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Preliminary study of dynamic modeling based on quaternion analysis for tricopter drone

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Abstract

Recently drone was used in many aspects, especially on military operation. Drone type three rotor, namely tricopter, was used for surveillance with stability motion needed too well operating. This study examines the dynamical aspects of a tricopter. A quaternion-based transformation method is developed to transition between reference coordinate systems. It forms a mathematical foundation for modeling tricopter dynamics. The quaternion formulation used as a mathematical tool to obtain equation of motion in translational and rotational. The result show that the derived equations provide a quaternion-based framework for modeling the tricopter's motion, enabling singularity-free transformations and accurate translational and rotational dynamics for real-time flight control and stability. These models form the basis for advanced navigation systems, offering precise trajectory planning and attitude control. Further research should focus on advanced control strategies, aerodynamic effects, and experimental validation to optimize tricopter's performance.

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INTRODUCTION

Unmanned Aerial Vehicles (UAV) or simply Drones, have grown rapidly and widely used in recent years (Edulakanti & Ganguly, 2023). Their applications range from precision agriculture (Liu & Li, 2023) and military operations (Udeanu et al., 2016) to surveillance (Chamola et al., 2021), environmental monitoring and aerial photography (Dronova et al., 2021). UAV's are classified into five primary types based on their design which are Aircraft (fixed-wing), Multirotor UAVs, Tailsitters, VTOL (vertical take-off and landing), and Balloons and Airships (Peksa & Mamchur, 2024). Based on the flight performance, UAV can be classified to several factors which are operating altitude, endurance, operating range, max take-off weight, and payload (PS & Jeyan, 2020). These factors will affect the flight dynamics of an UAV.

A Tricopter or three-rotor system and tilt servo mechanism provide a distinctive design among the several UAV types, enabling improved maneuverability and efficient yaw control (Sababha et al., 2015). The main advantage of tricopter is ability to balance different flight parameters, such as throttle, rudder, and aileron (Das et al., 2019). Understanding the dynamic aspect of the Tricopter was required in order to improve control algorithms, aerodynamic efficiency, and stability (Hassanalian & Abdelkefi, 2017). Studies on UAV dynamics primarily focused on multirotor systems such as quadcopters (Paz et al., 2021) and hexacopter (Ligthart et al., 2017) due to their symmetrical rotor configurations and stability. The investigation of dynamic model and control of quadrotors have provided insight to multirotor control strategies and stability considerations (Bouabdallah et

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al., 2004). The optimization of the multirotor power system has also been performed to maximize its endurance time in a high-altitude environment (Qin et al., 2023).

Recent developments of tricopter highlight the importance of optimizing flight control systems (Kaneko & Martins, 2024). For VTOL operations, a dynamic flight model for a tricopter integrated with fixed-wing aircraft was proposed (Salahudden et al., 2024). It uses PID control and smooth rotor tilting to provide stable transitions between forward flight modes and VTOL (Manzoor et al., 2020). A study of mathematical model for the dynamics of a tricopter analyzes its aerodynamic attitudes and shows that using LQG controller instead of PID control greatly increases response time and stability for precise attitude control (Yang Yang et al., 2013). Lagrange-Euler model and a modified torque calculation method was used to develop a mathematical model for tricopter's adaptive-robust control system (Nguyen et al., 2023). A proper mathematical model for tricopter dynamics will produce a proper simulation and experiment result (Chowdhury et al., 2021).

Nonlinear mathematical control system model analysis as quaternion was used on tricopter for minimalizing the parameters hovering control (Kataoka et al., 2011). Similar models for dynamics flying control already applied in quadrotor (Falconi & Melchiorri, 2012). Reduced parameter such as gyroscope was designed for tricopter experimentally, and effectively control the stability of movement in spin axis (Raj et al., 2021). Quaternion formulation can be modified to applied on quadcopter for stability of terminal slide (Serrano et al., 2023). Until now it can be developed to advanced stage of control system for many types of multirotor system (Iriarte et al., 2024).

This paper aims to provide a preliminary study of the dynamical aspects of tricopters. The mathematical modeling begins with the quaternion formulation for transformation between two reference systems of the tricopter, which consists of a body frame and an earth frame. Then, the equation of motion for translation and rotation aspects will be derived to obtain the dynamics of the tricopter.

METHOD

In this section, will describe the method of calculating the dynamical aspect of tricopter. The methods we are using are obviously mathematical analysis. The methods start from deciding coordinates and references system of the aircraft since it requires two references systems for the drone's body and the observer of drone (pilot or flight planner) (Cefalo & Mirats-Tur, 2011; Rico-Martinez & Gallardo-Alvarado, 2000). Afterwards, the dynamical aspects of the drone will be derived with the Newton-Euler methods for both translational and rotational motions. The translational and rotational aspects of the drone will be evaluated to show the six equations of motion of the aircraft.

Coordinates and References System of The Tricopter

Consider a tricopter with three arms of equal length and each having a rotor as shown in the Figure 1. The direction of rotation of the three rotors not necessarily in one the direction. It can be configured where two rotate in one direction and one rotor rotates in other direction. The angular velocity of these rotors denotes by ω_1, ω_2, and ω_3 in each rotor. This angular velocity produces an upward lift force denotes by f_1, f_2, and f_3 in each rotor that allows the tricopter to take-off.

First, the drone's body will be described by three Euler angles. These angles are easy to visualize and develop but unstable and have a gimbal lock (singularity) problem. Therefore, it is not a good method for the dynamic system of an aircraft (Groÿekatthöfer & Yoon, 2012). Quaternions can be the alternative to this problem since the quaternion is singular free and only has a single angle parameter around the rotation axes (Stengel, 2004). In the next sections, the aircraft's dynamic will be explained by using quaternion (Stengel & Berry, 1977).

To explain the motion of the tricopter, two frame references are necessary those are non-inertial frame (body frame) and the inertial frame (earth frame). It is necessary to include an earth frame since the motion of an aircraft in general is planned using geographical maps. The origin of the inertial frame is one point on earth with (x, y, z), which are north, east, and up, respectively. The body frame's origin (x_B, y_B, z_B) is centered at the body of the tricopter as shown in Figure 1.

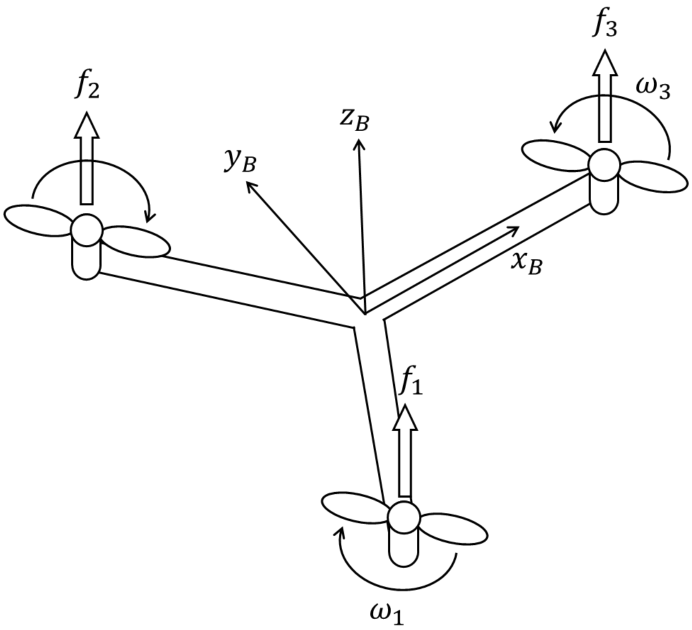

Figure 1. Parameter the motion of tricopter in body frame with origin axis (x_B, y_B, z_B), angular velocity (ω_1, ω_2, ω_3), and generated force (f_1, f_2, f_3).

Let us define the inertial position and angular position as matrices:

$$\xi = \begin{pmatrix} x \\ y \\ z \end{pmatrix}; \eta = \begin{pmatrix} \phi \\ \theta \\ \psi \end{pmatrix} \tag{1}$$

Besides the position, we define the angular velocity vector as

$$v = \begin{pmatrix} p \\ q \\ r \end{pmatrix} \tag{2}$$

The rotational matrix for transforming from a body frame to an inertial frame will use quaternion. Several fields are commonly using quaternion for computer game development, 3D virtual worlds, and the rotation of a rigid body. The definition of quaternion itself is

$$q = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{pmatrix} \tag{3}$$

where, $q_1 = \cos(\alpha/2)$, $q_2 = \sin(\alpha/2)u_1$, $q_3 = \sin(\alpha/2)u_2$, and $q_4 = \sin(\alpha/2)u_3$. The notation of are vector units in (x, y, z) axes respectively. The quaternion can be written in Q matrix for the transformation of the translational velocities from the body frame to the inertial frame as

$$\xi = Q\xi_{\mathrm{B}} \tag{4}$$

where the matrix Q is

$$Q = \begin{pmatrix} q_1^2 + q_2^2 - q_3^2 - q_4^2 & 2(q_2q_3 - q_1q_4) & 2(q_1q_3 + q_2q_4) \\ 2(q_2q_3 + q_1q_4) & q_1^2 - q_2^2 + q_3^2 - q_4^2 & 2(q_3q_4 - q_1q_2) \\ 2(q_2q_4 - q_1q_3) & 2(q_2q_3 + q_1q_4) & q_1^2 - q_2^2 - q_3^2 + q_4^2 \end{pmatrix} \tag{5}$$

As for the transformation matrix from the inertial frame to the body frame is $Q^{-1} = Q^T$. For the angular velocities, the transformation matrix can be written as

$$\dot{q} = Sv \tag{6}$$

where the matrix S is

$$S = \frac{1}{2}\begin{pmatrix} -q_2 & -q_3 & -q_4 \\ q_1 & -q_4 & q_3 \\ q_4 & q_1 & -q_2 \\ -q_3 & q_2 & q_1 \end{pmatrix} \tag{7}$$

In summary, the quaternion formulation for transformation between two reference systems will be used for our formulation in the dynamical model of tricopter in the following section. Besides that, the equation (7) is singular-free, because when the Euler angle is used the transformation matrix for angular velocity there will be a singularity factor (Alaimo et al., 2013) as shown in equation below

$$\dot{\eta} = \begin{pmatrix} 1 & \sin\phi\ \tan\theta & \cos\phi \tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sec\theta\ \sin\phi & \sec\theta \cos\phi \end{pmatrix} v \tag{8}$$

The equation has a singularity when $\theta = \frac{(2n-1)\pi}{2}$, $(n \in Z)$. The formulation of Euler angle cause this and will leads to a loss of degree of freedom, which is called as gimbal lock. Therefore, using quaternion as the transformation matrix could solve the singularity and gimbal lock problems.

RESULTS AND DISCUSSION

Dynamics of The Tricopter

The equations derived in the previous sections provide a comprehensive framework for understanding the motion and dynamics of the tricopter. The formulation uses a quaternion-based approach to overcome the limitations of Euler angles, ensuring singularity-free rotation representation. The use of two reference frames, body (non-inertial) and earth (inertial), is crucial for accurate motion modeling. The inertial frame provides a stable reference for planning motion relative to geographical coordinates, while the body frame allows analysis of forces and torques relative to the tricopter itself. The quaternion formulation ensures smooth and singularity-free transformations between the body and inertial frames. By defining rotation through a single angle around an arbitrary axis, the quaternion matrix Q enables precise translation of angular velocities and positions. This makes the model well-suited for real-time applications, such as flight control algorithms.

The motion of an aircraft can be divided into translational and rotational motions. The translational motion on the tricopter is provided by

$$\mathbf{F} = \frac{m d\mathbf{v_B}}{dt} + \mathbf{v} \times (m\mathbf{v_B}) \tag{9}$$

where the mass m is constant. Each rotor i generate angular velocities ω_i which generates a force, f_i, in z-direction

$$\boldsymbol{f_i} = (0 \quad 0 \quad f_i)^T = (0 \quad 0 \quad k\omega_i^2)^T \tag{10}$$

with k as the lift constant. The total thrust can be written as

$$T_B = \begin{pmatrix} 0 \\ 0 \\ T \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ k\sum_{i=1}^{6} \omega_i^2 \end{pmatrix} \tag{11}$$

The total force acting on the tricopter are summation of gravitational force and the total thrust,

$$\mathbf{F} = Q^{\mathrm{T}}\mathbf{F_g} + \mathbf{T_B} \tag{12}$$

The equation above can be substituted to the equation (8) and become

$$m\frac{\mathrm{d}\mathbf{v_B}}{\mathrm{d}t} + \mathbf{v} \times (m\mathbf{v_B}) = Q^T\mathbf{F_g} + \mathbf{T_B} \tag{13}$$

In the earth frame, the equation becomes

$$m\ddot{\xi} = \mathbf{F_g} + Q\mathbf{T_B} \tag{14}$$

where the centrifugal force $\mathbf{v} \times (m\mathbf{v_B})$ vanish because the inertial frame doesn't rotate.

The translational dynamics derived in equation (13) and (14) generated for the net forces acting on the tricopter. The thrust generated by the rotors is balanced against gravity, making it possible to compute the acceleration of the tricopter in the earth frame. This equation is essential for trajectory planning and altitude control.

The tricopter has a symmetrical structure with respect to (x_B, y_B, z_B) axes. Thus, the moment inertia I matrix has only a diagonal component

$$\mathbf{I} = \begin{pmatrix} I_{\mathrm{xx}} & 0 & 0 \\ 0 & I_{\mathrm{yy}} & 0 \\ 0 & 0 & I_{\mathrm{zz}} \end{pmatrix} \tag{15}$$

The moment inertia will be used in the total moment of the tricopter provided by

$$\mathbf{M} = \frac{\mathrm{d}}{\mathrm{dt}}(\mathbf{I}v) + \mathbf{v} \times (\mathbf{I}v) \tag{16}$$

In addition, angular velocity and acceleration of rotors create a torque $\tau_{M_i} = b\omega_i^2 + I_{M_i}\dot{\omega}_i$ around the rotor axis, where b is drag constant and I_{M_i} is the inertia moment of the rotor. This torque will be the one responsible for yawing.

From the geometry structure of the tricopter and from the forces and torque components over the aircraft frame, the roll, pitch, and yaw moment can be written as

$$\begin{pmatrix} \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{3}}{2}kl(-\omega_1^2 + \omega_2^2) \\ kl\left(-\frac{1}{2}\omega_1^2 - \frac{1}{2}\omega_2^2 + \omega_3^2\right) \\ b(-\omega_1^2 + \omega_2^2 - \omega_3^2) + I_m(\dot{\omega}_1 + \dot{\omega}_2 + \dot{\omega}_3) \end{pmatrix} \tag{17}$$

where l is the length between the rotors and the center of gravity of tricopter and dot on $\dot{\omega}_i$ denote the derivatife with respect to time t. Furthermore, the equation of motion in rotational dynamics can be written as

$$\frac{\mathrm{d}}{\mathrm{d}t}(\boldsymbol{I}v) + \boldsymbol{v} \times (\boldsymbol{I}v) = \tau_{\mathrm{B}} - \Gamma \tag{18}$$

where Γ represents the gyroscopic force caused by rotors and τ_{B} is external torque. With a little algebra, we can write the equation as

$$\dot{v} = \boldsymbol{I}^{-1}\left(\begin{pmatrix} p \\ q \\ r \end{pmatrix} \times \begin{pmatrix} I_{xx}p \\ I_{yy}q \\ I_{zz}r \end{pmatrix} - I_r \begin{pmatrix} p \\ q \\ r \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \omega_{\Gamma} + \tau_B \right) \tag{19}$$

with equation (19), we can easily deduce the angular velocity in quaternion term as

$$\ddot{q} = \frac{d}{dt}(Sv) \tag{20}$$

The rotational dynamics model, expressed in equations (18) and (19), incorporates moments of inertia, gyroscopic effects, and rotor-generated torques. The tricopter's symmetrical structure simplifies the inertia matrix to a diagonal form, allowing more straightforward computation of angular accelerations. The yaw moment is particularly influenced by the drag constant and rotor angular accelerations, providing insight into how to achieve stable yaw control.

The roll, pitch, and yaw moments from equation (17) are critical for maintaining the tricopter's attitude and stability. By adjusting the individual rotor forces and their placement relative to the center of gravity, precise control over these moments is achievable. The derived equations form the basis for implementing flight control algorithms. They allow for real-time calculation of required rotor speeds to achieve desired translational and rotational motions. Additionally, the quaternion-based approach can be directly integrated into advanced navigation systems, ensuring robust and reliable operation even under dynamic conditions (Mohammadkarimi et al., 2024)(Ghanizadegan & Hashim, 2025). While rotation matrices and Euler angles have been traditional choices as mentioned in Alaimo et al, the quaternion-based system in this paper represents a paradigm shift. It's not merely an alternative but a step forward in addressing the following limitations of efficiency, robustness, and ease of fusion.

CONCLUSION

We have calculated all the use of quaternion dynamics for modeling the tricopter which offers a significant advantage in terms of stability and computational efficiency. This study presents a preliminary analysis of the dynamical aspects of a tricopter. The quaternion formulation derived provides a robust mathematical foundation for transforming between reference systems. It enables precise modeling of tricopter dynamics. The dynamics of the tricopter offer critical insights into the interactions of forces, moments, and motion to set the stage for designing effective control systems. It provides a foundation for developing advanced control systems that can exploit the full potential of the tricopter's design.

Future research should focus on experimental validation of the proposed dynamical model and quaternion-based transformations through physical flight tests. Investigating aerodynamic effects and environmental disturbances, such as wind, will also be crucial for optimizing the performance and reliability of tricopter systems in diverse operational scenarios.

AUTHOR CONTRIBUTIONS

Conceptualization, methodology, simulation, and writing-original manuscript preparation, A.S. A, A. K.; formal analysis, A.S. A, A.T. O; validation, A.T. O.

CONFLICT OF INTEREST

The authors declare that they have no conflicts of interest.

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