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Bibliometrix research of noise removal techniques in digital images for defense

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Abstract

In modern defense applications, the accuracy and clarity of digital images are crucial, especially for tasks like surveillance, reconnaissance, and intelligence gathering. However, noise introduced during image acquisition or transmission significantly degrades image quality. This paper presents a comprehensive review of various noise removal techniques employed in digital image processing for defense systems. The review focuses on both linear and non-linear methods, including matrix decomposition, hybrid deep learning, Generative Adversarial Networks (GANs), and trimming filters. Emphasis is placed on the effectiveness of each technique in enhancing image quality while preserving critical details. The use of linear and non-linear methods such as deep learning-based approaches is shown to outperform traditional linear filters in handling complex noise patterns, particularly in scenarios requiring precise object detection and image restoration. The paper highlights a comprehensive overview of the researched literature and shows the latest trends and developments in the field. Finally, recommendations for future research and the development of more robust noise reduction methods are provided, aiming to improve operational effectiveness in defense applications.

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INTRODUCTION

In modern defense systems, the accuracy and clarity of digital images are critical for decision-making, particularly in applications such as surveillance, reconnaissance, and intelligence gathering (JosephNg et al., 2023; Onoja, 2023; Saxena et al., 2023; Sun et al., 2023). However, the presence of noise often introduced during image acquisition or transmission can significantly degrade the quality of these images, reducing their effectiveness in defense operations (Filippo Neri, 2018). This challenge has driven the development of advanced noise reduction techniques, with non-linier and linier filtering and matrix decomposition methods emerging as powerful tools for enhancing image quality. These methods are increasingly valued for their ability to handle complex, non-uniform noise patterns while preserving essential details like edges and textures (Cywińska et al., 2019).

Noise removal techniques have gained significant attention for their ability to reduce noise while maintaining image clarity, which is crucial in both defense and healthcare applications. Research by Vimala et al. (2023) and Khmag et al. (2022) highlights the importance of these methods

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in improving sensor data interpretation accuracy, essential for real-time defense operations, especially when combined with advanced technologies like Generative Adversarial Networks (GANs) and deep learning. In side-scan sonar image processing, for instance, He et al. (2023) employed low-rank sparse matrix factorization to separate targets from complex backgrounds, enhancing detection accuracy by effectively reducing noise without losing key details, which is vital for maritime defense. Similarly, in medical imaging, such as coronary angiography, noise from patient movement and equipment artifacts can degrade image quality. Kasai and Otsuka (2023) applied Singular Value Decomposition (SVD) to reduce noise in computed tomography angiography (CCTA) images, using Jensen-Shannon divergence to preserve essential structures. Both approaches demonstrate the effectiveness of matrix decomposition techniques in improving image quality in critical areas, where maintaining detail is crucial to avoid misinterpretations that could affect national security.

As defense strategies become increasingly reliant on high-quality, noise-free imagery, the development and refinement of these noise reduction methods are more urgent than ever. By offering a comprehensive review of linear and non-linear noise removal techniques, this paper aims to provide defense professionals and researchers with insights into the most effective methods for various defense-specific scenarios. Through a detailed comparison of existing techniques, this review sheds light on the practical applications and implications of noise reduction methods, ensuring that defense agencies can make informed choices in enhancing situational awareness and operational effectiveness.

METHOD

The main objective of this research is to explore noise removal techniques in digital imagery through bibliometric analyses that provide a new perspective with a focus on measured data related to publications, citations, and trends in this field. Noise removal is particularly important in the context of digital imagery in the defense sector, where analysis of images from satellites, drones, or other monitoring systems can be severely compromised by noise. This research will discuss various noise removal techniques, including linear techniques and non-linear techniques. Bibliometric analysis explores quantitative data that allows mapping of publication trends, citation frequency, and collaboration between authors and institutions in the field of noise removal techniques. This approach enables the identification of the most influential articles and authors, as well as the visualization of relationships between authors, institutions, and countries involved, which is important for understanding areas of rapid growth and research gaps that need to be further explored. Data will be collected from reputable journals and databases targeting studies published from 2023 to capture the latest advancements. Through bibliometric analysis, this research not only provides an up-to-date picture of the noise removal landscape but also facilitates a deeper understanding of the dynamics of knowledge creation, allowing us to draw insights into research trends and predict future directions. This is particularly relevant in the face of challenges in the defense sector, where high image quality is crucial for strategic decision-making, as well as enriching the understanding of the techniques used and their implications in digital image processing. The following steps were taken in this research to review the literature (Bindal et al., 2022).

Scope Determination

This research focuses on bibliometric research on linear and non-linear noise removal techniques in digital images relevant to defense applications. The scope of the research is limited to the types of non-linear noise, such as impulsive noise, speckle noise, and multiplicative noise, as well as various noise removal techniques that have been developed, including space-frequency methods, spatial methods, wavelet methods, sparse representation methods, and deep learning and linear methods such as low-rank sparse matrix factorization and singular value decomposition. This research reviews the application of non-linear noise removal techniques in various defense scenarios, such as surveillance systems, target recognition, and remote imaging, as well as linear techniques such as detection sonar and CCTA images. The focus is on articles published within the last 2 years (2023-2024), using credible and proven research methods, and published in accredited journals. With clear scope limitations, this research is expected to produce a bibliometric study that is focused, comprehensive, and relevant to the latest developments in the field of non-linear and linear noise removal in digital images for defense applications.

Literature Search

1. Literature Sources: The literature search was conducted through reputable scientific databases such as IEEE Xplore, ScienceDirect, Scopus, and Google Scholar. These databases were chosen because they provide access to accredited scientific journals and international conference publications in the fields of electrical engineering, computer, medicine and materials science, which are relevant to the research topic.
   Keywords: The keywords used in the search include:
   i. Noise Removal Techniques: “linear noise removal”, “non-linear noise removal”, “image denoising”, “digital image processing”, “image restoration”, “noise reduction”, “adaptive filtering”, “wavelet denoising”, “sparse representation”, “deep learning”.
   ii. Defense Applications: “defense applications”, ‘military imaging’, ‘surveillance systems’, ‘target recognition’, ‘object detection’, ‘remote sensing’, ‘infrared imaging’, ‘radar imaging’, ‘sonar imaging’.
2. Publication Time Limit: The time limit for publication of articles is determined based on relevance and the latest technological developments. The selected articles are those published within the last 2 years (2023-2024). This limitation aims to get the latest information and relevant to the latest technology.

Literature Selection

1. Topic Relevance: The selected articles should be relevant to the research topic, which is linear and non-linear noise removal techniques in digital images. Articles that only discuss linear and non-linear noise or noise removal techniques for other applications outside defense are not considered.
2. Research Methods: The selected articles should use credible and tested research methods. Commonly used research methods in the field of image processing include:
   i. Simulation Methods: This method involves using synthetic data or real images with added noise to evaluate the performance of noise removal algorithms.
   ii. Experimental Methods: This method involves using real images obtained from sensor systems or cameras to test the performance of noise removal algorithms in real-world scenarios.
3. Scientific Quality: The selected articles should be of high scientific quality, demonstrated through peer-review and publication in accredited journals. Accredited journals have strict scientific standards and peer-review processes that ensure the quality and validity of the research.

Literature Analysis

After conducting the literature search and selection, the next step is to process the selected literature. This process involves analyzing and synthesizing data from multiple sources to gain a comprehensive understanding of the research topic. Each article is analyzed to identify key information such as types of noise, noise removal techniques, algorithm performance, defense applications, and advantages and disadvantages of the discussed techniques. This key information was then systematically recorded in the form of tables or notes. The quality of each article is also assessed based on author credibility, research methods, and publication quality. Furthermore, articles that have similar topics or research methods are grouped together to identify trends and recent developments in the field of non-linear and linear noise removal techniques in digital images for defense. Based on the analysis and synthesis of the articles, a comprehensive framework on noise removal techniques is organized, covering the classification of techniques, performance comparison of algorithms, applications, and current trends and challenges. Finally, conclusions on the development of noise removal techniques are drawn and recommendations for future research and development are given. The results of the literature analysis and synthesis are then compiled in the form of a systematic and easy-to-understand written report, supplemented by tables, figures, and diagrams to clarify the presentation of data and analysis results.

Reporting the Data

In the final stage, the analysis of the sample and the characterization of the articles were compiled and reported. To characterize the literature covered in our review, we used five categories:

1. Year of Publication: Indicates the time span of publication of the article under study.
2. Journal and Field of Science: Indicates the journal in which the article was published and the field of science represented by the journal.
3. Article Type and Research Method: Describes the type of article (e.g., research article, literature review, case study) and the research methods used in each article.
4. Emerging Topics: Lists the main topics covered in the literature researched.
5. Content Systematization: Organizing the literature content based on relevant themes or categories.

By using these categories, we aim to provide a comprehensive overview of the researched literature and show the latest trends and developments in the field.

RESULTS AND DISCUSSION

Pre-requisite concept

Matrix decomposition in image noise removal is a technique used to break down an image into multiple components, making it easier to isolate and remove noise while preserving important details like edges and textures (Rajwade et al., 2013; Shi et al., 2019; Wang et al., 2020; Yang et al., 2021; Zeng et al., 2021). By decomposing an image into matrices, the noise can be separated from the underlying structure of the image (Zhang et al., 2022). Common methods include SVD, which breaks the image into singular values where lower values often contain noise, and Principal Component Analysis (PCA), which reduces the image to its most significant components, filtering out noise from less important ones (Škorić et al., 2022). SVD, by decomposing the image matrix into singular values and vectors, allows for noise reduction by setting small singular values to zero, effectively removing low-rank noise like blur or compression artifacts (Colace et al., 2022; Guzelbulut et al., 2023; Hartebrodt et al., 2024; Wahyulaksana et al., 2023). PCA, on the other hand, identifies the principal components of the image, representing the most significant variations in the data, and by focusing on these components, it can effectively filter out noise while preserving the essential features of the image (Dhiman et al., 2023; Kristanto et al., 2023; Montanaro et al., 2023; Setiawan et al., 2023; You et al., 2023). Non-Negative Matrix Factorization (NMF) is also used to decompose the image into non-negative parts allowing for better interpretation and noise separation. All of the common methods above are linear noise removal decomposition matrix method. All of those linear method claimed to be more simple and has more error than the non-linear ones although the computational time of the non linear ones claimed to be longer. Non-linear techniques help in enhancing image quality by effectively reducing noise without losing critical visual information (Bhateja et al., 2020).

The term "window" or "kernel" refers to the neighborhood around a pixel with a predefined size. Common window sizes in filtering processes are 3×3, 5×5, and 7×7 (Yang et al., 2022). In de-noising, larger window sizes reduce noise more effectively but can cause the image to lose important features. Smaller windows perform better at lower noise levels, while larger windows work well at higher noise densities but can introduce blurring (Thakur et al., 2021). Filtering enhances images by applying operations to both the noisy pixel and its neighboring pixels (Jiang et al., 2022; Wang et al., 2023). Common filtering operations include smoothing, sharpening, and edge enhancement. Filtering can be done in either the spatial or frequency domain: in the spatial domain, operations are directly applied to image pixels, while in the frequency domain, the image is transformed using methods like Fourier transforms before applying the filter (John et al., 2020). Various filtering techniques for removing salt-and-pepper noise, especially non-linear filters, are discussed in the following sections. Several non-linear filters are developed to provide a better performance over linear filters and overcome their shortcomings. As the name suggests, filters of this class do not satisfy superposition. However, it is also important to understand the advantages and disadvantages of each filter, both linear and non-linear. This section provides a comprehensive overview of the various linear and non-linear filters available in the literature.

Table 1. Literature Review

No	(Author)	Method	Type	Description	Advantages
1.	Vimala et al, 2023	Hybrid Deep Learning Technique	Non-Linear	Hybrid deep learning technique is used to remove local speckle noise from breast ultrasound images. The contrast of ultrasound breast images was first improved using logarithmic and exponential transforms, and then guided filter algorithms were used to enhance the details of the glandular ultrasound breast images. In order to finish the pre-processing of ultrasound breast images and enhance image clarity, spatial high-pass filtering algorithms were used to remove the extreme sharpening. In order to remove local speckle noise without sacrificing the image edges, edge-sensitive terms were eventually added to the Logical-Pool Recurrent Neural Network (LPRNN).	The time required to destroy local speckle noise is low, edge information is preserved, and image features are brought into sharp focus with high accuracy.
2.	Jana et al, 2023	Trimmed median filter	Non-Linear	The trimmed median filter is a non-linear image noise removal technique that works by sorting the pixel values within a neighborhood (or window) around a target pixel, discarding a certain percentage of the highest and lowest values (trimming), and then calculating the median of the remaining pixels. This method effectively reduces noise, especially impulse noise like salt-and-pepper noise, while preserving image edges better than traditional filters.	The trimmed median filter is experienced alongside various grayscale images and the color images and it exhibits a high Peak Signal-to-Noise Ratio, low mean square error and better Structural Similarity Index, image enhancement factor and Correlation Index.
3.	Khmag, 2023	Generative adversarial network (GAN)	Non-Linear	A Generative Adversarial Network (GAN) is used to extract the fine edge of the noised digital images in order to improve the actual signal in	The proposed method has better visual quality, and the proposed

				the high frequency components where the main parts of the clean pixels may consider as noise pixels, and as a result delete the unwanted noise from the tested image that might cause over smoothing to the resulted images. In order to further denoise the contaminated digital image, adaptive learning GAN model throughout scoring machine is exploited	method improves PSNR by 2.27 dB and 0.85 dB on average compared with state-of-the-art-denoising methods. In addition, the proposed method could shorten the processing time noticeably
4.	He et al., 2023	low-rank sparse matrix factorization and robust principal component analysis algorithm	Linear	This article discusses a method for small target detection in sonar images that uses low-rank and sparse matrix decomposition. This research focuses on the challenges faced in detecting small objects affected by environmental noise and complex backgrounds. The proposed method implements an improved robust principal component analysis (RPCA) algorithm to extract target information from sonar images. This process involves the simultaneous estimation of the background (low-rank) matrix, target (sparse) matrix, and noise matrix without requiring large training data. Furthermore, we apply morphological operations to eliminate noise and smoothen the target's edges, thereby enhancing detection accuracy.	The small target detection method proposed in this article offers a robust and efficient solution for detecting small objects in underwater environments. The approach shows high effectiveness under strong noise conditions, does not require large training data, and improves detection accuracy by using low-rank and sparse matrix decomposition.
5.	Kasai & Otsuka, 2023	SVD combined with Jensen-Shannon Divergence (JSD)	Linear	This paper introduces a new method to reduce noise in CCTA images, which is an important tool in the diagnosis of coronary artery disease. CCTA is often affected by noise due to the lack of X-ray photons, which can reduce image quality. The authors propose the use of SVD combined with JS divergence as	This research presents a novel noise reduction method combining SVD and JS divergence, which is shown to be effective in reducing noise, especially at

	an evaluation function to determine the optimal threshold for noise reduction. The method was tested on numerical phantoms (Shepp-Logan and water phantoms) with various noise levels, as well as on clinical CCTA images. The experimental results show that this method not only successfully reduces noise but also preserves important structures in the image, resulting in higher image quality compared to conventional noise reduction techniques.	higher noise levels, while maintaining image quality. The method was validated on CCTA clinical images, demonstrating potential applications in medical practice. This research also promises future developments targeting the impact of noise on machine learning training data.

Table 1 provides a comparative analysis of five different noise reduction techniques used in medical imaging, particularly focusing on ultrasound and digital images. The first method, proposed by Vimala et al. (2023), employs a hybrid deep learning technique to remove local speckle noise from breast ultrasound images. The experimental data was collected from the INbreast and CBIS-DDSM datasets. INbreast contained 120 instances (412 images), with 91 from women with both breasts (four images each) and 30 from women who had undergone mastectomy (two images each). The dataset included various inflammatory lesions, and expert-provided outlines in XML format. CBIS-DDSM, a large breast data collection, was divided into benign without call-backs, benign, malignant, and normal categories. A total of 1000 ultrasound images from these datasets were used, with 800 images for training a recurrent neural network and 200 for testing. The method enhances contrast using logarithmic and exponential transforms, followed by guided filtering to improve image details. Further preprocessing steps involve spatial high-pass filtering to sharpen the image while preserving edge information. The Logical-Pool Recurrent Neural Network (LPRNN) helps reduce noise without distorting important image features. This method is efficient, requiring minimal time to eliminate noise while maintaining image clarity, with a low error rate of 1.1%. Algorithm 1 shown a more detailed procedure that explains the Hybrid Deep Learning method from Vimala et al. (2023).

Algorithm 1. Noise Removal of the Local Speckle Noise

1: Begin
2: Logarithmic and computational transforms are used to improve the differentiation of the input ultrasound breast images; the algorithm (guided filter) is used to improve the details of the glandular ultrasound images; and the spatial high-pass filtering algorithm is used to denoise the over-sharpening of the ultrasound breast images, all based on their grayscale values
3: The pre-processed ultrasound breast images are fed into a local-speckle-noise destruction model of a logical-pool recurrent neural network
4: Ultrasound breast images are susceptible to losing image edge information during the local speckle noise reduction procedure. If we want to preserve the edge information after local speckle noise removal is applied, we will need to understand how that information is lost during processing. The meaning of “edge information loss”.

$$Loss_{edge}(P) = log\frac{I''_m(a,b)}{\sum_{i,j}|b_{i+1,j}-b_{i,j}|} \quad (1)$$

5: In order to construct ultrasound image gradients, we first analyze the aforementioned stages and then use edge loss pairs to compare the edges of canonical clear images of ultrasound breast images. The unique anatomy of the breast emphasizes the significance of the gradients in the vertical plane. That is why we first use contrast in the vertical direction to depict breast ultrasound images. Integrating edge loss LossEdge(P) and L1 distance with a recurrent neural network yields the following objective function:

$$P^*, c^* = argmin_p \max_c Loss_{HRNN}(P, C) + bLoss_{Loss1}(P) + \beta Loss_{Edge}(P) \quad (2)$$

6: Enhance the loss function to optimize the edge-specific improvement feature of the ultrasound images during training with the logical-pool recurrent neural network. The resulting model will be more responsive in edge local speckle noise destruction in ultrasound images, enhancing its effect on ultrasound breast images.

7: While noise removal reduces the local speckle noise of ultrasound breast images, the edge information is preserved by the action of the advantage term in the logical-pool recurrent neural network as described above

8: End

The process of reducing speckle noise in breast ultrasound images typically follows an additive noise model. One effective approach is using a Logical-Pool Recurrent Neural Network (LPRNN), which involves three main steps: pre-processing, training, and denoising. During pre-processing, ultrasound images are aligned and enhanced, with contrast adjustments to ensure clarity. This is followed by data augmentation to create training samples for the LPRNN model. The LPRNN, once trained, is applied to reduce speckle noise, although this process may result in edge blurring and data loss. To address this, the LPRNN algorithm specifically targets reducing edge information loss during noise reduction. Breast ultrasound image pre-processing plays a crucial role in reducing local speckle noise. This involves enhancing image contrast, applying image-guided filtering to improve detail, and using high-pass filtering for spatial analysis to prevent over-sharpening. Grayscale settings are adjusted using logarithmic and exponential transformations, which expand lower grey values and handle overly bright images, respectively. These methods improve image quality and reduce the noise prior to object identification and segmentation. To further enhance clarity, spatial filters are employed to adjust the image's frequency distribution, increasing contrast and using high-pass filters to limit low-frequency content. This helps reduce over-sharpening caused by image-guided filtering. The output of the ultrasound breast image is calculated using template parameters, ensuring edge data retention. In combination, the LPRNN method and pre-processing techniques work together to effectively reduce speckle noise while preserving crucial edge information, improving the quality of breast ultrasound images and making segmentation more accurate and reliable.

The second approach, by Jana et al. (2023), uses a pixel density-based trimmed median filter (PDBTMF) to remove noise, particularly effective for impulse noise like salt-and-pepper. The experimental data used in this research was a standard image with different density of noise range from 10 to 90%. It works by trimming the highest and lowest pixel values within a neighborhood and calculating the median of the remaining pixels. Below will be shown a more detailed PDBTMF flowchart that explains the method from Vimala et al. (2023) more clearly. This method reduces noise while preserving image edges better than traditional filters, especially for grayscale and color images. In the first stage, this filter can decide that the test pixel is degraded by SPN or not. For the identified corrupted pixel, this filter can check for the identified corrupted pixel is noisy one or not by checking all the pixels present in the selected mask. When test pixel is 255 and the maximum number of pixels present in the selected 3×3 window, then this filter can treat the present pixel 255 is a non-noisy one. The Pixel Density-Based Trimmed Median Filter (PDBTMF) works in several steps to restore noisy images. First, it reads the noisy image, identifying each pixel. If a pixel is 0 or 255, it is considered corrupted, and a 3x3 window around the pixel is selected. If all values in the window are either 0 or 255, two cases apply: (1) if the pixel is 0 and at least six surrounding pixels are 0, the pixel is considered non-corrupted and remains 0; otherwise, the mean value of the window replaces the pixel; (2) if the pixel is 255 and at least six pixels are 255, it remains 255; otherwise, the mean value is used. If the window has values other than 0 or 255, additional cases are considered: (1) if the window contains pixels within a specific range ($0 < N(i,j) < 10$ or $245 < N(i,j) < 255$), the most frequent value in the window is identified, and its median replaces the current pixel; (2) if no such values are found, all 0s and 255s are eliminated from the window, and the median of the remaining values is used. Finally, if a pixel falls between 0 and 255, it is considered uncorrupted and left unchanged. Applying this process to every pixel in the image results in a denoised output. However, results shows that this method has a higher error rate of 22%, although it shows improved metrics like Peak Signal-to-Noise Ratio (PSNR) and Structural Similarity Index, enhancing image quality overall from all the other kind of trimmed median filters. The flowchart of noise removal in the study can be seen in the Figure 1.

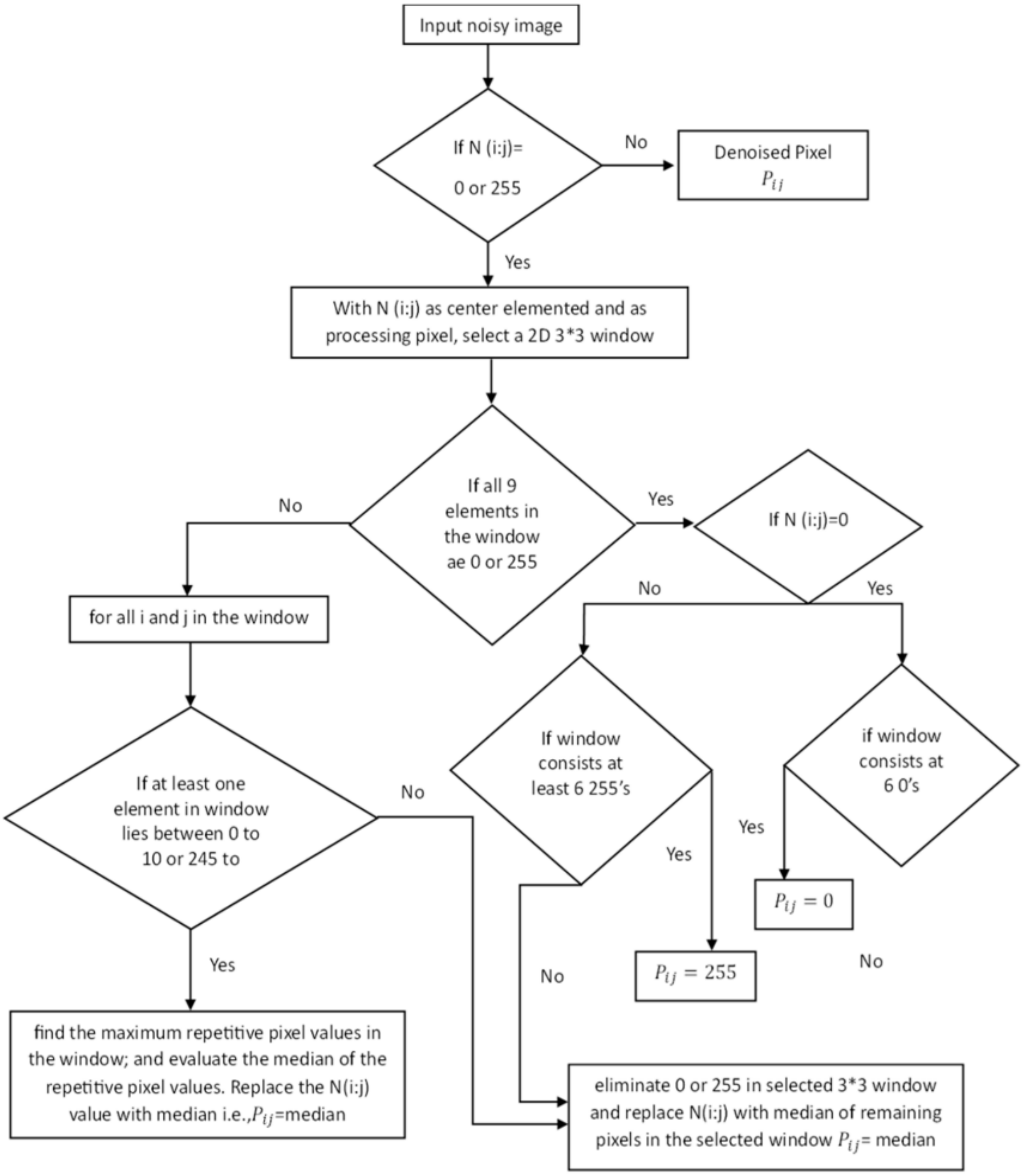

Figure 1. Noise Removal Flowchart by Jana et.al, 2023

The third method by Khmag (2023) utilizes a Generative Adversarial Network (GAN) to remove noise from digital images by focusing on high-frequency components and eliminating unwanted noise that causes over-smoothing. The experimental data used in this research were 12 images with different density of noise range from 10 to 70%. A self-adjusting generative adversarial network (GAN) is employed to classify the sharp edges of contaminated images using a semi-soft thresholding model. The noise removal technique in the study can be seen in the Figure 2.

Convolutional neural networks (CNNs) are used to build a generative adversarial network (GAN) for training digital image samples. GANs are particularly effective in generating digital images that closely resemble the original data distribution, creating non-realistic effects. The primary goal of the generator in a GAN is to produce images indistinguishable from real ones, while the discriminator aims to differentiate between the fake and real images. Through this adversarial process, the variance between generated and real data is minimized. The mathematical foundation of GANs involves an extraction function that aligns the distributions of generated data (PC) and actual data (Pdata). Despite their success, GANs often face challenges in training, such as convergence issues and pattern collapse. The proposed GAN model in this study addresses these problems, particularly the overlap between generated and real data, which can lead to gradient loss and difficulty in training. To improve performance, the generator network focuses on creating images that can deceive the discriminator, while the discriminator continuously refines its ability to detect fake images. The improved model demonstrates superior results compared to traditional GANs and can be applied to fields like machine learning and computer vision. The GAN method improves both visual quality and PSNR values compared to state-of-the-art denoising methods with an error rate of 1.4%. Additionally, it significantly reduces processing time while maintaining superior image enhancement.

The fourth approach, by He et al. (2023), entitled "Small Target Detection Method Based on Low-Rank Sparse Matrix Factorization for Side-Scan Sonar Images" proposes a new method to detect small targets in side-scan sonar images (He et al., 2023). This method uses low-rank sparse matrix factorization and robust principal component analysis to optimize the solution in sonar detection. Side-scan sonar image processing is a growing field, particularly in the effort to improve the accuracy of underwater target detection, which has important implications in the defense field. A major challenge in side-scan sonar image processing is the presence of significant noise, which comes from various sources such as environmental interference, electronic noise, and echo effects. This noise can obscure the presence of targets and complicate the detection process, which can impact the effectiveness of military operations and maritime security. Figure 3 displays the subsequent visualization of a side scan sonar picture. Figure 4 shows the low rank sparse matrix factorization algorithm.

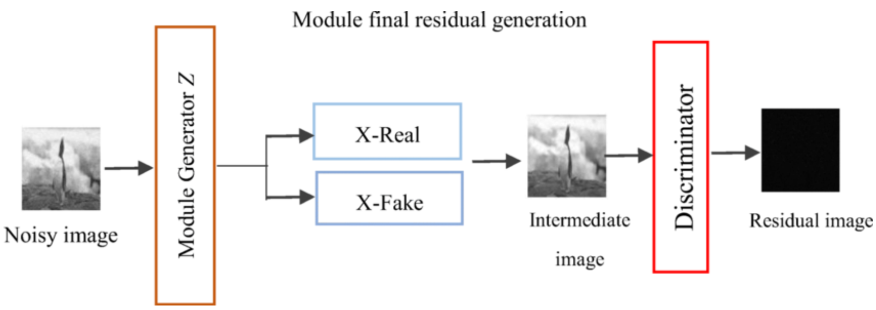

Figure 2. Illustration of Noise Removal Technique by Khmag, 2023

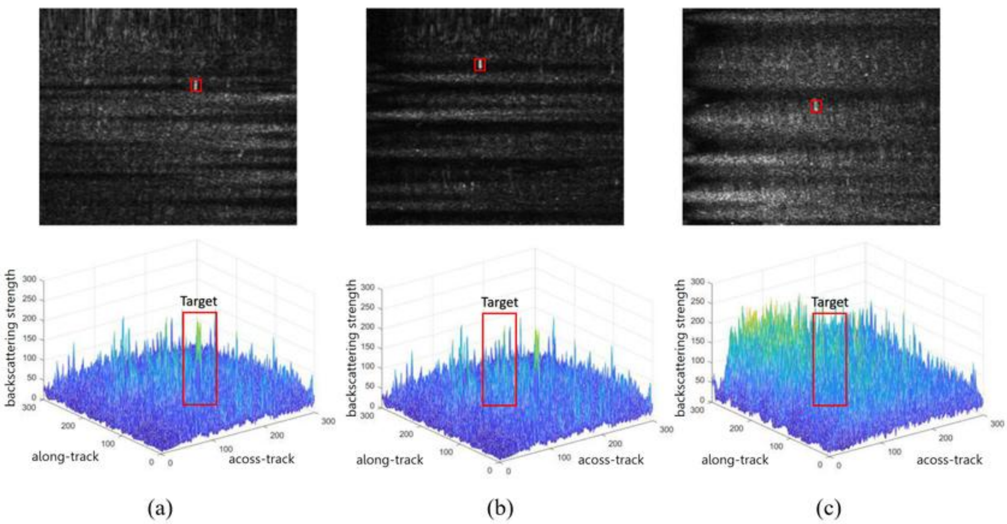

Figure 3. Side-scan Sonar Images and their 3D Visualization by He et.al, (2023)

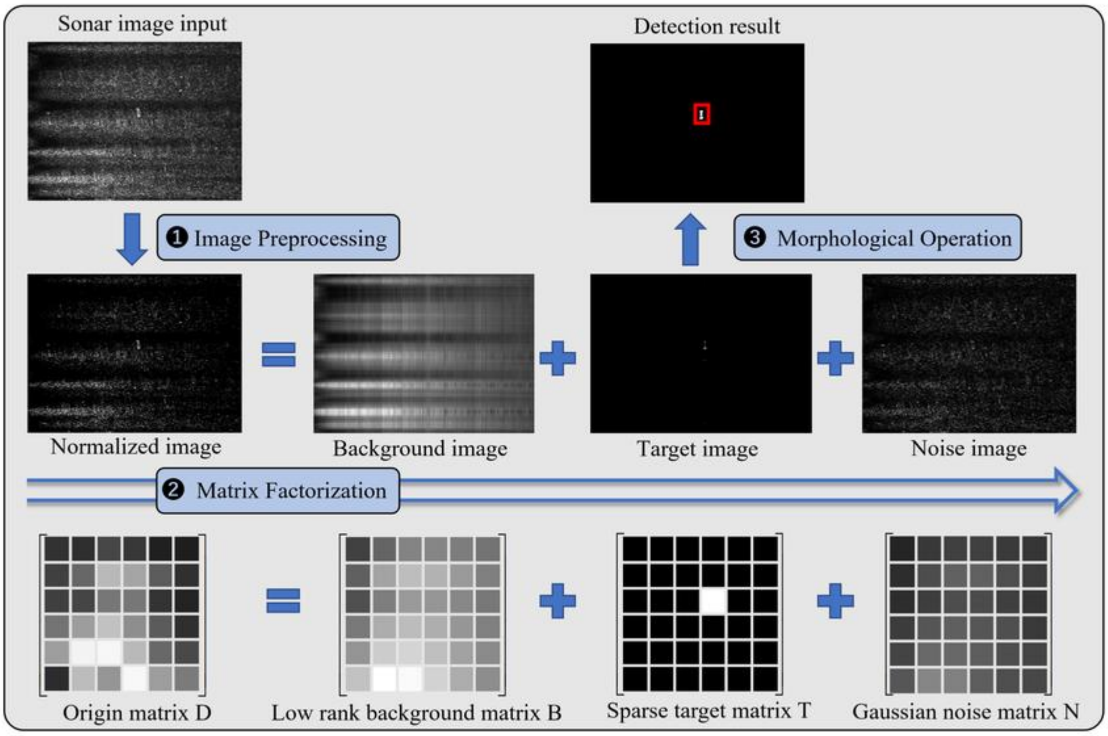

Figure 4. The Flowchart of our Proposed Algorithm Based on Low-Rank Sparse Matrix Factorization by He et.al, 2023

Complex side-scan sonar (SSS) images have unique characteristics that allow the use of matrix factorization to separate the target from the background. The complex background, which usually consists of constant seafloor reflections, has a low rank nature, meaning that the information in the background can be represented by a small number of principal components. In contrast, small targets to be detected tend to be sparse, occupying only a small portion of the entire image. Thus, matrix factorization can be used to separate the sparse target signal from the low-rank background. Based on this, the background image B, which has low-rank properties, is as follows:

$$rank(B) \leq r \tag{3}$$

where r is a constant, the input image is pixel-sized. In the next stage, the problem of target feature extraction and noise removal is transformed into a matrix decomposition problem, where an improved robust principal component analysis (RPCA) algorithm is used to extract the target information. The optimization method used is the fast proximal gradient method to refine the solution, which allows simultaneous estimation of the low-rank background matrix, sparse target matrix, and noise matrix. Finally, morphological operations are applied to filter out the noise and smoothen the target edges in the target matrix, thus improving the target detection accuracy.

The fifth approach, by Kasai & Otsuka (2023), discusses the examination of the blood vessels of the heart. Cardiac vascular examinations, such as coronary angiography, are important procedures for diagnosing and monitoring heart disease (Kasai & Otsuka, 2023). The quality of the resulting image is critical for accurate interpretation by the doctor but is often compromised by noise originating from various sources such as patient motion, device artifacts, and electronic noise. This noise can obscure important details in the image, making it difficult to identify blockages or abnormalities in the blood vessels. One approach to reducing noise in cardiac blood vessel examination images is to use signal processing techniques such as singular value decomposition (SVD). SVD is a linear algebra technique that can decompose an image matrix into simpler components, allowing for selective identification and removal of noise. The main function of SVD in this context is to break the image matrix into its simpler components. Mathematically, SVD can be expressed as:

$$A = U\Sigma\, V^T \tag{4}$$

where **U** is the orthogonal matrix representing the *right-singular* vectors, Σ is the diagonal matrix containing the singular values of matrix **A**, and **V** is the orthogonal matrix representing the *left-singular* vectors. Furthermore, Jensen-Shannon Divergence (JSD) is used to measure the similarity between the probability distributions before and after noise reduction. JSD can be expressed as:

$$\begin{aligned} JS(p,q) &= \frac{1}{2}KL\left(p,\frac{1}{2}(p,q)\right) + \frac{1}{2}KL\left(q,\frac{1}{2}(p,q)\right) \\ &= \frac{1}{2}\sum_{i=1}^{n} p_i log\frac{p_i}{\frac{1}{2}(p_i+q_i)} + \frac{1}{2}KL\left(q,\frac{1}{2}(p,q)\right) \end{aligned} \tag{5}$$

where, p and q are two non-negative vectors.

The technology combining SVD and JSD to reduce noise (CCTA) images has significant implications for military defense, particularly in the area of soldier health. The enhanced clarity of CCTA images can aid in a more accurate diagnosis of combat injuries, enable faster and more effective treatment, and improve soldier survival rates. The outcomes of noise reduction with k = 66 are displayed in Figure 5. Normalized scaling is used to display the density profiles, with the greatest value at 1 and the minimum value at 0. Additionally, clearer CCTA images can assist battlefield medics in making faster and more accurate assessments, allowing them to prioritize treatment and allocate resources effectively. In remote deployments, where access to specialists is limited, noise-free CCTA images can assist in telemedicine consultations, enabling remote physicians to make more accurate diagnoses and guide treatment strategies. Moreover, this technology can aid in long-term health monitoring of soldiers, detecting early signs of health issues related to combat exposure, such as cardiovascular problems or lung damage, allowing for timely intervention and preventative measures. By improving the accuracy and clarity of CCTA images, this technology can significantly enhance the medical care provided to soldiers, both in the field and during follow-up care. This ultimately contributes to improved soldier health, readiness, and overall defense capabilities.

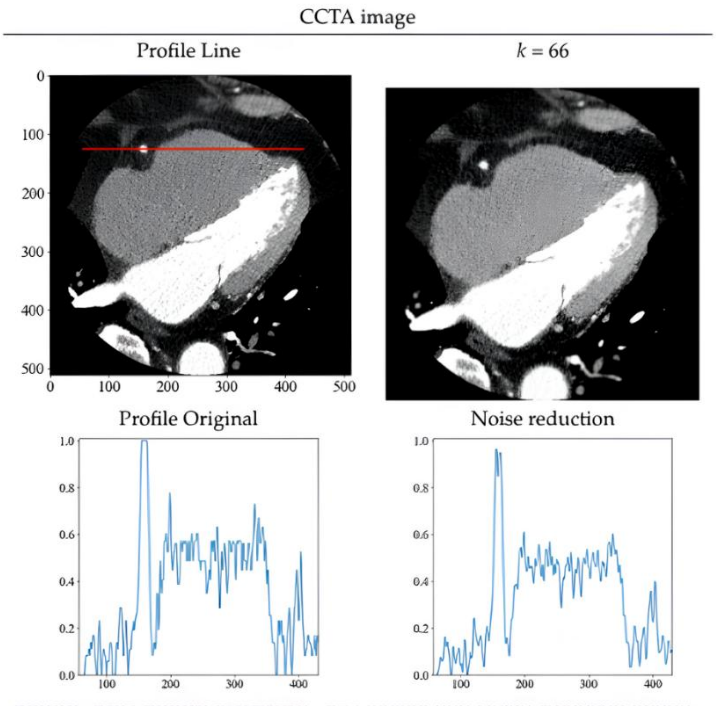

Figure 5. Original and Noise-Reduced Image, and Their Respective Density Profiles Before and After by Kasai & Otsuka, 2023

Each noise reduction technique offers a unique balance between efficiency, image clarity, and error rate. The hybrid deep learning technique (Vimala et al., 2023) is particularly effective in removing local speckle noise from ultrasound images while preserving edge information through the use of logarithmic and exponential transforms alongside edge-sensitive filter algorithms, maintaining high accuracy and sharpness without sacrificing image detail. On the other hand, the GAN-based approach (Khmag, 2023) focuses on reducing noise in specific image components while enhancing finer details, delivering better visual quality and performance, as evidenced by improvements in PSNR and reduced noise distortion. Additionally, the trimmed median filter (Jana et al., 2023)is successful in reducing salt-and-pepper noise by trimming extreme values and calculating the median, leading to enhanced Peak Signal-to-Noise Ratio and lower error rates in grayscale images. Furthermore, methods like SVD and low-rank sparse matrix factorization excel in removing specific noise types such as blur or compression artifacts. SVD, by decomposing the image matrix into singular values and vectors, allows for noise reduction by setting small singular values to zero (He et al., 2023). Low-rank sparse matrix factorization, particularly with the Robust Principal Component Analysis (RPCA) approach, decomposes the image into a low-rank matrix representing the original image and a sparse matrix representing the noise, effectively separating the noise from the original signal (Kasai & Otsuka, 2023). Overall, these advanced techniques such as GANs, trimmed median filters, hybrid deep learning, SVD, and low-rank sparse matrix factorization prove to be superior to traditional filtering methods by offering enhanced noise reduction, image enhancement, and error minimization.

Bibliometric Study

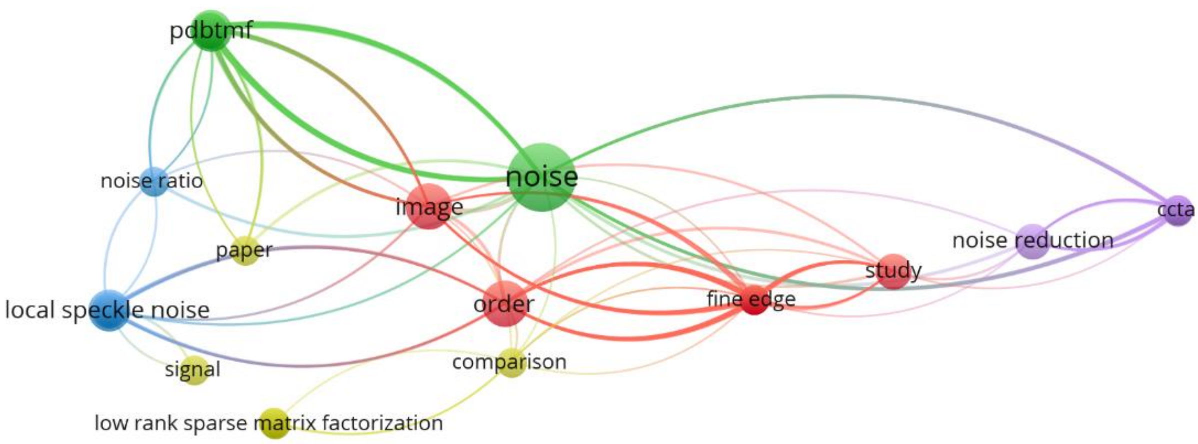

Figure 6. Bibliometric Analysis of Noise Removal Methods Visualized Using VOSviewer

The bibliometric study presented in Figure 6 visualizes the research landscape surrounding noise removal methods using VOSviewer. Each node (circle) in the image represents a keyword or term frequently appearing in studies related to noise removal. The size of the nodes indicates the frequency of the keywords, with larger nodes representing higher occurrences. For example, the term "noise" is the largest and central node, highlighting its significance as the core topic in this research area. Other notable keywords include "local speckle noise", "noise reduction", "image", "order", and "pdbtmf", each representing relevant subtopics or methods. The colors of the nodes reflect clusters formed by VOSviewer, where nodes within the same color group are closely related. The green cluster, for instance, centers around noise and technical approaches like pdbtmf. The blue cluster focuses on specific types of noise, such as local speckle noise and noise ratio. The red cluster addresses topics like order, fine edge, and study, while the purple cluster highlights noise reduction and its applications in CCTA (Computed Tomography Angiography). The yellow cluster includes terms like signal, comparison, and low rank sparse matrix factorization, indicating a focus on signal processing and comparison-based studies. The edges (lines) connecting the nodes signify the relationships or co-occurrence of keywords, with thicker lines denoting stronger connections. The term "noise" has multiple strong connections to keywords like image, noise reduction, and pdbtmf, emphasizing its central role in this research field. Links between clusters, represented by lines crossing different colors, indicate interdisciplinary studies or connections between various research themes. Overall, Figure 6 reveals key themes in noise removal research. The central focus is on addressing noise, particularly through methods like matrix factorization and pdbtmf, while specific noise types such as local speckle noise and applications in medical imaging, such as CCTA, are also prominent. Additionally, terms like comparison and study suggest a significant portion of the research involves evaluating and comparing different noise removal techniques. This visualization effectively captures the interconnectedness of methods, applications, and topics within the field of noise reduction.

CONCLUSION

Matrix decomposition techniques like Singular Value Decomposition (SVD), Principal Component Analysis (PCA), and Non-Negative Matrix Factorization (NMF) are widely used for image noise removal, each offering different strengths in separating noise from key image features. Non-linear methods, such as those based on Generative Adversarial Networks (GANs) and hybrid deep learning, outperform traditional linear approaches by preserving edge information and reducing noise more effectively, especially in high-noise environments. Advanced techniques like the trimmed median filter and low-rank sparse matrix factorization provide more accurate noise reduction with improved image clarity. The latest developments indicate a trend towards hybrid and non-linear methods, which offer enhanced performance and reduced error rates while maintaining essential image details.

By these results, recommendations for future research focused on developing more robust and efficient noise reduction techniques, which are essential for real-time applications. By refining these methods, the aim is to further improve operational effectiveness, particularly in fields like defense where clarity and accuracy in image data are critical to mission success.

AUTHOR CONTRIBUTIONS

Each author of this article played an important role in the process of method conceptualization, simulation, and article writing.

CONFLICT OF INTEREST

The authors declare that they have no conflicts of interest.

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