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The conceptual model to improve failure risk management water distribution system using ordinary differential equation model to support water resilience in military residential facilities

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Abstract

Water resilience is still big problem in Indonesia. In border and underdeveloped areas in Indonesia, the use of water sources is still considered not resilience. Especially in military context, where water needs are bigger and also more fundamental, this water resilience problem demanding a comprehensive solution. To address this issue, this research proposes the use of ordinary differential equations as a mathematical tool to model the dynamics of system damage over time, consumption, maintenance scheme, water crisis scheme, and other factors affecting water distribution resilience in military facilities. This journal presents a conceptual model of failure risk management water distribution system using a differential equation model approach to support water resilience. Specifically, the derivation of failure equation in the "reliability and maintenance system technical" textbook will be the basic reference for generating mathematical model. It is used because our model will be focused in improving failure risk management. By using the model, there are a lot of problem will be tackled such as Identify and manage failure risks in the water supply system, design an efficient water distribution maintenance scheme, and predict how strong the system to face water crisis. But before the model applied, the prediction of model will be tested by applying it in form of computer program. The case study of this research will be focused in testing the model in form of computer program with some simplicity and assumption. Through this approach, it is expected to find solutions that improve water usage efficiency, support the well-being of military personnel, and contribute to national water resilience to bolster national defense especially in case of water crisis happened. This research holds significant benefits for scientific advancement by providing a conceptual model that can serve as a reference for future research. It has the potential to make a tangible contribution but also still need so much development especially for application in real data, adding others variables that can be included for next research, conducting the interpretation, and better defining the measurement boundaries.

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INTRODUCTION

Background

Water is a fundamental resource in the daily lives of humans (Wardani, 2021). Essentially, water is the supporting element for all human activities, from eating, learning, to exercising (Kamaruddin, 2022). In developing countries like Indonesia, each individual requires at least 90 to 140 liters of water per day. As time passes and the population continues to grow, the demand for water supply also increases (Asuma, 2021). In general, Indonesia, as a vast and diverse country, often faces challenges in efficiently and sustainably managing its water resources (Arto, 2021). This is evident in the continued difficulty in accessing water in some border and underdeveloped areas or often known as 3T regions (Disadvantaged, Frontier and Outermost Areas) (Lestari, 2021). In the context of the living conditions of soldiers, who have higher water needs than the general population (Koeriyanto, 2023), the water resilience index in military facilities becomes crucial to enhance. This plays a pivotal role in the sustainability of life within military facilities. In this context, water resilience refers to the ability of a region or community to sustainably meet the clean water needs, including aspects of availability, accessibility, quality, and sustainable water management (Jannah, 2020). This involves efforts to preserve water resources, support fair distribution and reliable services to all residents, and address challenges such as drought, water pollution, and climate change that can affect water availability and accessibility (Sulistyo, 2022). Particularly in the case of military facilities located in border and underdeveloped areas (3T regions) of Indonesia, where water needs are still unmet, ensuring sufficient water resilience in these locations is crucial for the well-being and health of the soldiers residing there. Furthermore, guaranteed water resilience in every military facility ultimately plays a strategic role in supporting national defense. In this regard, one way to improve water resilience in an area is through improving water management system (Bagusrama, 2023). Therefore, a comprehensive approach is needed in modeling failure risk of water distribution systems to enhance water management system in military facilities. This approach should integrate principles of sustainability, efficiency, and wise water management.

Literature Review

Generally, the analysis of water system failures is divided into three categories: long-term, medium-term, and short-term (Oladipupo, 2019). The Distribution Process Failure Analysis (DPFA) referred in this study as one of the water distribution system analyses that falls under the purview of medium-term and short-term water distribution analyses. Figure 1 shows the management problems in Water Distribution Network (WDNs).

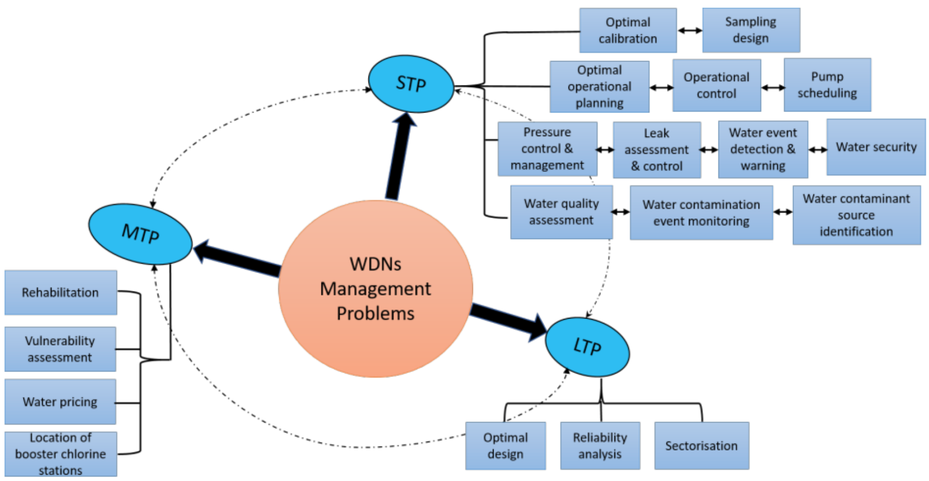

Figure 1. Classifications of management problems in WDNs

Within it, the analysis is constrained to the failure analysis of pump systems, pipe failures, extreme weather conditions, and distribution leaks. Generally, DPFA is divided into two types: failures that only reduce distribution efficiency and, subsequently, failures that cause distribution interruptions. The difference in types of failure analysis is determined based on the proposed model solutions. For the first type of failure, the proposed solution involves initially gathering reliability data of distribution equipment (pipes, pumps, connections, etc.). Data collection occurs through scheduled and periodic monitoring management and historical data from technicians. After data collection, the reliability model for each component will be modeled using the Weibull distribution model (Charles, 2003). This model fundamentally analyzes the deterioration process with non-constant failure rates over time (Charles, 2003). This means that by adapting to the collected historical data and periodic data points, the rate of deterioration over time most suitable for predicting the probability of failure will be selected for application to a component. Subsequently, with knowledge of these probability distribution values, they will be considered in establishing a maintenance schedule for the water distribution system itself.

Start from the review of DPFA, it is known that data collection plays essential role in modelling failure in water distribution system. But to avoid data collection difficulty it is interesting to develop the DPFA concept by coupling it with concept of differential equation in reliability and maintenance engineering. The concept of differential equations is utilized to model the impact of water distribution process failures on the distribution of water over time. This model will also be evaluated considering the reliability and the failure as probability (Charles, 2003). This is the reason that differential equation approach becomes important part to model failure function overtime. From this model, the risk of system failure will be predicted with some probability density function and from that prediction efficient maintenance plan and water stockpile determined by simulation just with assumption. Finally, a thorough analysis will be conducted on the simulation results to serve as considerations for developing standards of maintenance and standard operating procedures for the distribution components of the water distribution system to prevent failures and enhance water usage efficiency by consumers. Then, a model depicting the system's ability to withstand extreme conditions will be devised. This model signifies the differential equation model that outlines the distribution of water usage by personnel compared to the available water when extreme conditions occur. The results from this model are to simulate these extreme conditions so that incorrect decisions can be prevented in case of such disasters.

In connection with the above, this research suggests a differential equation approach, which is a powerful mathematical method for modeling system changes over time (Aisyiyah, 2023). In this context, differential equations can be used to model dynamics of system damage over time, consumption, and other factors affecting resilience of water distribution system (Ikhtiar, 2023). By understanding the dynamics of failure water distribution system in military facilities, areas where water resilience can be improved, leaks reduced, and water distribution ready to face water crisis. It's also important to note that this approach can provide predictions and risks for the modeled irrigation system under extreme conditions (Nadwah, 2019). Ultimately, risk management in military housing can contribute to sustainable water resource management at the national level. With this differential equation approach, water distribution system problem solving by a novel approach has been developed. This research recognizes the vital role of modelling in ensuring the water resilience of military housing. And it extends this concept by developing conceptual model and predict the failure of distribution process from beginning. Tabel 1 shows several related existing researches.

Table 1. Related journals

No.	Title	Result	Similiarity
1.	Application of Finite Difference Method in groundwater modeling groundwater modeling in mining cases: literature review (Trijayanti, 2022)	The Visual Modflow application uses the Finite Difference method to design hydrological models, including model conceptualization, constraints, and calibration, which can be applied in hydrogeological contexts, such as in mines, for groundwater management and protection.	Models and Issues
2.	Socialization of Internet of Things (IoT) on Faucets Water as an Effort to Minimize Interaction Personal Hygiene (Parawangsa, 2022)	Socialization of the presence of IoT-based water faucets with body sensors and automatic control is very important in reducing interactions and preventing the spread of Covid-19, especially in public places such as schools to improve personal hygiene.	Data and Issues
3.	Thermal Physical System Analysis Study with Stirrer Tank Heater Modeling Using Simulink MATLAB (Surindra, 2021)	Analyze the Thermal Stirrer Tank Heater System Model as a first-order system with transfer function for heat rate change.	Methods and Models

4.	Groundwater Modeling Using the Finite Difference Method in Pre-Coal Mining in Rawas Ilir, North Musi Rawas Regency, South Sumatra (Salma, 2022)	Assess the water layers in the study area, calibrate the model, and estimate the movement of water direction to the lowest elevation in the hilly area.	Problems and Data
5.	Application of Linear Differential Equation System to the Simulation of Water Discharge in Pipes (Efendi, 2021)	Comparing Experimental and Numerical methods in the application of a system of linear differential equations in modeling the volume and rate of change of water.	Model and Method
6.	Water Pollution Monitoring System and Flood Early Warning Based on Internet of Things-Based Water Levels (Prakoso, 2021)	Use of IoT to monitor water levels and pollution remotely with the development of connecting devices to smartphones.	Problems
7.	Internet of Things-based Smart Automatic Water Filler (SAWF) of Things (Sukarta, 2021)	Application of Internet of Things Based Technology to monitor the water filling process to prevent water wastage.	Problems

Theoretical Framework

The theoretical framework encompasses several critical components relevant to the study. First and foremost, the water distribution system plays a pivotal role in ensuring reliable and safe access to clean water for communities, including military personnel (Hariyani, 2020). This complex infrastructure involves an intricate network of pipes, pumps, storage tanks, and various equipment working in harmony to deliver quality water to end-users (Astuti, 2022). Key elements include the distribution pipelines connecting water sources to homes and businesses, and pumps that facilitate the flow, particularly in challenging terrains (Eko, 2022). Storage tanks act as backup reserves to ensure a stable water supply, especially during periods of high demand (Wicaksono, 2023). Regular water quality monitoring, routine maintenance, and emergency planning are essential for sustaining the system's reliability and public health (Juwono, 2022). Safety and sustainability are paramount, particularly for military communities with higher water requirements. Thus, water distribution systems are adapted to cater to the unique needs of military personnel, who often necessitate greater water access than the general population.

For that, the military personnel's housing facilities need a good water distribution system for smooth training and mission execution (Nugroho, 2019). In this case, military personnel's housing facilities varies significantly based on rank, duties, and responsibilities (Syarifudin, 2023). These accommodations can range from communal dormitories to more comfortable military dorms with private rooms and enhanced amenities (Martani, 2020). Family-housing units are provided for those with dependents. Military bases encompass a wide array of facilities, including living quarters, headquarters, airfields, naval ports, and army bases (Faqih, 2023). The type of housing allocated depends on factors such as rank and family status, and it plays a crucial role in ensuring the well-being and operational effectiveness of military units (Siswanto, 2021). And also, different type of housing needs different type of water distribution scheme.

The study will involve risk management analysis using differential equation model approach (Satria, 2021). A systematic process used to find the best solutions to complex water distribution problems using available resources named dynamic optimization. Dynamic optimization, in particular, focuses on time-dependent decision-making, considering changing conditions and constraints (Nursyanti, 2023). Ordinary Differential equations are integral in dynamic optimization as they help model the system's behavior over time (Raming, 2023). These equations describe the relationship between derivatives of one or more dependent variables with respect to one or more independent variables. The order of differential equations signifies the complexity of the system, and linear differential equations represent the linear relationship between variables, while nonlinear differential equations encompass more intricate relationships (Sinaga, 2021). A deeper understanding of these equations facilitates effective problem-solving and decision-making (Rizkayanti, 2023).

As the purpose of this study is to guarantee water resilience by conceptual model, in this part definition of the water resilience and also its relation to country defense will be explained. Water resilience in this study refers to a system or community's capacity to sustainably manage, maintain, and utilize water resources (Mulyono, 2019). It encompasses efficient water management, equitable access, and the preservation of water resources to meet current needs without compromising those of future generations (Jannah, 2020). Water resilience addresses social, economic, and environmental aspects, striving to create long-term water resilience while facing challenges like climate change, increased water demand, environmental degradation, and water resource conflicts (Sutrisno, 2019). In big picture, application of this conceptual model will strengthen Country defense and makes Indonesia more ready to face the dynamic threat.

In conclusion, this theoretical framework incorporates various elements essential to this study. It spans the water distribution system's complexity, housing for military personnel, optimization methods, dynamic optimization with differential equations, stability analysis, and water resilience. These components collectively contribute to understanding the intricacies of water supply systems and their significance. But in this study, every component is seen as one whole system. Treating it this way will simplify the model but on the other hand also make the model very applicable. Because in the context of military it is necessary to get more fast and applicable solution even though the accuracy may decrease.

Contribution

From study of the theoretical framework above, this research introduces a novel perspective on modelling water distribution system by implementing the model in form computer program while considering the automatization of the system. While previous research has focused on improving method and applying different method in similar problem, this study integrates several methods into one conceptual model and implement that model so that the model could be simulate freely. The use Ordinary Differential Equation by applying some laws and theorems from maintenance and reliability engineering allows comprehensive evaluation of the water distribution system failure. By making the conceptual model, this research provide a clear understanding of water distribution system, contributing to the improvement of ensuring water resilience in military practices. This innovative approach not only enhances the water resilience in particular but also aligns with the defense country focus towards more preparedness military operations. Of course, all these modelling processes are limited by assumptions and limitations that make them relevant to real-world problems (Zhang, 2021).

In the next chapter, the method to model will be perform comprehensively step by step, and the considered variables will be explained. And after that in chapter result and discussion, the equation will be generated and the case study will be made in form of simulation. Lastly, in last chapter the conclusion of the model and some recommendation will be performed.

METHOD

Research Framework

Modelling a water distribution system using an ordinary differential equation system to manage failure risk will consist of several stages (Roziqin, 2023). The initial stage will involve identifying the right probability density function (PDF) of failure distribution in system. PDF considered to be the function to describe failure rate is because failure rate itself must meet 2 characteristics $f(t) \geq 0$ and $\int_0^\infty f(t)dt = 1$ which PDF fulfills it. After knowing choosing the right PDF with preference and parameter to be considered, the failure distribution will be known and variables those are correlated with the distribution can be defined. In this case, those variables are energy losses **(En)** due to friction in pipes (Stanislas, 2021), head loss **(Hl)** in a pipe or conduit through which a fluid is flowing (Achour, 2020), the flow regime **(Fr)** and characteristics of fluid flow within a particular system (Krishnan, 2021), elevation of a fluid **(Ev)** within a streamline (Kaushik, 2019), Pipe Design **(Pd)**, Weather **(We)**, maintenance cost **(Mc),** water consumption **(Wc)** by users affect the amount of water present in the system (Mashaly, 2020). And after that, parameters from the distribution are developed by those variables. The step-by-step framework has been developed is presented as follows:

a) Identify probability density function (PDF) of failure distribution in system by data or dummy data.
b) Test with characteristic test.
c) Define variables those are correlated.
d) Developing parameter by that particular data.
e) Governing the model.
f) Test the model.

Surely, in order to do that it is necessary to collect real data so that the development of the distribution still represent real case. But it would not done in this study because this study will be just limited in the conceptual model. In the other hand, to complete the analysis of risk management as mentioned in the first section, this study will also model the maintenance scheme **(E)** and also water crisis scheme. To model the maintenance scheme, it is important to consider the maintenance cost **(Mc)**, intensity of maintenance **(In)**, impact of maintenance **(Im)**, and also repair time **(Rt)**. Those are the main three variables are considered to control the maintenance cost. And there will be others variables affect those main variables such as technician skills **(Ts)**, tools for repair **(Tr)** will also play a role (Charles, 2003). And for water crisis scheme, the model will be focused in determining recovery capability time **(Rct)** if there where water crisis happened. Water crisis means situation where consumer don't get any water that usually they get. It could happen in several days and escalating from regular to dangerous. In this case, ability to survive without water will control by the recovery time **(Rt)**. And then, of course how big the water reserve **(Wrb)**, how good the water management **(Wn)**, how trained the consumer **(Tnc)**, how available the water reserve **(Awr)** and the technician skills **(Ts)** will affect the ability to survive.

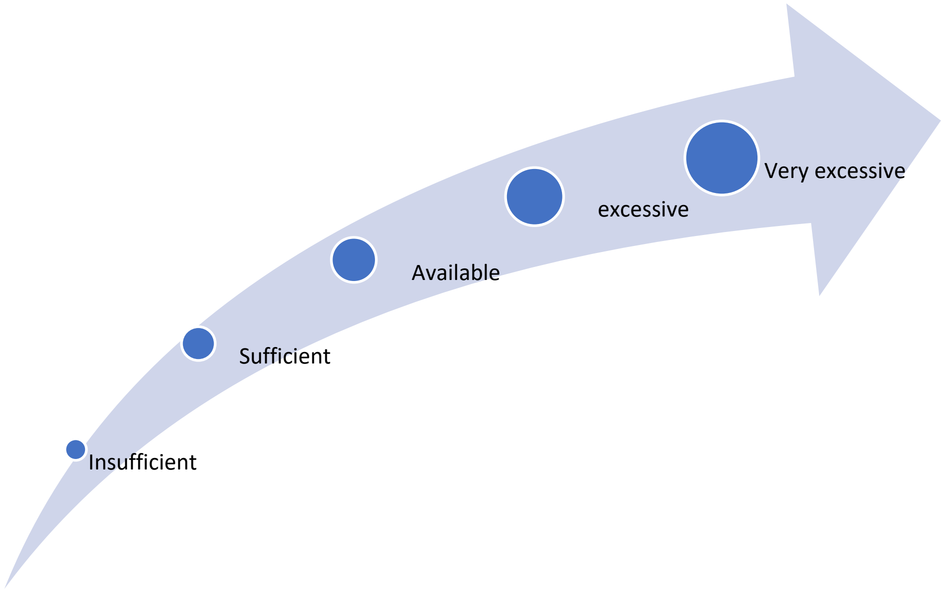

Figure 2. Water Resilience Index Scale

Through the integration of all three concepts above, a model was developed that can describes water resilience. In order to measure the water resilience, first the failure probability that integrated with maintenance will be measured how hazard it is. If the model predict that in time **(t)** the failure probability tend to occurs, water resilience index will be how hazard it is in time **(t)**. But not just that, if the failure occurs there will be another different measurement of water resiliece to measure the rediness of water distribution system to recovery and avoid collapse occur (Renn, 2022). Water resilience index value consisting of an ordinal scale of insufficient-sufficient-available-excessive-very excessive (see Figure 2).

Variables Declaration

In this case, the following variables and parameters will be considered by the literature review as shown in Table 2 and Table 3,

Table 2. Respond Variables

Variables	Description	Unit	Value
A	Water Resilience Index	System	$1 \leq A \leq 5$
B	Failure Rate	System	$0 \leq B \leq 1$
C	Maintenance Scheme	System	$1 \leq C \leq 5$
D	Recovery Capability	System	$1 \leq D \leq 5$

Equations containing these variables and parameters will first be quantified. Appropriate units will be created based on previous research for some parameters that cannot be directly measured. And then, the data will be processed using the basics of dynamical systems and mathematical modeling. In the next section, the analysis of critical situation will be explained by conceptual model Governing Equation.

Table 3. Explanatory Variables

Variables	Description
A1	Failure Rate **(B)**
A2	Maintenance Scheme **(C)**
A4	Recovery Capability **(D)**
B1	Energy Loss **(En)**
B2	Head Loss **(Hl)**
B3	Flow Regime **(Fr)**
B4	Elevation of Fluid **(Ev)**
B5	Pipe Design **(Pd)**
B6	Weather **(We)**
B7	Maintenance Cost **(Mc)**
B8	Water Consumption **(Wc)**
C1	Intensity of Maintenance **(In)**
C2	Impact of Maintenance **(Im)**
C3	Technician Skills **(Ts)**
C4	Tools for repair **(Tr)**
D1	How Big Water Reserve **(Wrb)**
D2	How well Water Management **(Wn)**
D3	How trained the consumer **(Tnc)**
D4	How Available the Water Reserve **(Awr)**

Development of Conceptual Model Governing Ordinary Differential Equation

This section will show how the governing ordinary equation model developed; first step of development will be defining the three main variables in term of Water Resilience Index (WRI) as the WRI is the basis measurement in this research.

$$\boldsymbol{WRI}(t) = ((\boldsymbol{WRI1}(t) + \boldsymbol{WRI2}(t) + \boldsymbol{WRI3}(t) + \ \dots\ + \boldsymbol{WRI} - \boldsymbol{n}(t))/\mathrm{n}) \tag{1}$$

For WRI-1 it used conditional function such as, *(*upper bound determined by preference)*

$$\boldsymbol{WRI1}(t) = \begin{cases} 1\,, & \dfrac{\boldsymbol{B}\ (\mathrm{t})}{\left(1 - \int_0^t \boldsymbol{B}(t)dt\right)} \geq \left(upper\ bound\ \times \dfrac{5}{5}\right) \\ 2\,, & \dfrac{\boldsymbol{B}\ (\mathrm{t})}{\left(1 - \int_0^t \boldsymbol{B}(t)dt\right)} \geq \left(upper\ bound\ \times \dfrac{4}{5}\right) \\ 3\,, & \dfrac{\boldsymbol{B}\ (\mathrm{t})}{\left(1 - \int_0^t \boldsymbol{B}(t)dt\right)} \geq \left(upper\ bound\ \times \dfrac{3}{5}\right) \\ 4\,, & \dfrac{\boldsymbol{B}\ (\mathrm{t})}{\left(1 - \int_0^t \boldsymbol{B}(t)dt\right)} \geq \left(upper\ bound\ \times \dfrac{2}{5}\right) \\ 5\,, & \dfrac{\boldsymbol{B}\ (\mathrm{t})}{\left(1 - \int_0^t \boldsymbol{B}(t)dt\right)} \geq \left(upper\ bound\ \times \dfrac{1}{5}\right) \end{cases} \tag{2}$$

For WRI2 it used conditional function such as,

$$\boldsymbol{WRI2}(t) = \begin{cases} 1, & (\boldsymbol{C}(\mathrm{t}) \leq (\max(\boldsymbol{C}_n(\mathrm{t})) - \min(\boldsymbol{C}_n(\mathrm{t}))/5 \leq \left(upper\ bound \times \frac{5}{5}\right) \\ 2, & (\boldsymbol{C}(\mathrm{t}) \leq (\max(\boldsymbol{C}_n(\mathrm{t})) - \min(\boldsymbol{C}_n(\mathrm{t}))/5 \leq \left(upper\ bound \times \frac{4}{5}\right) \\ 3, & (\boldsymbol{C}(\mathrm{t}) \leq (\max(\boldsymbol{C}_n(\mathrm{t})) - \min(\boldsymbol{C}_n(\mathrm{t}))/5 \leq \left(upper\ bound \times \frac{3}{5}\right) \\ 4, & (\boldsymbol{C}(\mathrm{t}) \leq (\max(\boldsymbol{C}_n(\mathrm{t})) - \min(\boldsymbol{C}_n(\mathrm{t}))/5 \leq \left(upper\ bound \times \frac{2}{5}\right) \\ 5, & (\boldsymbol{C}(\mathrm{t}) \leq (\max(\boldsymbol{C}_n(\mathrm{t})) - \min(\boldsymbol{C}_n(\mathrm{t}))/5 \leq \left(upper\ bound \times \frac{1}{5}\right) \end{cases} \quad (3)$$

For WRI3 it used conditional function such as, *(*n is number of different ranks of Recovery Capability from the best to the worst)*

$$\boldsymbol{WRI3}(t) = \begin{cases} 1, & (\boldsymbol{D}_n(\mathrm{t}) \leq (\max(\boldsymbol{D}_n(\mathrm{t})) - \min(\boldsymbol{D}_n(\mathrm{t}))/5 \leq \left(upper\ bound \times \frac{5}{5}\right) \\ 2, & (\boldsymbol{D}_n(\mathrm{t}) \leq (\max(\boldsymbol{D}_n(\mathrm{t})) - \min(\boldsymbol{D}_n(\mathrm{t}))/5 \leq \left(upper\ bound \times \frac{4}{5}\right) \\ 3, & (\boldsymbol{D}_n(\mathrm{t}) \leq (\max(\boldsymbol{D}_n(\mathrm{t})) - \min(\boldsymbol{D}_n(\mathrm{t}))/5 \leq \left(upper\ bound \times \frac{3}{5}\right) \\ 4, & (\boldsymbol{D}_n(\mathrm{t}) \leq (\max(\boldsymbol{D}_n(\mathrm{t})) - \min(\boldsymbol{D}_n(\mathrm{t}))/5 \leq \left(upper\ bound \times \frac{2}{5}\right) \\ 5, & (\boldsymbol{D}_n(\mathrm{t}) \leq (\max(\boldsymbol{D}_n(\mathrm{t})) - \min(\boldsymbol{D}_n(\mathrm{t}))/5 \leq \left(upper\ bound \times \frac{1}{5}\right) \end{cases} \quad (4)$$

where $B(t)$ =Failure Rate, C =Maintenance Scheme, and D =Recovery Capability.

All the term is using conditional function is because by that the value can be normalized and pair to pair each other. And after those equation claimed, next the three main variables are derived one by one. For failure rate function $B(t)$ it is actually free to choose what distribution used as long as the distribution is probability density function. But every distribution has its own characteristic that uniquely arise. The reason of choosing the distribution will be very affected the result. In this cas for conceptual model it is Weibull Distribution used for giving example.

$$\boldsymbol{B}(t) = \frac{\beta}{\theta} * \left(\frac{t}{\theta}\right)^{\beta-1} * e^{-\left(\frac{t}{\theta}\right)^{\beta}};\ \boldsymbol{B}(t) \text{ is Weibull Distribution} \quad (5)$$

So, parameter μ and σ will affected by $(A1, A2, A3, A4)$

$$\boldsymbol{\beta} = \boldsymbol{\beta}(B1, B2, B3, B4, B5, B6, B7, B8)$$
$$\boldsymbol{\theta} = \boldsymbol{\theta}(B1, B2, B3, B4, B5, B6, B7, B8)$$

The function will be adjusted with the characteristic of each parameter. For example, parameter $\boldsymbol{\beta}$ in Weibull distribution has characteristic that effect the shape of distribution. For $\boldsymbol{\beta} < 1$, the PDF similar to exponential distribution, for large value $\boldsymbol{\beta}$ the shape will be similar to normal distribution, and for $\boldsymbol{1} < \boldsymbol{\beta} < \boldsymbol{3}$ the PDF will be skewed (Charles, 2003). It is more likely to explanatory variables *B6 Pipe Design, B7 Weather, B8 Maintenance Cost* effect $\boldsymbol{\beta}$ because those variables are base setting to system distribution that must be affected $\boldsymbol{\beta}$. Different parameter needs different treatment and also different variables. There must be another research for that by collecting real data first and plot it so that the interpolation of the plot can lead to the right model. This research limited just in testing the model with case study in form of computer program but not collecting real data and analyzing the variables for real case.

For determining Maintenance Scheme $\boldsymbol{C}(t)$, the discussion will be very related to the failure distribution $\boldsymbol{B}(t)$. It is clearly like that because it is assumed that how the failure distributes affected how people will determine maintenance scheme to tackle it risk. so $(\boldsymbol{C}(t))$ can be expressed like this.

$$\boldsymbol{C}(t) = \boldsymbol{C}(t; \boldsymbol{C}1, \boldsymbol{C}2, \boldsymbol{C}3, \boldsymbol{C}4, \dots, \boldsymbol{Cn})$$

where n is a category of different rank of maintenance scheme

It means for different time it will be different Maintenance scheme used. For the maintenance scheme, the model will be simple, because all the parameters are depended on user preference. So, the model for that is kind of WRI model like this:

$$\mathbf{C}\,(\mathrm{t}) = \begin{cases} \boldsymbol{C_1}(t)\,, & (n = 1, \quad 0 \le t < t_1) \\ \boldsymbol{C_2}(t)\,, & (n = 2, \quad t_1 \le t < t_2) \\ \boldsymbol{C_3}(t)\,, & (n = 3, \quad t_2 \le t < t_3) \\ \boldsymbol{C_4}(t)\,, & (n = 4, \quad t_3 \le t < t_4) \\ & \dots \\ \boldsymbol{C_n}(t)\,, & (n = n, \quad t_{n-1} \le t < t_n) \\ \boldsymbol{C_{n+1}}(t)\,, & (n = n+1, t_{n-1} \le t < t_{n+1}) \end{cases} \tag{6}$$

For C1 intensity of maintenance it is used conditional function for normalizing data. It means because of this function $(\sum_{t=t0}^{t}(1 - \int_0^t \boldsymbol{B}(t)dt)/t)$ value of $\boldsymbol{C_1}(t)$ will be between one and zero. It is the mean reliability value between repairment used to quantify $\boldsymbol{C_1}(t)$ intensity of maintenance.

$$\boldsymbol{C_1}(t) = \begin{cases} 1\,, & \left(\left(\sum_{t=t0}^{t}(1 - \int_0^t \boldsymbol{B}(t)dt)/t\right) \le\ 0.2\right) \\ 2\,, & \left(\left(\sum_{t=t0}^{t}(1 - \int_0^t \boldsymbol{B}(t)dt)/t\right) \le\ 0.4\right) \\ 3, & \left(\left(\frac{\sum_{t=t0}^{t}(1 - \int_0^t \boldsymbol{B}(t)dt)}{t}\right) \le\ 0.6\right) \\ 4, & \left(\left(\frac{\sum_{t=t0}^{t}(1 - \int_0^t \boldsymbol{B}(t)dt)}{t}\right) \le\ 0.8\right) \\ 5, & \left(\left(\frac{\sum_{t=t0}^{t}(1 - \int_0^t \boldsymbol{B}(t)dt)}{t}\right) \le\ 1.0\right) \end{cases} \tag{7}$$

For C2 impact of maintenance we do it the same way but we look the maintenance value after maintenance treatment. If it can restore the maintenance value higher so the maintenance scheme must be good.

$$\boldsymbol{C_2}(t) = \begin{cases} 1\,, & \left(\int_{tmaintenance+dt}^{t} 1 - \boldsymbol{B}(t)\,dt\right) \leq 0.2) \\ 2\,, & \left(\int_{tmaintenance+dt}^{t} 1 - \boldsymbol{B}(t)\,dt\right) \leq 0.4) \\ 3, & \left(\int_{tmaintenance+dt}^{t} 1 - \boldsymbol{B}(t)\,dt\right) \leq 0.6) \\ 4, & \left(\int_{tmaintenance+dt}^{t} 1 - \boldsymbol{B}(t)\,dt\right) \leq 0.8) \\ 5, & \left(\int_{tmaintenance+dt}^{t} 1 - \boldsymbol{B}(t)\,dt\right) \leq 1.0) \end{cases} \tag{8}$$

For C_3 and C_4, the model will be follows the pattern with different time interval.

Next, for recovery capability **D** the vector will be linear function because all the variables are not dynamic over time. It all are set as the initial condition as **D1** is How Big Water Reserve (Wrb), **D2** is How well Water Management (Wn), **D3 is** How trained the consumer (Tnc) and **D4** is How Available the Water Reserve (Awr). It is actually there are some variables that could change over time, but for simplicity of the model it is used like that. Because **D** is not dynamic over time, the normalization is done manually using norm of vector **D**. Next, **D** expressed like this:

$$\boldsymbol{D}(t) = \begin{cases} 1\,, & \mathbf{D}(D1, D2, D3, D4)/||\mathbf{D}|| \leq 0.2 \\ 2\,, & \mathbf{D}(D1, D2, D3, D4)/||\mathbf{D}|| \leq 0.4 \\ 3, & \mathbf{D}(D1, D2, D3, D4)/||\mathbf{D}|| \leq 0.6 \\ 4, & \mathbf{D}(D1, D2, D3, D4)/||\mathbf{D}|| \leq 0.8 \\ 5, & \mathbf{D}(D1, D2, D3, D4)/||\mathbf{D}|| \leq 1.0 \end{cases} \tag{9}$$

After the model governing model is made, ordinary differential equation of **WRI** (Water Resilience Index will be modeled a simulated in computer program. The solution of the model will be in form of numerical solution as the analytical form wrote down in the next chapter.

RESULTS AND DISCUSSION

Analytical Form of Ordinary Differential Equation Derivation

For the sake of simulation of the model with dummy data, first of all the main function WRI is derived.

$$\begin{aligned} \frac{\textbf{WRI}\ (\mathrm{t})}{dt} &= \frac{d\left(cond.function\left(\frac{(\boldsymbol{WRI1}(t)+\boldsymbol{WRI2}(t)+\boldsymbol{WRI3}(t))}{3}\right)\right)}{dt} \\ &= \frac{1}{3} * \frac{d\left(cond.function\left((\boldsymbol{WRI1}(t)+\boldsymbol{WRI2}(t)+\boldsymbol{WRI3}(t))\right)\right)}{dt} \\ &= \frac{1}{3} * \left(\frac{d(cond.function(\boldsymbol{WRI1}(t))}{dt} + \frac{d(cond.function(\boldsymbol{WRI2}(t))}{dt} + \frac{d(cond.function(\boldsymbol{WRI3}(t))}{dt}\right) \end{aligned} \tag{10}$$

and this equation arise so that it needs to solve

$$\int \mathrm{d}(\textbf{WRI}\ (\mathrm{t}))\,dt = \frac{1}{3} * \int \left(\frac{d(cond.function(\boldsymbol{WRI1}(t))}{1} + \frac{d(cond.function(\boldsymbol{WRI2}(t))}{1} + \frac{d(cond.function(\boldsymbol{WRI3}(t))}{1}\right) dt \tag{11}$$

To solve that equation, we solve it in by one from WR1 until WR3. In this case the weibull distribution chosen for **WRI1**

$$\frac{d(cond.function(\mathbf{WRI1}\ (t))}{dt} = cond.\,function\left(\frac{d(\mathbf{B}(t))}{dt} * \frac{1}{d\left(\left(1-\int_0^t \boldsymbol{B}(t)dt\right)\right)}\right), \text{ for weibull PDF}$$

$$= \frac{d\left(\frac{\beta}{\theta}*\left(\frac{t}{\theta}\right)^{\beta-1}*e^{-\left(\frac{t}{\theta}\right)^{\beta}}\right)}{dt} * \frac{1}{d((1-\int_0^t \boldsymbol{B}(t)dt))}, \text{ As the parameter } \boldsymbol{\beta} \text{ and } \boldsymbol{\theta} \text{ is constant, it can be derived}$$

$$= \frac{d\left(C1*t^{C2}*e^{-((C3t)^{C2+1})}\right)}{dt} * \frac{1}{d((1-\int_0^t \boldsymbol{B}(t)dt))}$$

$$= \frac{C1*d\left(t^{C2}*e^{-((C3t)^{C2+1})}\right)}{dt} * \frac{1}{((1-\int_0^t \boldsymbol{B}(t)dt))}$$

$$= \frac{C1*d\left(t^{C2}*e^{-((C3)^{C2+1}(t)^{C2+1})}\right)}{dt} * \frac{1}{((1-\int_0^t \boldsymbol{B}(t)dt))}$$

$$= \frac{C1*d\left(t^{C2}*e^{-(C3)^{C2+1}(t)^{C2+1})}\right)}{dt} * \frac{1}{((1-\int_0^t \boldsymbol{B}(t)dt))}$$

$$= \frac{C1*d\left(t^{C2}*e^{-(C4*(t)^{C2+1})}\right)}{dt} * \frac{1}{((1-\int_0^t \boldsymbol{B}(t)dt))}$$

$$d(\mathbf{WRI1}\ (t)) = \left(\frac{C1*d\left(t^{C2}*e^{-\left(C4*(t)^{C2+1}\right)}\right)}{dt} * \frac{dt}{\left((1-\int_0^t \boldsymbol{B}(t)dt)\right)}\right)$$

$$\left(\frac{1}{d(\mathbf{WRI1}\ (t))}\right) = \left(\frac{dt}{C1*d\left(t^{C2}*e^{-(C4*(t)^{C2+1})}\right)} * \frac{\left((1-(\int_0^t C1*d\left(t^{C2}*e^{-\left(C4*(t)^{C2+1}\right)}\right)dt)\right)}{dt}\right)$$

And then we got this equation,

$$\int d(\text{cond. function}(\mathbf{WRI1}\ (t)) = \int \frac{\left((1-(\int_0^t C1*d\left(t^{C2}*e^{-\left(C4*(t)^{C2+1}\right)}\right)\right)}{C1*d\left(t^{C2}*e^{-(C4*(t)^{C2+1})}\right)dt} \tag{12}$$

And that equation will normalize by the conditional function from previous chapter. After that **WRI2** solved with similar way,

$$\frac{d(\mathbf{WRI2}\ (t))}{dt} = \frac{d(\mathbf{C}(t))}{dt},$$

$$= \frac{d(\text{ mean}(\boldsymbol{C1}(t),\boldsymbol{C2}(t),\boldsymbol{C3}(t),\boldsymbol{C4}(t)))}{dt}, \text{ It is known that C3(t) and C4(t) are constant parameter in C2(t)}$$

$$= \frac{\left(\frac{d((\boldsymbol{C1}(t))}{dt}+c\frac{d((\boldsymbol{C2}(t)))}{dt}\right)}{2}$$

$$= \frac{d\left((\sum_{t=t0}^{t}(1-\int_0^t \boldsymbol{B}(t)dt)/t\right)}{dt} * \frac{d(\int_0^t(1-\boldsymbol{B}(t+dt))\ dt)}{dt}$$

$$d(\mathbf{WRI2}\ (t)) = \left(\frac{d\left((\sum_{t=t0}^{t}(1-\int_0^t \boldsymbol{B}(t)dt)/t\right)}{dt} * \frac{d(\int_0^t(1-\boldsymbol{B}(t+dt))\ dt)}{dt}\right)dt$$

$$d(\mathbf{WRI2}\ (t)) = \left(\frac{(\sum_{t=t0}^{t}(1-\int_0^t \boldsymbol{B}(t)dt)/t)}{t} * \frac{\int_0^t(1-\boldsymbol{B}(t+dt))\ dt)}{1}\right)dt$$

And then we got this equation,

$$\int d(\text{cond. function}(\mathbf{WRI2}\ (t)) = \int\left(\frac{(\sum_{t=t0}^{t}(1-\int_0^t \boldsymbol{B}(t)dt)/t)}{t} * \frac{(\int_0^t(1-\boldsymbol{B}(t+dt))\ dt)}{1}\right)dt \tag{13}$$

And that equation will normalize by the conditional function from previous chapter. After that **WRI3** solved with similar way,

$$\frac{d(\mathbf{WRI3}\ (t))}{dt} = \frac{d(\mathbf{D}(t))}{dt}, \text{for Weibull PDF}$$
$$\frac{d(\boldsymbol{WRI3}\ (t))}{dt} = C, \text{Because D is not Dynamic Over Time}$$
$$d(\boldsymbol{WRI3}\ (t)) = (C)dt$$
$$\int d(\mathbf{WRI3}\ (t)) = \int C\, dt$$
$$\text{cond. function}(\mathbf{WRI3}\ (t)) = (t * C)$$

And that equation will normalize by the conditional function from previous chapter. After the derivation of equation (10) obtained,

$$d(\mathbf{WRI}\ (t))\, dt = \frac{1}{3} * cond.function\left(\int \frac{\left((1-(\int_0^t C1*d\left(t^{C2}*e^{-\left(C4*(t)^{C2+1}\right)}\right)\right)}{C1*d\left(t^{C2}*e^{-(C4*(t)^{C2+1})}\right)dt} + \left(\int\left(\frac{\sum_{t=t0}^{t}(1-\int_0^t \boldsymbol{B}(t)dt)/t)}{t} * \frac{(\int_0^t(1-\boldsymbol{B}(t+dt))\, dt)}{1}\right)dt\right) + (t*C)\right)$$
$$(\mathbf{WRI}\ (t)) = \frac{1}{3} * cond.function\left(\int \frac{\left((1-(\int_0^t C1*d\left(t^{C2}*e^{-\left(C4*(t)^{C2+1}\right)}\right)\right)}{C1*d\left(t^{C2}*e^{-(C4*(t)^{C2+1})}\right)dt} + \left(\int\left(\frac{(\sum_{t=t0}^{t}(1-\int_0^t C1*d\left(t^{C2}*e^{-\left(C4*(t)^{C2+1}\right)}\right)dt)/t)}{t} * \frac{(\int_0^t(1-C1*d\left((t+dt)^{C2}*e^{-\left(C4*(t+dt)^{C2+1}\right)}\right))\, dt)}{1}\right)dt\right) + (t*C)\right) \quad (14)$$

From the generated models above, researcher decide to solve it numerically. The simulations and analyses will be conducted to visualize the intended outcomes. Next Section, will be the numerical solution by some simplicity assumption to simulate the model using jupyter notebook. The setup and the process will be explained in the next section. The numerical solver is chosen to visualize the model better and also make the model variables more flexible to tune so that the effect from each variable can be studied easier.

Numerical Setup for Solving the Governing Equation

First of all, import all the package used. There are *numpy*, *matplotlib* and *math* package which are installed before with *pip*. After that, the variables above define in term of scalar or array. If the value is constant, variable used. If its dynamic, array used. In this experiment, the variables used are, energy loss (e1), Head loss (h2), Flow Regime (f3), Elevation of Fluid (e4), pipe Design (p5), Weather (w6), maintenance Cost (m7), Water Consumption (w8), Recovery Skill (D), length of experiment (L), number of step (n), intensity of repairment, time repairment, beta and theta as constant of failure function. The working details of the code are presented in pseudocode format below:

Algorithm 1. The numerical simulation procedure

Start
input *package and constant*
define *variables and function*
define *Maintenance scheme*
Plot *All the function without maintenance*
Get *time to failure*
For all *Maintenance scheme:*
    Plot *All the function with maintenance*
    Calculate *min (time to failure)*
Return *best maintenance scheme with minimum time to failure:*
End

It is defined all of the function will be used in this experiment such as, failure function ($B(t)$), integration of failure function ($integrate\_B(t)$), linear function ($lf(y2, y1, x2, x1, t)$), failure function with maintenance scheme ($Bmaintenance3(t)$) and integration for failure function with maintenance ($integrate\_Bmaintenance3(t)$). And after that, all of the function will be generated and plotted by matplotlib. First, the plot of failure function will be generated as shown in Figure 3.

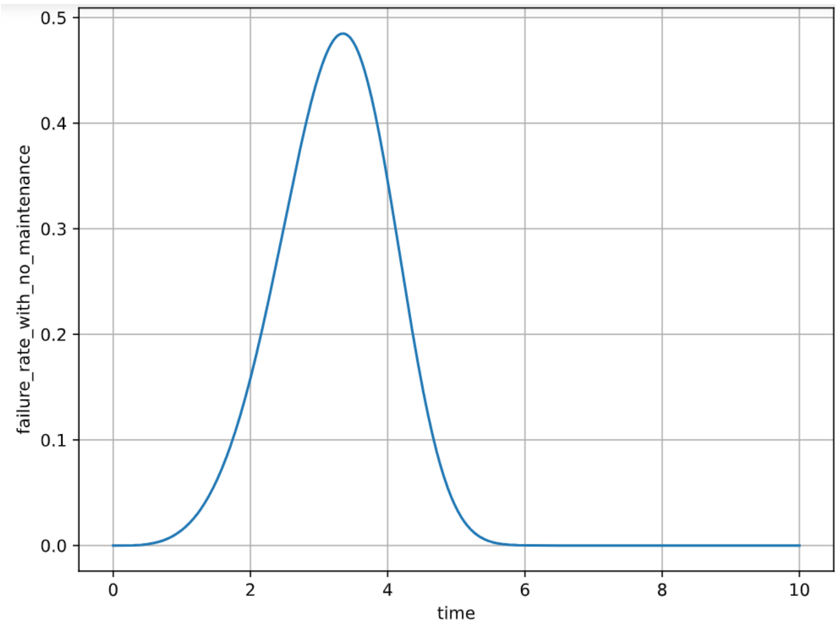

Figure 3. the plot of failure rate function (weibull)

By the constant those are chosen for this experiment, the plot of the failure rate function generated like this. It means that the Weibull distribution is similar to normal distribution that are distorted to the left. Which is means that the failure is more likely to happen in the first time of usage. But with this failure rate distribution, we got the reliability function or $(1 - integrate\_B(t))$ as show in Figure 4.

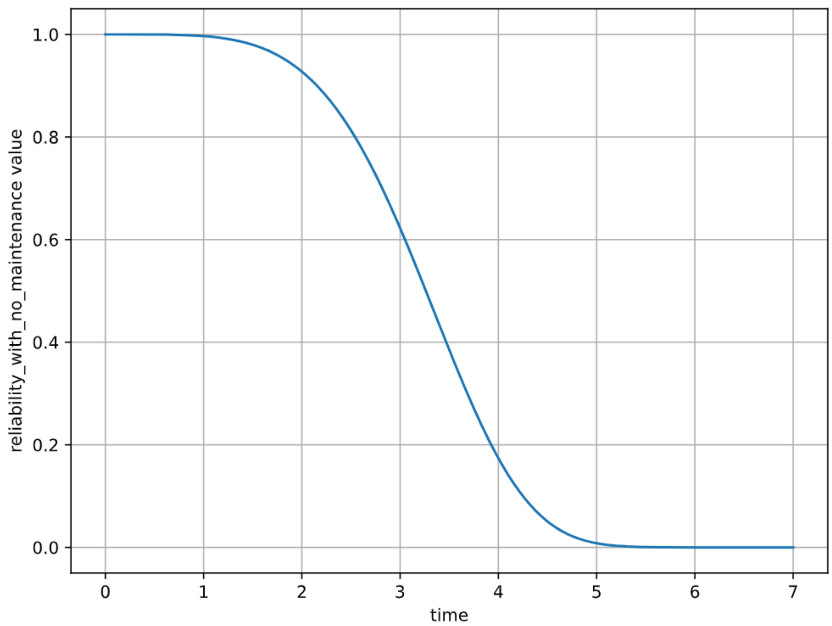

Figure 4. the plot of water distribution system reliability

It is means that the reliability of the system will be zero in time six to seven (this experiment time does not represent some unit of time). But in the next experiment with some maintenance scheme, it is got that the time to longer

Numerical Solution

From the simulation above it is known that the main problem of improving failure risk for water management system is how to make the reliability function last longer before it reaches zero or failed. In this research it will be simulated a maintenance scheme so that the maintenance function will be perform better. The maintenance scheme is by doing extreme maintenance simulation.

Extreme maintenance simulation means it used assumption that every time the system maintained, the reliability is back as good as the first time but, the failure rate also back as big as the first term. This extreme maintenance scheme used in this simulation because by that we expect that the maintenance will perform as good as we want. In this case, four different extreme maintenance schemes used to perform better reliability. And the best scheme will be plotted and explained in Figure 5.

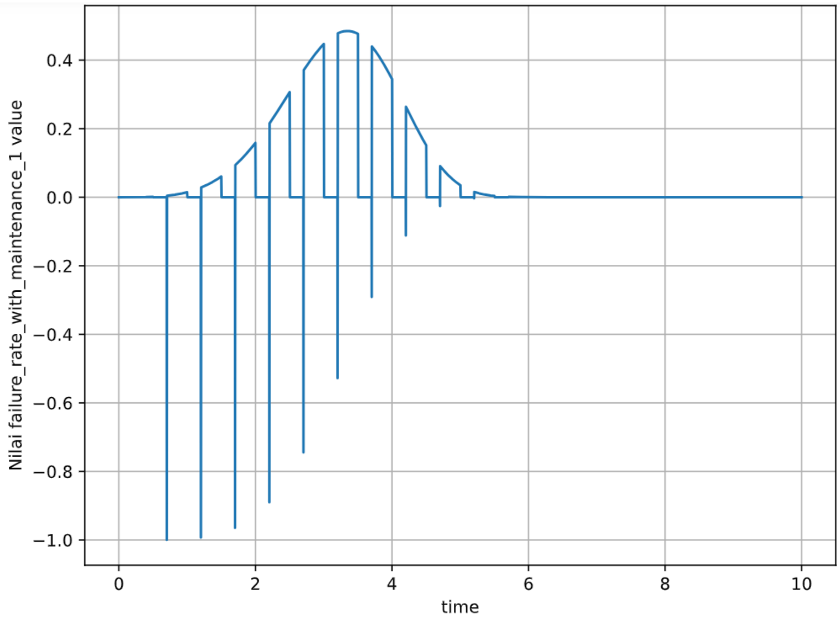

Figure 5. the plot of failure function (Weibull) by maintenance scheme 1

This failure rate plot distribution used assumption that the maintenance just occurs twelve times every 0.5 time. And in every maintenance, there are assumed maintenance interval 0.2 times long (the time units are assumption so in real case it will be easily set up). And the consequence of this failure rate distribution is good reliability function as shown in Figure 6.

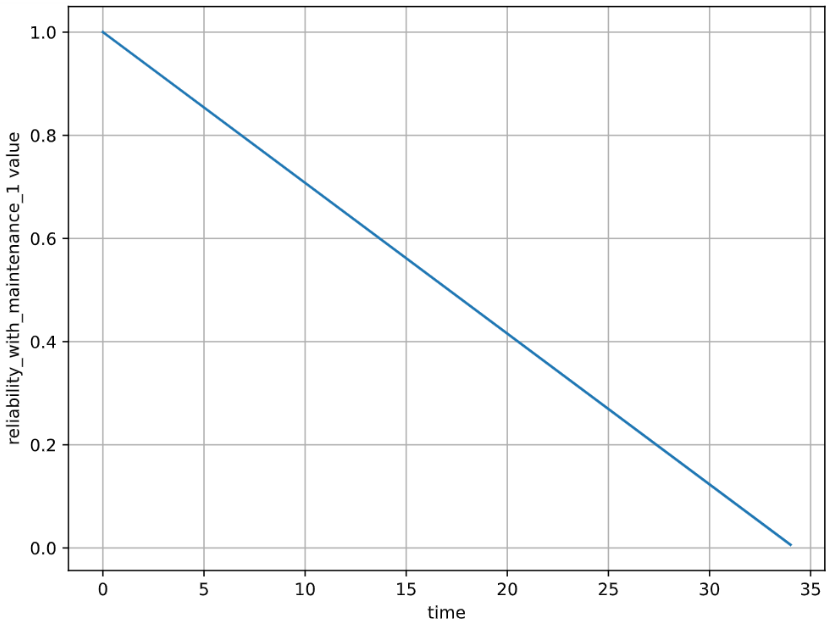

Figure 6. the plot of water distribution system reliability by maintenance scheme 1

Apparently, the reliability distribution function tends to be linear function. It is maybe occurring because of the non-continuous function made by maintenance scheme. The integral function read the value as the expected value but it still gives us right approximation of solution. As we expected, the performance of the reliability significantly better. The reliability of system turns to zero or failed in this case around 33.510 which is almost five times longer that the basic case. It means that by this numerical modelling and right assumption of variables and testing the model with real case, it is indeed the model will be reliable enough to use to predict the better maintenance scheme so that the reliability of the system long last.

CONCLUSION

Through this study, it can be concluded that a conceptual model has been developed to improve failure risk management the water distribution system using a differential equation system approach in order to enhance water resilience in military residential facilities. The contribution of this research is by making the conceptual model using differential equation based on reliability and maintenance engineering and implement it in term of experiment. And also as the groundwork to make more accurate and applicable failure risk model. Even though it still just conceptual model, from this model it can be studied how one parameter affect the others and also it can be designed by the user how it will be as their own preference and research to continue developing failure model of distribution system by exploring more in the experiment. With the right development, this model could help improving standard policy procedures, maintenance protocols, and component standards. The actual model can only be put into practical use once field data collection and measurements have been carried out. It is anticipated that more robust evaluation methods can be devised based on this model, which can be universally applied to mitigate the risk of future failures in water distribution systems.

However, several limitations in this study should be acknowledged. Firstly, the analysis conducted in the model was based on simulated data, which may outline the framework required to create an actual model. Consequently, the results need to be cautiously interpreted until a real model is constructed. These limitations could be addressed in future research endeavors by creating and enhancing the validity and reliability of the conceptual model. Subsequent studies should focus on gathering data from a representative sample of water distribution systems to ensure more resilient and applicable outcomes for the intended user base. Lastly, clearly defining measurement boundaries will be crucial in establishing the context, interpretation, and scope of the model.

AUTHOR CONTRIBUTIONS

F.K.A.: Conceptualization, data curation, investigation, visualization, & writing – original draft. M.S.: Methodology, resources, & writing – review & editing. S.S.: Formal analysis, validation, & writing – original draft. N.I.: Methodology & formal analysis. F.F.: Visualization & writing – review & editing.

CONFLICT OF INTEREST

The authors declare that they have no conflicts of interest.

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