurate Actual Learning Trajectory (ALT), reflecting how students progressed from contextual understanding to formal mathematical abstraction. This phase allowed for the identification of learning patterns, and it informed necessary adjustments to the instructional strategies and the overall trajectory.

Instruments

To collect and analyze data, several instruments were designed specifically for this study. Student worksheets (LKPD) were used to guide students through the learning activities. These worksheets contained tasks that involved partitioning the melemang context into parts, representing percentages using bar models and 100-grids, and solving related percentage problems. Additionally, observation protocols were used by both the teacher and researcher to record student interactions and participation during the lessons. These protocols helped document the strategies students used, their engagement levels, and any emerging misconceptions. To gain deeper insights, interviews were conducted with a selected group of students, exploring their understanding of percentages and the difficulties they encountered during the lessons. Video recordings were also made during the classroom sessions. These recordings allowed for a detailed review of the interactions between students and the teacher, providing valuable data for the retrospective analysis. The videos helped to capture specific moments that were later used to refine the learning trajectory and better understand the students' reasoning.

Data Analysis

Data analysis in this study followed a qualitative descriptive approach, with the primary objective being to understand how students' percentage understanding developed during the teaching experiment. The analysis involved coding and categorizing the student responses, classroom interactions, and student work. This process allowed the researchers to identify key patterns in students' reasoning and the progression of their understanding from concrete to formal mathematical representations. A comparative analysis was conducted by comparing the initial Hypothetical Learning Trajectory (HLT) with the final Actual Learning Trajectory (ALT). This comparison helped to assess the effectiveness of the HLT and provided insights into how the students' actual learning aligned with the planned trajectory. Any discrepancies or challenges observed during the teaching experiment were addressed and used to refine the trajectory further. Triangulation was employed to ensure the reliability and validity of the findings. By cross-referencing data from different sources student worksheets, classroom observations, interviews, and video recordings the researchers were able to verify the consistency of the students' understanding and engagement. This triangulated approach ensured that the findings were robust and reflective of the actual learning process that occurred during the study.

RESULTS AND DISCUSSION

Results

Overview of Student Performance

The results of this study demonstrate how students' understanding of percentages evolved throughout the teaching experiment. These changes were assessed through various data sources, including student worksheets, observations, video recordings, and interviews. Across these data sets, key shifts were observed in how students approached percentage problems initially relying on procedural methods and later transitioning to a deeper, more relational understanding.

At the beginning of the experiment, students encountered difficulty with the abstract concept of percentages. Many of them focused primarily on the procedural aspects of solving percentage problems, without understanding the underlying meaning. However, as the lessons progressed and students engaged with the cultural context of melemang, their comprehension shifted. They began to recognize percentages not just as standalone numbers, but as a relationship between parts and the whole. This shift was most evident when students used the bar model and 100-grid representations to illustrate percentages as parts of a larger whole.

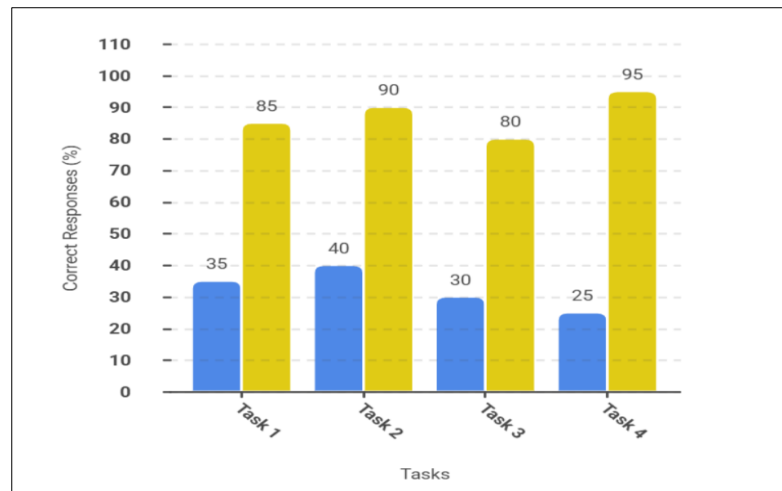


Figure 2. Comparison of Initial and Final Correct Responses Across Tasks

Key Findings

Conceptual Progression

As the students worked with the melemang context, they gradually moved from a concrete understanding of dividing a whole into smaller parts to representing percentages using more abstract models. Initially, many students struggled to understand that 1% represents one part out of 100, but with time, they were able to visualize and apply this concept. By using melemang as a starting point, students not only divided the food into portions but also translated those portions into percentage terms through visual models like the bar model and 100-grid.

Misconceptions Identified

While significant progress was made, some students still faced challenges, especially when tasked with applying their understanding of percentages to word problems. Even though they grasped the relational aspect of percentages, they had trouble solving problems that involved multiple steps or comparisons. This highlights the need for further scaffolding in problem-solving, particularly for more complex scenarios.

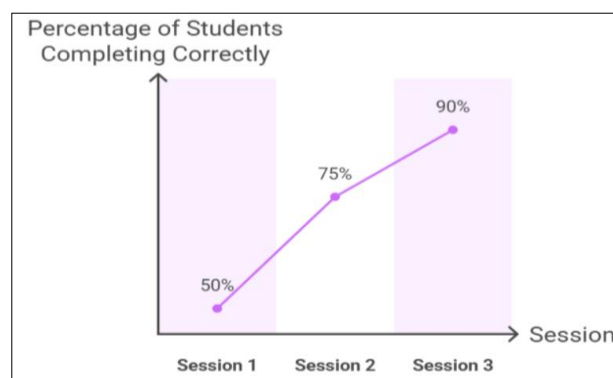


Figure 3. Students' Progression Across Sessions

Role of Contextual Learning

The melemang context was instrumental in helping students develop a deeper understanding of percentages. This culturally relevant context enabled students to see percentages as a comparison between parts and the whole, which they could visualize and manipulate more easily. As the experiment progressed, students demonstrated a clearer understanding of percentage calculations, and their ability to explain their reasoning improved.

Group Work and Collaboration

Collaboration among students also played a crucial role in reinforcing their learning. During group activities, students were able to discuss their approaches to solving percentage problems,

helping each other clarify misunderstandings. This collaborative process encouraged deeper engagement and allowed students to articulate their reasoning in a supportive environment.

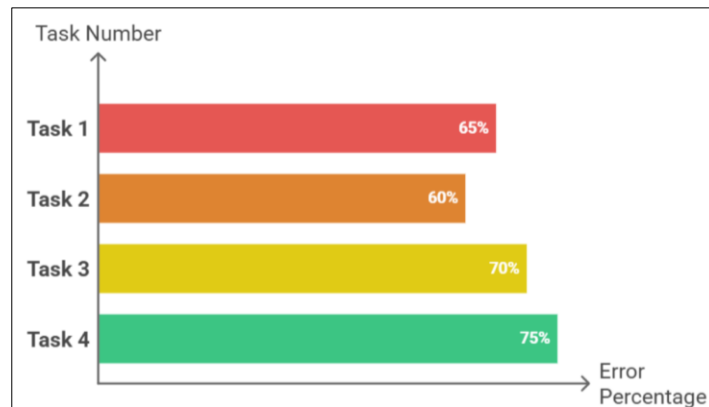


Figure 4. Error Percentage Across Tasks

Data from Student Worksheets

The student worksheets revealed a clear improvement in students' ability to correctly solve percentage problems. Initially, many students struggled with expressing percentages as parts of 100, but by the end of the experiment, most students could solve problems confidently and explain their reasoning.

Table 1. Improvement in Student Performance Based on Task Completion

Task Number	Initial Correct Responses (%)	Final Correct Responses (%)
Task 1	35%	85%
Task 2	40%	90%
Task 3	30%	80%
Task 4	25%	95%

Table 1 highlights the improvement in student performance from initial responses to final responses across each task. This significant increase in correct responses indicates that the learning trajectory, supported by PMRI principles and the melemang context, was effective in helping students better understand the concept of percentages.

Observational Data and Video Recordings

The video recordings and observational data provided additional insights into the students' engagement throughout the experiment. Early on, students were hesitant and relied heavily on procedural methods. However, as the sessions progressed, they became more confident in using the melemang context to explain their answers and make connections between the visual models and the mathematical concepts of percentages.

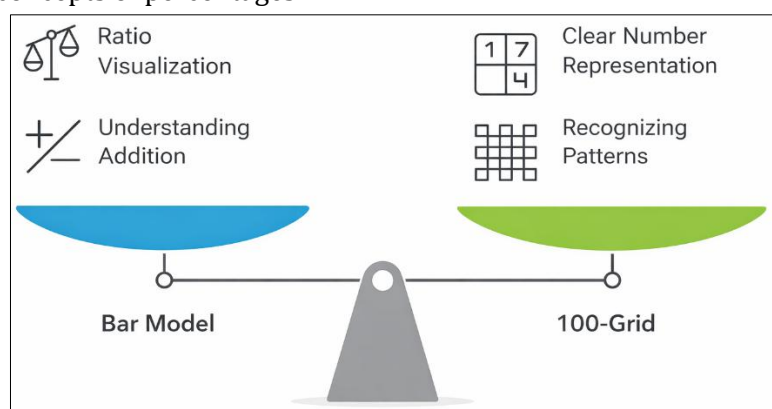


Figure 5. Student Engagement During the Teaching Experiment

The video recordings also captured significant moments of student interaction, where they collaboratively worked through percentage problems using bar models and 100-grids. These interactions were vital in helping students solidify their understanding of percentage as a relational concept.

Interviews with Students

Interviews with a select group of students provided valuable feedback on their understanding of percentages. Most students reported that the melelang context was particularly helpful in clarifying the concept of percentages. They explained how physically partitioning the melelang helped them visualize percentage as a "part of the whole," making it easier to understand.

However, some students still faced difficulties in applying their knowledge to word problems, especially those involving comparisons between different percentages. This suggests that while students developed a solid foundational understanding of percentages, they still require additional practice and support in applying this knowledge to more complex situations.

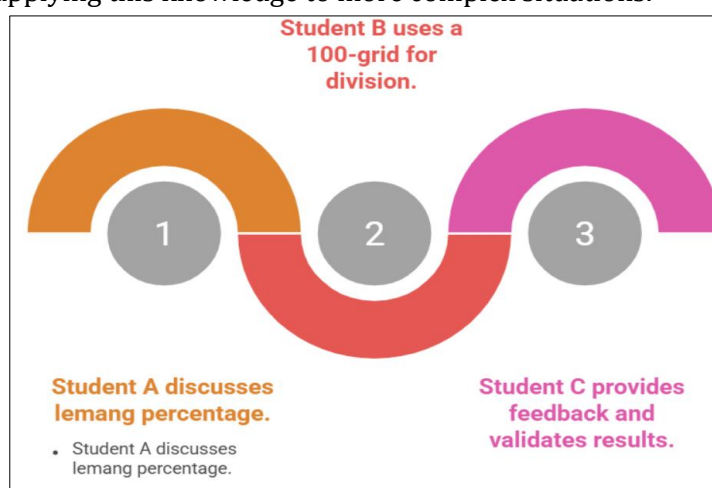


Figure 6. Diagram of Student Group Discussion and Collaboration

The results of this study show that embedding a PMRI-based learning trajectory in a culturally relevant context, such as the melelang practice, can effectively enhance students' conceptual understanding of percentages. Through the use of visual models like the bar model and 100-grid, students transitioned from a procedural understanding of percentages to a more relational one. Although some students continued to struggle with applying their knowledge to word problems, the overall impact of this culturally integrated approach was clear. The study highlights the importance of using real-world contexts in mathematics education to foster deeper conceptual understanding and improve problem-solving abilities.

Discussion

The results from this study provide compelling evidence for the effectiveness of a PMRI-guided learning trajectory, enriched by the melelang cultural context, in improving elementary students' understanding of percentages. The research demonstrates that incorporating real-world, culturally relevant contexts can significantly enhance students' conceptual grasp of mathematical concepts (Kolovou, 2023; Montesdeoca, 2023). In this case, the integration of melelang, a familiar cultural element, helped students bridge the gap between abstract percentage calculations and their everyday experiences, promoting both engagement and deeper learning.

One key takeaway from this study is the role of concrete, culturally grounded experiences in fostering students' understanding of abstract mathematical concepts like percentages. Through the hands-on experience of partitioning a lelang and visualizing it using models, students were able to make sense of percentages not as isolated numbers, but as relationships between parts and a whole.

This aligns with the idea that learning mathematics through real-world contexts facilitates the development of meaningful mathematical understanding, as students see the relevance of what they are learning in their daily lives (Hidayana & Lianingsih, 2025).

The transition from concrete experiences to abstract mathematical representations, such as bar models and 100-grids, was crucial in helping students make connections between fractional relationships and percentages. By visually representing the relationship between parts and wholes, students were able to internalize the concept of percentage as "per one hundred" and apply it to solve mathematical problems. This visual approach aligns with Dual Coding Theory, which emphasizes the importance of using both verbal and visual representations in supporting students' understanding of abstract concepts (Gayathri & Vijayalakshmi, 2025; Gray & Holyoak, 2021). The success of these models highlights their effectiveness in making complex concepts more accessible to students, particularly those in elementary education (Relmasira et al., 2023; Ziatdinov & Valles, 2022).

Despite the successes observed, the study also revealed that misconceptions persist, particularly when students were faced with word problems. Although students were able to understand percentages conceptually and apply their understanding to simple tasks, they struggled when it came to translating their knowledge into problem-solving scenarios that involved comparisons or multiple steps. This indicates that while a solid foundation of conceptual understanding was established, there is a need for continued practice in applying these concepts to more complex problems. It also suggests that further scaffolding is necessary to help students bridge the gap between simple calculations and the more complex, real-world applications of percentages (Barile, 2024; Czocher et al., 2025).

Another interesting finding was the significant role of group work and collaboration in reinforcing students' understanding of percentages. Throughout the experiment, students who worked together were able to clarify each other's misunderstandings and strengthen their reasoning. The social interactions observed during group activities were an essential part of the learning process, supporting the idea that mathematical understanding can be deepened through collaborative discourse. This supports Vygotsky's sociocultural theory, which argues that social interactions are vital in the development of cognitive skills (Alkhudiry, 2022; Daramola et al., 2024). By working together, students were able to articulate their thought processes, which not only reinforced their own learning but also helped their peers solidify their understanding (Cahusac de Caux & Pretorius, 2024).

The use of a gradual learning trajectory, beginning with concrete experiences and progressing through visual models to abstract concepts, was another key finding of this study. This step-by-step approach allowed students to build on their existing knowledge and internalize the concept of percentage before attempting more complex tasks. The importance of scaffolded learning in mathematical instruction cannot be overstated, as it allows students to progress at a pace that aligns with their cognitive development. This method is in line with Cognitive Load Theory, which suggests that learning is most effective when instructional content is presented in manageable steps, avoiding cognitive overload (Partarakis & Zabulis, 2024; Parveen, 2025).

However, while the PMRI-based learning trajectory was successful in improving students' understanding of percentages, this study also highlights areas for further improvement. Although the students were able to apply their knowledge to basic tasks, they struggled with more complex, multi-step word problems that required higher-level thinking and reasoning. This suggests that while the foundational understanding of percentages was well established, additional practice and instructional support are needed to help students navigate more advanced problem-solving tasks (Burke & Stewart, 2024). Future research could focus on developing strategies to support students in applying their knowledge to such complex problems, perhaps through additional contextual tasks or practice with varied problem types (Ding et al., 2024).

Moreover, the findings of this study underscore the value of culturally responsive teaching. The integration of the *melemang* context was not just an example or an activity but a core component of the learning trajectory that anchored students' understanding of percentages. By drawing upon students' cultural knowledge and everyday experiences, the study demonstrates how culturally relevant contexts can significantly improve the teaching and learning of abstract mathematical concepts (Acharya et al., 2021). This approach not only makes the learning process more engaging but also allows students to see the practical applications of mathematics in their lives, thus reinforcing the relevance of what they are learning.

Finally, while this study contributes to our understanding of how culturally grounded learning trajectories can enhance students' understanding of percentages, it also points to several areas for future exploration. For instance, research could investigate the long-term impact of such culturally contextualized learning trajectories on students' overall mathematical proficiency (Biernacki et al., 2023). Additionally, it would be valuable to explore how this approach can be extended to other areas of mathematics or applied to different cultural contexts to determine whether these findings are generalizable beyond the specific scope of this study.

Implications

The findings of this study underline the importance of integrating culturally relevant contexts, such as the *melemang* practice, into mathematics instruction. This approach significantly enhances students' conceptual understanding of percentages by connecting abstract mathematical concepts to real-world, familiar experiences. By incorporating PMRI principles and using visual models like bar models and 100-grids, educators can provide a solid foundation for students to develop relational understanding, making mathematical learning both engaging and meaningful. As such, schools and curriculum developers should consider embedding culturally relevant practices into their teaching strategies to foster deeper engagement and improve student learning outcomes.

Limitations

One limitation of this study is its relatively small sample size, consisting of only 28 students from a single school, which may not be representative of the wider population. This limits the generalizability of the findings to other educational contexts. Additionally, the duration of the study was limited to just a few sessions, which restricts the ability to assess the long-term effects of the culturally contextualized learning trajectory. Lastly, the study focused only on percentages, leaving questions about how well this approach can be applied to other mathematical concepts, such as fractions or ratios, unresolved.

Suggestions

Future research could explore the long-term impact of the PMRI-guided learning trajectory to determine if the improvements in students' understanding of percentages are sustained over time. Furthermore, extending this approach to different educational levels, such as middle or high school, could provide insights into its applicability for older students. Research should also focus on developing targeted strategies to help students apply their understanding of percentages to more complex, multi-step word problems, as this remains a key challenge. Lastly, expanding the use of culturally responsive teaching to incorporate various cultural contexts could help broaden the scope and impact of contextual learning in mathematics.

CONCLUSION

This study highlights the positive impact of integrating a PMRI-guided learning trajectory with the *melemang* cultural context on elementary students' understanding of percentages. By beginning with hands-on experiences rooted in a familiar cultural context, students were able to

connect abstract percentage concepts to real-life situations, enhancing their conceptual understanding. The use of visual models like the bar model and 100-grid facilitated the transition from concrete understanding to abstract reasoning, allowing students to develop a relational view of percentages. While some challenges, such as applying knowledge to multi-step word problems, remain, the overall effectiveness of the culturally contextualized approach shows its potential to foster deeper student engagement and comprehension. This study underscores the value of using culturally responsive teaching strategies in mathematics to create more meaningful and relevant learning experiences. Future research should explore the long-term effects of this approach and investigate its applicability across other mathematical topics and diverse student populations.

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AUTHOR CONTRIBUTIONS STATEMENT

All authors have significantly contributed to the development of this study. Arbella Sri Marleny conceptualized the research design and led the study implementation, including the development of the learning trajectory and analysis of data. Zulkardi provided key theoretical insights and methodological guidance throughout the research process. Ratu Ilma Indra Putri contributed to the design and refinement of the learning trajectory, as well as the interpretation of findings related to the integration of cultural context. Yusuf Hartono supported the data analysis and provided critical feedback on the manuscript. All authors reviewed and approved the final manuscript, ensuring its accuracy and validity.

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