



## Approximation of BPS Skyrme model using modified Lagrangian Skyrmion

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### Abstract

One of the nuclear atomic models represented by Skyrme model was the Skyrme model. This model was a modified nonlinear sigma model with a Skyrme field where the classical solution use generalized sixth order terms and potential terms. The binding energy that will be studied in the Skyrme SU(2) model is to generalize the second order nonlinear sigma model terms with sixth order derivative terms. The Lagrangian will be obtained for these two terms to find the BPS (Bogomolny Prasad Sommerfield) solution for the profile function numerically. The result of numerical calculation will be used to calculate static energy and rotational energy, where the characteristic of the nucleus can be observed from these two energies. Furthermore, the value of the coupling constant in the Lagrangian Skyrme will be calculated from the static energy and rotational energy obtained previously. These values are expected to help in the application of Skyrme model for many research physics field.

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## INTRODUCTION

Generally the universe follows nonlinear phenomena. In this category, the nonlinear phenomena solutions from nonlinear equations are solutions that describe the nonlinear dynamics of the system. For solve this case, soliton is one of the nonlinear equation solution used ([Blanco-Pillado, 2009](#)).

Soliton generated in solitary waves (a wave packet or pulse) that maintain their shape and propagate at a constant speed. Soliton founded in many physics phenomena, which appeared as solutions of partial differential equations of dispersive effects and weak nonlinear effects that describe physical system. Mathematically, a soliton is a classical solution of a nonlinear partial differential equation that has finite total energy, localized in space, stable, and does not spread. The energy density distribution profile corresponds to a pulse centered in a finite spatial range ([Hadi et al., 2004](#)).

Soliton phenomenon emerged when the quantum field theory that was developing at that time became popular. Physicists and mathematicians began to study classical field equation in nonlinear form, and interpreted some solutions as strong candidates for particle theory. Some of the particles

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studied are different from previous particles, where their characteristics determined by classical equations and can be systematically treated using quantum corrections ([Atmaja & Ramadhan, 2014](#)).

One of the soliton in three dimension is a Skyrmion, which are candidates for the soliton description of the nucleus with the number of soliton identified as baryon number ([Wong, 2002](#)). The Skyrme model is a modified pion nonlinear sigma model with Skyrme field,  $U(t;x)$ , and its classical solution known only through numerical computation. Although it does not include quarks, this model can be considered an approximation of low effective energy QCD theory so that the quark color number becomes very large ([Naya & Sutcliffe, 2018](#)). This became Skyrme's motivation in developing this model.

Most of the Skyrmion are applied in electromagnetic theory. One application is the concept of magnetic fields in an electronic device ([Finocchio et al, 2016](#)). Quantum-based telecommunication devices, radar, or devices related to semiconductors makes Skyrmion one of the strong candidate concepts in the development of advanced materials ([Luo, 2021](#)).

The Skyrme model proposed by T.H.R. Skyrme does not have a BPS (Bogomolnyi-Prasad-Sommerfield) solution. However, by generalizing the Lagrangian model added a squared baryon current term  $L_6$  and a potential term  $L_0$ , the results of analytical calculations from the Skyrme model obtained a solution that is close to BPS. These results make the model can be called as BPS Skyrme Model. The BPS Skyrme model describes the existence of an energy contribution that analogous to the binding energy in the nuclear model, where the contribution of the nuclear number A in that model is analogous to the baryon number ([Hadi & Wospakrik, 2004](#)).

The Skyrmion model attempts to explain certain properties of baryons such as their masses and magnetic moments. These Skyrmions have intriguing applications in magnetism, particularly in spintronics (an area focused on utilizing electron spin for technology). In potential uses like magnetic sensors, they could be pivotal for military navigation systems. These systems would boast highly sensitive magnetic sensors due to the distinct stability and properties of Skyrmions ([Li et al, 2023](#)). Such capabilities could be essential in various navigation scenarios, particularly in environments where GPS signals might be unreliable, like marine or specific land-based navigation ([Burt, 2002](#)).

This research uses analytical calculation to find out how phenomena occur when calculating the Skyrme SU(2) model using a generalized Lagrangian, and approaching it with only two terms ( $L_2$  and  $L_6$ ). The hedgehog ansatz solution for the matrix  $U(r)$  on SU(2) is used for the calculation. From those analytical calculation, results will be obtained in the form of static energy and rotational energy. From the static energy and rotational energy obtained, it can be calculate the coupling constant  $\lambda$  in the Lagrangian.

## METHOD

The Lagrangian Skyrmion for the Skyrme SU(2) model defined by ([Marleau & Rivard, 2000](#)):

$$\mathcal{L}_{sky} = L_2 + L_4 + L_6 + L_0 = -\alpha Tr[R_\mu, R^\mu] + \beta [R_\mu, R_\nu]^2 - \frac{3}{2} \frac{\lambda^2}{16^2} Tr([R_\mu, R_\nu][R^\nu, R^\rho][R_\rho, R^\mu]) - \mu^2 V \quad (1)$$

with constant parameters  $\alpha = \frac{f_\pi^2}{16}$  and  $\beta = \frac{1}{32e^2}$ .

The BPS solution will be calculated by setting the parameter value  $\alpha = \beta = 0$ . From this condition, the Skyrmion equation only consists of two terms, namely the 6th order derivative term  $L_6$  which is represent the nonlinear sigma term and Skyrme term, and the pion mass term  $L_0$  often referred as the potential term ([Marleau, 2004](#)). Our calculation refers to common BPS solution calculation using modified Lagrangian, where only use the sigma nonlinear 2th order derivative term  $L_2$  and the 6th order derivative term  $L_6$  (only  $\beta = 0$ ). Calculation of the modified Lagrangian Skyrmion with  $L_2$  and  $L_6$  terms indicate that the  $L_6$  term has the function of stabilizing the soliton solution, where this function in the original Lagrangian indicated by the 4th order derivative Skyrme term  $L_4$ .

### Calculation of Static Skyrme Equation

Set the parameter value  $\beta = 0$  and the pion mass term  $L_0$  considered too small that it can be ignored from equation (1), it will generalize to:

$$\mathcal{L} = -\alpha \text{Tr}[R_\mu, R^\mu] - \frac{3}{2} \frac{\lambda^2}{16^2} \text{Tr}([R_\mu, R_\nu][R^\nu, R^\rho][R_\rho, R^\mu]) \quad (2)$$

Static energy equation can be calculated by deriving in the related energy-momentum density tensor:

$$T^{\chi\sigma} = -g^{\chi\sigma} \mathcal{L} - 2\alpha g^{\mu\chi} g^{\sigma\nu} \text{Tr}[R_\mu, R_\nu] - 3 \frac{\lambda^2}{16^2} (g^{\nu\chi} g^{\sigma\alpha} g^{\rho\beta} g^{\mu\gamma} + g^{\nu\alpha} g^{\rho\chi} g^{\sigma\beta} g^{\mu\gamma} + g^{\nu\alpha} g^{\rho\beta} g^{\mu\chi} g^{\sigma\gamma}) \text{Tr}[f_{\mu\nu} f_{\alpha\beta} f_{\rho\gamma}] \quad (3)$$

Where  $f_{\mu\nu} = [R_\mu, R_\nu]$ ,  $f_{\alpha\beta} = [R_\alpha, R_\beta]$ ,  $f_{\rho\gamma} = [R_\rho, R_\gamma]$ . From equation (3) obtained:

$$T^{00} = -\mathcal{L} - 2\alpha \text{Tr}[R_0^2] - 3 \frac{\lambda^2}{16^2} g^{ab} g^{cd} \text{Tr}[6f_{c0} f_{0b} f_{ad}] \quad (4)$$

Definition of  $\mathcal{L}$  changed into:

$$\mathcal{L} = -\alpha(R_0 R_0 + g^{ab} R_a R_b) - 3 \frac{\lambda^2}{16^2} (3g^{ab} g^{cd} f_{a0} f_{0d} f_{cb} + g^{ab} g^{cd} g^{ef} f_{ea} f_{bd} f_{cf}) \quad (5)$$

Explicitly the static Skyrmion equation:

$$E_{sta} = - \int d^3x \text{Tr} \left[ \alpha R_a^2 + 3 \frac{\lambda^2}{16^2} g^{ab} g^{cd} g^{ef} f_{ea} f_{bd} f_{cf} \right] \quad (6)$$

And it can define the rotation Skyrmion equation:

$$E_{rot} = - \int d^3x \text{Tr} \left[ \alpha R_a^2 + 9 \frac{\lambda^2}{16^2} g^{ab} g^{cd} f_{a0} f_{0d} f_{cb} \right] \quad (7)$$

Derive the static equation using the coordinate transformation of spherical coordinates with axial symmetry:

$$\begin{aligned} 0 = & \partial_r \left( R_r - \frac{1}{4} \left[ \frac{1}{r^2} [R_\theta, [R_r, R_\theta]] + \frac{1}{r^2 \sin^2 \theta} [R_\phi, [R_r, R_\phi]] \right] \right) \\ & + \partial_\theta \left( \frac{1}{r^2} R_\theta - \frac{1}{4} \left[ \frac{1}{r^2} [R_r, [R_\theta, R_r]] + \frac{1}{r^4 \sin^2 \theta} [R_\phi, [R_\theta, R_\phi]] \right] \right) \\ & + \partial_\phi \left( \frac{1}{r^2 \sin^2 \theta} R_\phi - \frac{1}{4} \left[ \frac{1}{r^2 \sin^2 \theta} [R_r, [R_\phi, R_r]] + \frac{1}{r^4 \sin^2 \theta} [R_\theta, [R_\phi, R_\theta]] \right] \right) \end{aligned} \quad (8)$$

Where time independent chiral current  $R_0 = U^\dagger \partial_0 U = 0$  and using same ansatz of BPS Skyrme model  $U(r)$  ([Bonenfant & Marleau, 2010](#)) obtained:

$$E_{sta} = 4\pi \int r^2 dr \left[ \frac{f_\pi^2}{8} \left( g'^2 + (n^2 + 1) \frac{\sin^2 g}{r^2} \right) + \frac{9\lambda^2}{16} n^2 g'^2 \frac{\sin^4 g}{r^4} \right] \quad (9)$$

In spherical coordinates, the hedgehog ansatz solution was used with the baryon number value  $n = 1$  (Beaudoin & Marleau, 2014). The static energy on equation (9) becomes:

$$E_{sta} = 4\pi \int r^2 dr \left[ \frac{f_\pi^2}{8} \left( g'^2 + 2 \frac{\sin^2 g}{r^2} \right) + \frac{9\lambda^2}{16} n^2 g'^2 \frac{\sin^4 g}{r^4} \right] \quad (10)$$

To find the profile function  $g(r)$ , a scale transformation conducted by eliminating the parameters contained in the equation (10) with value of  $r$  scaled in space coordinates:

$$r' \equiv \left( \frac{f_\pi \sqrt{2}}{3\lambda} \right)^{\frac{1}{2}} r \quad (11)$$

The static energy transforms into:

$$E_{sta} = \kappa \int dr \left[ r^2 g'^2 + 2 \sin^2 g + g'^2 \frac{\sin^4 g}{r^2} \right] \quad (12)$$

Where  $\kappa = \frac{4\pi f_\pi^2}{8} \left( \frac{3\lambda}{f_\pi \sqrt{2}} \right)^{\frac{1}{2}}$  and define  $\varepsilon_{sta} = \int dr \left[ r^2 g'^2 + 2 \sin^2 g + g'^2 \frac{\sin^4 g}{r^2} \right]$ . Equation (12) varied to  $g(r)$  function and condition of stationary action variation  $\delta_g E_{sta} = 0$  will obtained related Euler-Lagrange equation:

$$g'' \left( 1 + \frac{\sin^4 g}{r^4} \right) + g'^2 \sin^2 g \sin 2g + g' \frac{2}{r} \left( 1 - \frac{\sin^2 g}{r^2} \right) - \frac{\sin 2g}{r^2} = 0 \quad (13)$$

### Shooting method and Skyrme Static Energy Solution

Equation (13) is a nonlinear second order ordinary differential with static energy solution for Skyrme Lagrangian calculated numerically using shooting method. The boundary condition from equation (13) state by set finite value for static energy at  $r = 0$  and  $r = 1$  and  $g(r)$  which satisfies  $g(0) = \pi, g(\infty) = 0$ . This method uses trial and error by suppose  $g'(r) = a$  where  $a$  can be predicted from those calculation. Forth order Runge-kutta used in shooting method for calculation of these solution with iteration steps:

$$\begin{aligned} r(j+1) &= r(j) + h \\ g(j+1) &= g(j) + hg'(j) \\ k_1 &= hf(r(j), g(j), g'(j)) \\ k_2 &= hf\left(r(j) + \frac{h}{2}, g(j) + g'(j)\frac{h}{2}, g'(j) + \frac{k_1}{2}\right) \\ k_3 &= hf\left(r(j) + \frac{h}{2}, g(j) + g'(j)\frac{h}{2}, g'(j) + \frac{k_2}{2}\right) \\ k_4 &= hf(r(j) + h, g(j) + hg'(j), g'(j) + k_3) \\ g'(j+1) &= g'(j) + \frac{(k_1 + 2k_2 + 2k_3 + k_4)}{6} \end{aligned} \quad (14)$$

Equation (13) become:

$$\frac{d^2 g}{dr^2} = \frac{r^4 g'^2 \sin^2 g \sin 2g + 2r^3 g'}{\sin^4 g - r^4} \quad (15)$$

The result of these calculation shown in Figure 1.

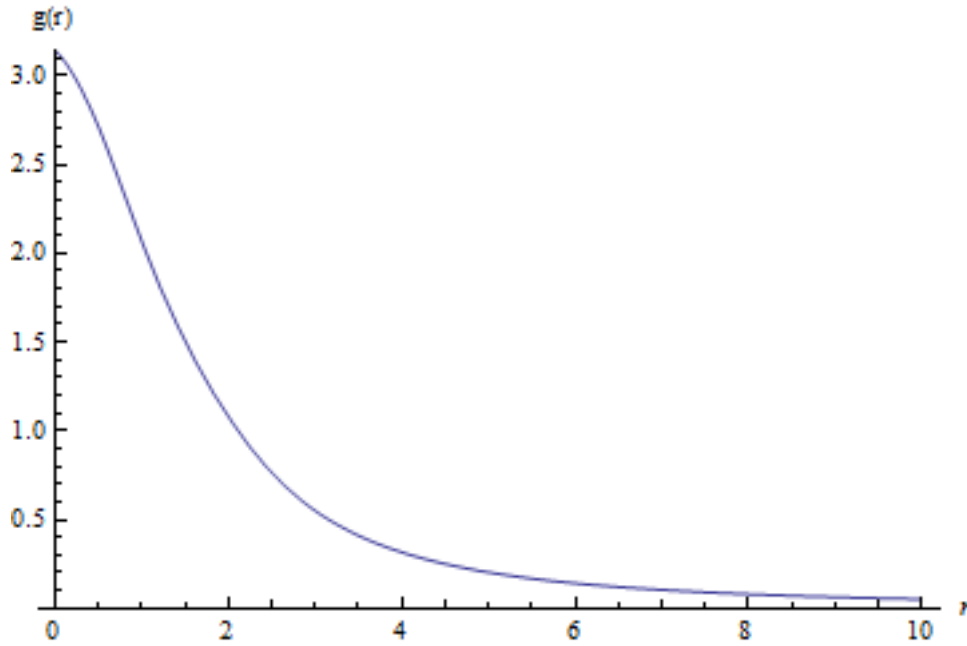


Figure 1. Profile function  $g(r)$  for equation (13)

**Calculation of Skyrmion Quantization and Quantized Rotational Energy**

Consider the group element Skyrmion quantization which involves time dependence:

$$\tilde{U}(\mathbf{r}, t) = A(t)U(D(C(t))\mathbf{r})A^\dagger(t) \tag{16}$$

With the chiral current  $R_0 = U^\dagger \partial_0 U \neq 0$ , so the matrix  $A(t)$  and  $C(t)$  are time-dependent unitary matrices (Bonenfant et al, 2012) where:

$$A(t), C(t) \in SU(2)_{internal} \tag{17}$$

$$AA^\dagger = A^\dagger A = CC^\dagger = C^\dagger C = I \tag{18}$$

Insert equation (16) to equation (2) will be given:

$$E_{rot} = \frac{1}{2}\Omega_i U_{ij}\Omega_j - \Omega_i W_{ij}\omega_j + \frac{1}{2}\Omega_i V_{ij}\omega_j \tag{19}$$

For inertia tensor  $U_{ij}, V_{ij}, W_{ij}$  defined:

$$U_{ij} = - \int d^3x \left( 2\alpha Tr[T_i T_j] + \frac{9\lambda^2}{16} Tr \left[ [T_i, R_k], [R_k, R_n], [R_n, T_j] \right] \right) \tag{20}$$

$$AV_{ij} = -\epsilon_{ikl}\epsilon_{jmn} \int d^3x x_k x_m \left( 2\alpha Tr[R_l R_n] + \frac{9\lambda^2}{16} Tr \left[ [R_l, R_p], [R_p, R_q], [R_q, R_n] \right] \right) \tag{21}$$

$$W_{ij} = \epsilon_{jkl} \int d^3x x_k \left( 2\alpha Tr[T_i R_l] + \frac{9\lambda^2}{16} Tr \left[ [T_i, R_m], [R_m, R_n], [R_n, R_l] \right] \right) \tag{22}$$

Where  $T_i = iU^\dagger [\sigma_i, U]$ . Using the hedgehog ansatz solution on each inertia tension and considering the index  $i, j = 1, 2, 3$  as Pauli matrices on  $SU(2)$  will be given:

$$U_{11} = U_{22} = \frac{4\pi}{3} \int r^2 dr \sin^2 g \left( 8\alpha + \frac{9\lambda^2}{4} \frac{(3n^2 + 1)}{4} g'^2 \frac{\sin^2 g}{r^2} \right) \tag{23}$$

$$V_{11} = V_{22} = \frac{4\pi}{3} \int r^2 dr \sin^2 g \left( 2(n^2 + 3)\alpha + \frac{9\lambda^2}{4} n^2 g'^2 \frac{\sin^2 g}{r^2} \right) \tag{24}$$

$$U_{33} = \frac{4\pi}{3} \int r^2 dr \sin^2 g \left( 8\alpha + \frac{9\lambda^2}{4} g'^2 \frac{\sin^2 g}{r^2} \right) \tag{25}$$

$$V_{33} = \frac{4\pi}{3n^2} \int r^2 dr \sin^2 g \left( 8\alpha + \frac{9\lambda^2}{4} g'^2 \frac{\sin^2 g}{r^2} \right) \tag{26}$$

$$W_{33} = \frac{4\pi}{3n} \int r^2 dr \sin^2 g \left( 8\alpha + \frac{9\lambda^2}{4} g'^2 \frac{\sin^2 g}{r^2} \right) \tag{27}$$

Equation (25), (26) and (27) can be written  $n^2 U_{33} = nW_{33} = V_{33}$  on condition  $n \geq 2$ , and  $U_{33} = W_{33} = V_{33}$  on condition  $n = 1$ . For  $n \geq 2$   $W_{11} = W_{22} = 0$ , and for  $n = 1$ ;

$$W_{11} = \frac{4\pi}{3} \int r^2 dr \sin^2 g \left( 8\alpha + \frac{9\lambda^2}{4} g'^2 \frac{\sin^2 g}{r^2} \right) \tag{28}$$

Rotational energy on equation (19) rewritten as:

$$E_{rot} = \frac{1}{2} \left( V_{11} - \frac{W_{11}^2}{U_{11}} \right) (\omega_1^2 + \omega_2^2) + \frac{1}{2} (\Omega_3 - \omega_3)^2 U_{33} + \frac{1}{2} \left[ \left( \Omega_1 - \frac{W_{11}^2}{U_{11}} \omega_1^2 \right)^2 + \left( \Omega_2 - \frac{W_{11}^2}{U_{11}} \omega_2^2 \right)^2 \right] \tag{29}$$

Calculate equation (29) by condition  $\mathbf{K}^2 = \mathbf{I}^2 = i(i + 1)$  and  $\mathbf{L}^2 = \mathbf{J}^2 = j(j + 1)$ , will be obtained:

$$E_{rot} = \frac{1}{2} \left[ \frac{j(j + 1)}{V_{11}} + \frac{i(i + 1)}{U_{11}} + \left( \frac{1}{U_{33}} - \frac{1}{U_{11}} - \frac{n^2}{V_{11}} \right) k_3^2 \right] \tag{30}$$

**Calculation of the coupling constant on hadron mass**

Skyrmion static energy equation and rotational energy equation that obtained in previous calculation will be used to find both finite energy values by considered the profile function  $g(r)$ . Based on Einstein energy-mass equation (Adam et al. 2013), by set speed of light  $c = 1$  as a natural unit, will be given:

$$m = M_{sta} + M_{rot} = E_{sta} + E_{rot} \tag{31}$$

$$m = \left( \frac{3f_\pi^2 \lambda}{\sqrt{2}} \right)^{\frac{1}{2}} \pi \varepsilon_{sta} + \frac{1}{2} \left[ \frac{j(j + 1)}{V_{11}} + \frac{i(i + 1)}{U_{11}} + \left( \frac{1}{U_{33}} - \frac{1}{U_{11}} - \frac{n^2}{V_{11}} \right) k_3^2 \right] \tag{32}$$

For find the value of coupling constant  $\lambda$ , considered  $f_\pi = 123$  MeV experimentally,  $m_{nucleon} = 938.9$  MeV and  $m_{delta} = 1232$  MeV as a values from ordinary atomic binding energy (Adam et al. 2011), for  $i$  and  $j$  using Wess-Zumino quantitation (Kopeliovich, 2006) which states Skyrmion as fermion so  $j = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \dots$  and  $k_3 = 0$ , lastly for the nucleon spin is  $\frac{1}{2}$  and for the delta spin is  $\frac{3}{2}$ . The results of those calculation can be seen in table 1.

**Table 1.** Coupling constant ( $\lambda$ ) with set value of nucleon mass and delta mass

Parameter	$\lambda_1$	$\lambda_2$
Nucleon Mass: 989.9 MeV	$7.575 \times 10^{-5}$	$8.303 \times 10^{-5}$
Delta Mass; 1232 MeV	$1.979 \times 10^{-4}$	$1.387 \times 10^{-3}$

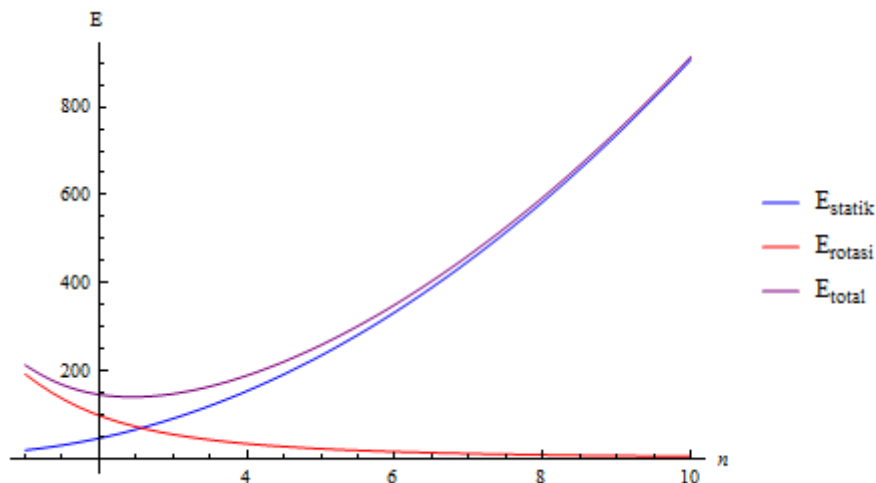
## RESULTS AND DISCUSSION

From equation (13) which produce the profile function graph in figure (1) numerically, shown that the exact value of the static energy calculation for equation (12). The baryon number used corresponds to the BPS solution, where  $E = |B|$  and  $B = |n| = 1$ . The  $n$  value is an energy solution in spherical coordinates which is called a hedgehog solution in the profile function solution to obtain the minimum energy value for the calculated static energy (Houghton & Magee, 2006). The energy yield of the profile function is  $E = 1.086$  MeV, which exceeds the BPS binding energy around 8%. Although this value is not exact as a condition for the BPS solution where  $E = 1$ , the result can be used as BPS Skyrme approximation because it has a lower error rate than the exact energy value where  $E = 1.232$  MeV.

Equation (30) is similar to the Skyrmion quantization with all the Lagrangian terms in BPS solution. Only the definition for the Skyrmion moment of inertia at  $U$ ,  $V$ , and  $W$  are different, since the Skyrme term will disappear because we have set the parameter value to be considered very small. These results indicate that the Skyrmion equation (2) shows the characteristics of rotational motion in the rigid-body nucleus. However, from these quantization yet not know the characteristics of each proton and neutron for the nuclear model because rotation and isorotation effects are described in nucleon (Fortier & Marleau, 2013).

The static energy equation (12) and rotational energy equation (30) can be graphically plotted for the two respective energies and the total energy from the sum of the two energies with the relationship of the baryon number  $n$  using the numerical solution of the profile function  $g(r)$  in Figure 1.

Figure 2 shown that static energy is directly proportional to  $n$ . The greater  $n$  then the greater the static energy, so that the binding energy of the nuclear nucleus will be greater in the case of the Skyrme model. Whereas the effect of rotational energy has characteristics that are inversely proportional to static energy. The total energy of static energy and rotational energy show that the effect of static energy is greater than the effect of rotational energy. Based on nuclear model theory and classical mechanics, rotational energy describes the movement of the rigid body in the nucleus where the static energy generated from Lagrangian Skyrme has a role in maintaining the shape of the nucleus with the minimum possible energy momenta.



**Figure 2.** Relation of static energy ( $E_{\text{statik}}$ ), rotational energy ( $E_{\text{rotasi}}$ ) and total energy from both ( $E_{\text{total}}$ ) in MeV unit to Baryon number ( $n$ )

The coupling constant parameter  $\lambda$  related to the sixth order derivative term  $L_6$ . The constant  $\lambda$  is a dimensionless constant that appeared when we stabilize the expanded Lagrangian Skyrmion equation. In a physical description,  $\lambda$  means the interaction that occurs in each nucleon for the given Lagrangian (Gudnason, 2023).

From Table 1, the best  $\lambda$  value which is close to the delta mass value for the nucleon mass set experimentally can be defined by set  $\lambda_1$  or  $\lambda_2$  to equation (32). The calculation shown that  $\lambda_2$  has small error compared to  $\lambda_1$ , because the delta mass and nucleon mass obtained has literature error

approximately 17; 9% and 27.8%. If we make a range the best value of  $\lambda$  is  $8.303 \times 10^{-5} \leq \lambda \leq 1.387 \times 10^{-3}$ . By taking the best  $\lambda$  value from this range using the mean calculation method, obtained  $\lambda_{best} = 1.108 \times 10^{-3}$ , with  $m_{nucleon} = 1075.6$  approximately error of 14.6% and  $m_{delta} = 1122.8$  approximately error of 8.8%.

## CONCLUSION

The results of static energy calculation using the Skyrme SU(2) model with Lagrangian terms  $L_2$  and  $L_6$  have a solution that is close to BPS, where the solution has an error 8% approximately. For the profile function that produces a solution close to the BPS, we can use it in calculating the rotational energy.

The rotational energy obtained has the same form as the previous calculation using all terms, with a different definition of the moment of inertia due to the loss of the  $L_4$  term. Rotational energy also describes rotational and isorotational movement on the rigid body of the nuclear nucleus in the Skyrme model, though the characteristics of the nucleons described in this model from protons and neutrons have not yet separated.

The static energy and rotational energy obtained from the modified Lagrangian are the same physical meaning as using the BPS solution as a description of the binding energy on atomic nucleus. The effect of static energy is greater than rotational energy in Skyrme model, so that can be use a representation of the rigid body system in nuclear model with those conditions.

The coupling constant value obtained from those calculation is the best value because it has the lowest error in calculating the nucleon mass and delta mass. The coupling constant value will later be compared with the coupling constant value in the Skyrme model binding energy fitting calculation.

However, calculation of binding energy studied for the Skyrme model are still not enough if considering the binding energy in the nuclear nucleus does not only consist of static energy and rotational energy, and all data fitting to obtain the model parameters has not been conducted. The effect of the calculated Coulomb energy and isospin symmetry breaking energy on the Lagrangian Skyrme model not included in this calculation. Moreover the surface tension energy in the Skyrme model has never been discussed before, so further research need to be conducted to study further binding energy in the SU(2) Skyrme model. It can be provide the information to contribute the research in some advanced physics application (such as condensed matter material or nuclear and particle physics).

## AUTHOR CONTRIBUTIONS

Conceptualization, methodology, simulation, and writing-original manuscript preparation, A.T. Oktaviana, A.R. Alfarasyi; formal analysis, A.T. Oktaviana, A.R. Alfarasyi, T.G. Huy, K. Fahmi; validation, T. G. Huy, K. Fahmi.

## CONFLICT OF INTEREST

The authors declare that they have no conflicts of interest.

## REFERENCES

- Adam, C., Sanchez-Guillen, J. & Wereszczynski, A. (2011). A BPS Skyrme model and phenomenology of nuclei. *AIP Conference Proceedings*. 1343 (2011), 598–600. <https://doi.org/10.1063/1.3575106>
- Adam, C., Naya, C., Sanchez-Guillen, J. & Wereszczynski, A. (2013). Nuclear binding energies from a Bogomol'nyi-Prasad-Sommerfield Skyrme model. *Physical Review C*. 88(5), 054313. <https://doi.org/10.1103/PhysRevC.88.054313>
- Atmaja, A. N. & Ramadhan, H. S. (2014). On Bogomol'nyi equations of classical solutions. *Physical Review D*. 90(10), 105009. <https://doi.org/10.1103/PhysRevD.90.105009>
- Beaudoin, M. O. & Marleau, L. (2014). Near-BPS Skyrme solitons: constant baryon density. *Nuclear Physics B*. 883(2014), 328-349. <https://doi.org/10.1016/j.nuclphysb.2014.03.025>

- Blanco-Pillado, J. J., Ramadhan, H. S., & Shiiki, N. (2009). Skyrme Branes. *Physical Review D*. 79(8), 085004. <https://doi.org/10.1103/PhysRevD.79.085004>
- Bonenfant, E. & Marleau, L. (2010). Nuclei as near BPS-Skyrmions. *Physical Review D*. 82(5), 054023. <https://doi.org/10.1103/PhysRevD.82.054023>
- Bonenfant, E., Harbour, L. & Marleau, L. (2012). Near-BPS Skyrmions: Nonshell configurations and Coulomb effects. *Physical Review D*. 85(11), 114045. <https://doi.org/10.1103/PhysRevD.85.114045>
- Burt, B. (2002). Storage technologies and issues for military suppliers. *19th Digital Avionics Systems Conference. Proceedings*. 1(2002), 6858852. <https://doi.org/10.1109/DASC.2000.886945>
- Finocchio, G., Buttner, F., Tomasello, R., Carpentieri, M., & Klaui, M. (2016). Magnetic Skyrmions: from fundamental to applications. *Journal of Physics D: Applied Physics*. 49(42), 423001. <https://doi.org/10.48550/arXiv.2308.11811>
- Fortier, J. & Marleau, L. (2013). The quantization of the B=1 and B=2 Skyrmions. *Physical Review D*. 77(5), 054017. <https://doi.org/10.1103/PhysRevD.77.054017>
- Gudnason, S. B. (2023). Nonlinear rigid-body quantization of Skyrmions. In arXiv: hep-th. <https://doi.org/10.48550/arXiv.2311.11667>
- Hadi, M. & Wospakrik, H.J. (2004). SU(2) Skyrme model for hadron. In arXiv: hep-ph. <https://doi.org/10.48550/arXiv.1007.0888>
- Hadi, M., Nurdin, I., & Hermawanto, D. (2004). Analytical analysis and numerical solution of two flavours Skyrme model. In arXiv: hep-ph. <https://doi.org/10.48550/arXiv.1006.5601>
- Houghton, C. & Magee, S. (2006). A Zero-mode quantization of the Skyrme model. *Physics Letters B*. 632(4), 593-596. <https://doi.org/10.1103/PhysRevD.61.114024>
- Kopeliovich, V. B., Piette, B. & Zakrzewski, W. J. (2006). Mass terms in the Skyrme model *Physical Review D*. 73(1), 014006. <https://doi.org/10.1103/PhysRevD.73.014006>
- Li, S., Wang, X., & Rasing, T. (2023). Magnetic Skyrmions: Basic properties and potential applications. *Interdisciplinary Materials*. 2(2), 260-289. <https://doi.org/10.1002/idm2.12072>
- Luo, S. & You, L. (2021). Skyrme devices for memory and logic applications. *APL Materials*. 9, 064035. <https://doi.org/10.1063/5.0042917>
- Marleau, L. & Rivard, J. F. (2000). A generating function for all-orders skyrmions. *Physical Review D*. 63(3), 036007. <https://doi.org/10.1103/PhysRevD.63.036007>
- Marleau, L. (2004). Solving Skyrmions. In arXiv: hep-ph. <https://doi.org/10.48550/arXiv.hep-ph/0403292>
- Naya, C. & Sutcliffe, P. (2018). Skyrmions and clustering in light nuclei. *Physical Review Letters*. 121(23), 232002. <https://doi.org/10.1103/PhysRevLett.121.232002>
- Wong, S.M.H. (2002). What exactly is a Skyrme model? In arXiv: hep-ph. <https://doi.org/10.48550/arXiv.hep-ph/0202250>

